Find The Critical Point Of The Function F(x,y)=2e^xxe^y C = ________ Enter Your Solution In The Format

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Understanding how to find critical points of multivariable functions is a foundational skill in calculus, especially in the field of multivariable calculus and optimization problems. In this article, we will explore the process of finding the critical point of the given function:

$$ F(x, y) = 2e^{x} \times x e^{y} $$

Our goal is to determine the critical points—points where the function's partial derivatives with respect to both variables are zero—and then interpret what these points signify in the context of the function's behavior.

Understanding the Function

Before delving into derivatives and critical points, it's essential to understand the structure of the function.

The Given Function

$$ F(x, y) = 2e^{x} \times x e^{y} $$

This can be simplified for easier differentiation:

$$ F(x, y) = 2x e^{x} e^{y} $$

Recall that:

$$ e^{a} \times e^{b} = e^{a + b} $$

Hence:

$$ F(x, y) = 2x e^{x + y} $$

This simplified form reveals that F is a product of a polynomial term $2x$ and an exponential term $e^{x + y}$.

Step 1: Find the Partial Derivatives

To locate critical points, we need to compute the partial derivatives of F with respect to x and y.

Partial derivative with respect to x (F_x)

Given:

$$F(x, y) = 2x e^{x + y}$$

Apply the product rule:

$$F_x = \frac{\partial}{\partial x} (2x e^{x + y})$$

Using the product rule:

$$F_x = 2 \times e^{x + y} + 2x \times e^{x + y}$$

because:

- Derivative of $2x$ with respect to x is 2.
- Derivative of $e^{x + y}$ with respect to x is $e^{x + y}$.

Simplify:

$$F_x = 2 e^{x + y} + 2x e^{x + y} = 2 e^{x + y} (1 + x)$$

Partial derivative with respect to y (F_y)

Similarly, differentiate F with respect to y:

$$F_y = \frac{\partial}{\partial y} (2x e^{x + y})$$

Since $2x$ is treated as a constant with respect to y:

$$F_y = 2x \times e^{x + y}$$

Step 2: Set the Partial Derivatives Equal to Zero

Critical points occur where both partial derivatives are zero:

\[
\begin{cases}

F_x = 0 \\
F_y = 0
\end{cases}
\]

Expressed explicitly:

\[
\begin{cases}
2 e^{x + y} (1 + x) = 0 \\
2x e^{x + y} = 0
\end{cases}
\]

Since \(e^{x + y} \neq 0 \) for all real \(x, y \), the exponential term doesn't contribute to zeros; the zero occurs when the factors involving \(x \) and \((1 + x) \) are zero.

Step 3: Solve the System of Equations

Let's analyze each equation separately.

From \(F_x = 0 \):

\[2 e^{x + y} (1 + x) = 0 \]

Since \(2 e^{x + y} \neq 0 \):

\[1 + x = 0 \Rightarrow x = -1 \]

From \(F_y = 0 \):

\[2x e^{x + y} = 0 \]

Again, \(e^{x + y} \neq 0 \), so:

\[2x = 0 \Rightarrow x = 0 \]

Contradiction?

Notice that the critical point conditions for \(F_x \) and \(F_y \) lead to different \(x \)-values:

- \(x = -1 \) from \(F_x = 0 \)
- \(x = 0 \) from \(F_y = 0 \)

Since both partial derivatives must simultaneously be zero at a critical point, there is no point where both conditions are satisfied unless the conditions are compatible.

But in this case, they are not: the conditions require $x = -1$ and $x = 0$, which cannot happen simultaneously.

Step 4: Conclusion — Critical Points

Because both partial derivatives cannot be zero at the same point simultaneously, the only possibility is to check whether either derivative can be zero independently at different points, but for a critical point, both must be zero together.

Alternatively, observe that:

- $F_y = 0$ when $x = 0$
- $F_x = 0$ when $x = -1$

Therefore, the function has no critical point where both derivatives vanish simultaneously.

Special Cases & Boundary Conditions

While the standard critical point analysis shows no solutions, it's insightful to analyze the behavior of the function at specific lines or points.

When $x = 0$:

- $F_x = 2 e^{0 + y} (1 + 0) = 2 e^{y} \neq 0$

unless $y = -\infty$, but this is not a finite point.

- $F_y = 2 \times 0 \times e^{0 + y} = 0$

Thus, at $x=0$, $F_y=0$, but $F_x \neq 0$. So no critical point here.

When $x = -1$:

- $F_x = 2 e^{-1 + y} (1 - 1) = 0$

- $F_y = 2 \times (-1) \times e^{-1 + y} \neq 0$ unless $y = -\infty$.

Again, partial derivatives are not both zero at the same point.

Final Summary: Critical Points of the Function

Given the above analysis, the function $F(x, y) = 2x e^{x + y}$ does not have any critical points in the finite plane where both partial derivatives vanish simultaneously.

Interpreting the Results

The absence of critical points suggests that the function does not have any local maxima, minima, or saddle points in the finite plane. Instead, its behavior is dominated by the exponential growth or decay, depending on the variables.

Additional Insights: Analyzing the Behavior of F

While no critical points exist, understanding the function's behavior involves examining its limits and possible asymptotic tendencies.

Behavior as $x \to \infty$ or $y \to \infty$:

- $F(x, y) \to \infty$ when both x and y tend to infinity.
- $F(x, y) \to 0$ when $x \to -\infty$ or $y \to -\infty$.

Behavior along specific lines:

- Along $x = 0$, $F(0, y) = 0$ for all y.
- Along $y = -x$, $F(x, -x) = 2x e^{x - x} = 2x$. This is a linear function in x.

Summary and Final Solution

Given the function $F(x, y) = 2x e^{x + y}$, the critical points are solutions where both partial derivatives are zero.

- $F_x = 2 e^{x + y} (1 + x) = 0 \Rightarrow x = -1$
- $F_y = 2x e^{x + y} = 0 \Rightarrow x = 0$

Since these conditions cannot be satisfied simultaneously, the function has no critical points in the finite plane.

Answer Format

C = None or No critical point

Alternatively, if required to enter the solution in a specific format, write:

C = None

Conclusion

In conclusion, the process of finding critical points involves computing partial derivatives, setting them equal to zero, and solving the resulting system. In this case, the analysis shows that no points satisfy both conditions simultaneously, meaning the function has no critical points. Understanding such behavior is crucial in multivariable calculus, especially when analyzing functions for extrema or saddle points.

Remember: Always interpret the derivatives carefully, consider the domain, and analyze whether the

Frequently Asked Questions

How do I find the critical points of the function F(x,y)=2e^x x e^y?

To find the critical points, compute the partial derivatives F_x and F_y, set them equal to zero, and solve the resulting system of equations.

What are the steps to determine the critical points of F(x,y)=2e^x x e^y?

First, find partial derivatives with respect to x and y, set each to zero, and then solve the equations simultaneously to find the critical points.

How do I compute the partial derivatives of F(x,y)=2e^x x e^y?

Compute F_x = 2e^x e^y (x + 1) and F_y = 2e^x x e^y. Then set these derivatives equal to zero to find critical points.

What is the significance of the critical points in the context of this function?

Critical points indicate locations where the function may have local maxima, minima, or saddle points, which are important for understanding its behavior.

How do I determine the value of C for the critical point in the given function?

After finding the critical point (x,y), substitute these values into the function F(x,y) to compute C = F(x,y). Enter your solution as C = ... in the specified format.

Can you provide an example solution for the critical point of F(x,y)=2e^x x e^y?

Yes. For example, setting F_x=0 gives x = -1, and F_y=0 for all x. At x=-1, F(-1,y)=2e^{-1} (-1) e^y. To find C, choose y arbitrarily or solve further depending on context.

Is it necessary to check second derivatives after finding the critical point?

Yes, to classify the critical point as a local maximum, minimum, or saddle point, compute the second derivatives and use the second derivative test.

What is the final step after finding the critical points and calculating C?

The final step is to enter the value of C into the specified format, e.g., "C = [value]".

How do I ensure my solution is correctly formatted as per instructions?

After calculating C, write your answer exactly as "C = [your value]" without additional symbols or explanations.

Are there multiple critical points for this function? How can I find all of them?

Yes, multiple critical points may exist. Find all solutions to the system of equations derived from setting the partial derivatives to zero to identify all critical points.

Additional Resources

Critical Point Analysis of the Function \(F(x, y) = 2e^{x} \times x e^{y} \)

Introduction

Mathematics often challenges us with functions that combine multiple exponential and polynomial expressions, demanding a precise approach to uncover their critical points. The function $F(x, y) = 2e^{x} \times x e^{y}$ exemplifies such complexity, blending exponential growth with polynomial factors. Recognizing the critical points of this function is crucial in understanding its behavior—such as identifying local maxima, minima, or saddle points—which has applications across fields like optimization, physics, and economics.

In this detailed exploration, we will dissect the process of finding the critical points of $F(x, y)$, guiding you through the calculus techniques involved, including partial derivatives, setting them to zero, and solving the resulting equations. By the end, you'll not only identify the critical point(s) but also gain insights into the function's structure and how to approach similar problems.

Understanding the Function $F(x, y)$

Before diving into derivatives, it's essential to understand the structure of $F(x, y)$:

$$
F(x, y) = 2e^{x} \times x e^{y}
$$

This can be simplified by combining common factors:

$$
F(x, y) = 2 e^{x} x e^{y} = 2 x e^{x} e^{y}
$$

This form highlights the multiplicative nature of the exponential functions and the polynomial factor x. It suggests that the critical points depend on the interplay between x and y, particularly where the derivatives with respect to each variable are zero.

Step 1: Calculating Partial Derivatives

To locate critical points, we need to compute the partial derivatives of F with respect to x and y:

$$

\frac{\partial F}{\partial x} \quad \text{and} \quad \frac{\partial F}{\partial y}
\]

Partial derivative with respect to \(x \):

Given

\[
F(x, y) = 2 x e^{x} e^{y}
\]

treat \(y \) as a constant during differentiation:

\[
\frac{\partial F}{\partial x} = 2 e^{y} \frac{\partial}{\partial x} (x e^{x})
\]

We need to differentiate \(x e^{x} \). Using the product rule:

\[
\frac{\partial}{\partial x} (x e^{x}) = e^{x} + x e^{x} = e^{x}(1 + x)
\]

Therefore,

\[
\frac{\partial F}{\partial x} = 2 e^{y} \times e^{x}(1 + x) = 2 e^{x + y} (1 + x)
\]

Partial derivative with respect to \(y \):

Treat \(x \) as a constant:

\[
\frac{\partial F}{\partial y} = 2 x e^{x} \frac{\partial}{\partial y} e^{y} = 2 x e^{x} e^{y}
\]

Step 2: Setting Partial Derivatives to Zero

Critical points occur where both partial derivatives are zero simultaneously:

\[
\frac{\partial F}{\partial x} = 0 \quad \text{and} \quad \frac{\partial F}{\partial y} = 0
\]

Solve \(\frac{\partial F}{\partial x} = 0 \):

$$
2 e^{x + y} (1 + x) = 0
$$

Since $e^{x + y} \neq 0$ for all real x, y, the only way this expression equals zero is when:

$$
1 + x = 0 \Rightarrow x = -1
$$

Solve $\frac{\partial F}{\partial y} = 0$:

$$
2 x e^{x + y} = 0
$$

Similarly, $e^{x + y} \neq 0$, so:

$$
x = 0
$$

Step 3: Analyzing the System of Equations

The two conditions derived from the derivatives are:

- From $\frac{\partial F}{\partial x}$: $x = -1$
- From $\frac{\partial F}{\partial y}$: $x = 0$

These are conflicting unless both are satisfied simultaneously, which is impossible.

Implication:

- The only critical points occur at $x = -1$ when $\frac{\partial F}{\partial x} = 0$.
- When $x = 0$, $\frac{\partial F}{\partial y} = 0$, but at $x=0$, $\frac{\partial F}{\partial x} \neq 0$ unless $x = -1$.

Conclusion:

The critical points occur where both derivatives vanish simultaneously. Since the only common solution is when both conditions are met, and these conditions are mutually exclusive, the only critical points are at the intersection where both derivatives are zero.

But from above, the only candidates are:

- For $x = -1$, check $\frac{\partial F}{\partial y}$:

$$
\frac{\partial F}{\partial y} = 2 x e^{x + y} = 2 (-1) e^{-1 + y} = -2 e^{-1 + y}
$$

Set to zero:

$$
-2 e^{-1 + y} = 0 \Rightarrow e^{-1 + y} = 0
$$

But the exponential function is never zero, so no solution here.

- For $x = 0$:

$$
\frac{\partial F}{\partial x} = 2 e^{y} (1 + 0) = 2 e^{y}
$$

Set to zero:

$$
2 e^{y} = 0 \Rightarrow e^{y} = 0
$$

Again, no solution because $e^{y} \neq 0$ for any real y.

Final observation:

Despite the derivatives not simultaneously equaling zero for any real x, y, the function's structure suggests potential boundary critical points or points where derivatives are undefined. But in this case, the derivatives are defined everywhere.

Step 4: Reassessing the Critical Point Condition

The above analysis indicates no points satisfy both equations simultaneously. But perhaps the approach needs refinement. Recall that critical points can also occur where the gradient vector is zero, which we've attempted to find via partial derivatives.

Alternative approach:

- Since the partial derivatives do not vanish simultaneously, the function might not have critical points in the traditional interior sense.
- However, critical points might occur where the partial derivatives are zero individually, indicating potential local maxima, minima, or saddle points.

Step 5: Solving for Critical Points — The Flat-Point Analysis

Re-examining the derivatives:

$$
\frac{\partial F}{\partial x} = 2 e^{x + y} (1 + x) = 0
$$

$$
\frac{\partial F}{\partial y} = 2 x e^{x + y} = 0
$$

Given $e^{x + y} \neq 0$, the solutions are:

- $1 + x = 0 \Rightarrow x = -1$
- $x = 0$

These are incompatible. Therefore, the only candidate critical points are at the intersection:

$$
x = -1, \quad x = 0
$$

which do not coincide.

Conclusion:

The critical points are located where either:

- $x = -1$ (from the first partial derivative), regardless of y, but only if $\frac{\partial F}{\partial y}$ is also zero at that point.
- Or $x = 0$ (from the second derivative), regardless of y, but only if $\frac{\partial F}{\partial x}$ is also zero at that point.

Since neither condition satisfies both derivatives simultaneously, the function F has no critical points in the interior.

Step 6: Summary and Final Solution

Given the above analysis, and considering the original problem statement:

> Find the critical point(s) of the function $F(x, y) = 2 e^{x} x e^{y}$. C = _________ Enter Your Solution In The Format

The critical points are the solutions to the system:

$$
\frac{\partial F}{\partial x} = 0 \quad \text{and} \quad \frac{\partial F}{\partial y} = 0
$$

which reduces to:

$$
2 e^{x + y} (1 + x) = 0
$$

$$
2 x e^{x + y} = 0
$$

and since \(

Find The Critical Point Of The Function F X Y 2e Xxe Y C Enter Your Solution In The Format

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