Find The Dimension Of A Rectangle Whose Width Is 6 Miles Less Than It Width And Whose Area Is 72 Square

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Introduction

Understanding how to determine the dimensions of a rectangle based on given parameters is a fundamental aspect of geometry and real-world problem solving. Whether you're a student working on homework, a professional dealing with land measurements, or simply someone interested in mathematical puzzles, grasping how to find the length and width of rectangles under various constraints is essential.

In this article, we will explore a specific problem: finding the dimensions of a rectangle whose width is 6 miles less than its length and whose area is 72 square miles. This problem combines algebraic expressions with geometric concepts, providing a comprehensive example of how to translate word problems into mathematical equations and solve them systematically.

By the end of this discussion, you'll not only learn how to approach similar problems but also gain insights into the importance of setting up proper equations, solving quadratic equations, and interpreting your solutions in a real-world context.

Context and Real-World Applications

Before diving into the problem-solving process, it’s useful to understand why such problems matter. Calculating the dimensions of a rectangle based on area and relative measurements can be applied in various fields, including:

- Land surveying and real estate: Determining property dimensions when given total area and relative measurements.
- Construction and architecture: Designing spaces where certain proportions are required.
- Agriculture: Planning plots of land with specific area and dimension constraints.
- Mathematical education: Strengthening skills in algebra and geometry through practical problems.

In these applications, understanding how to model the problem mathematically enables accurate and efficient planning.

Understanding the Problem Statement

Let's carefully analyze the problem:

> Find the dimensions of a rectangle whose width is 6 miles less than its length and whose area is 72 square miles.

Breaking it down into key points:

- The width of the rectangle is 6 miles less than its length.
- The area of the rectangle is 72 square miles.
- The goal: Determine the specific measurements of length and width.

This problem involves two variables: length and width. Our task is to find their numerical values satisfying both conditions.

Step 1: Define Variables

To formulate the problem mathematically, assign variables:

- Let L = length of the rectangle (in miles).
- Let W = width of the rectangle (in miles).

According to the problem:

> The width is 6 miles less than the length:

$$ W = L - 6 $$

> The area is 72 square miles:

$$ \text{Area} = L \times W = 72 $$

Step 2: Set Up the Equation

Using the expressions above, substitute $W = L - 6$ into the area formula:

$$ L \times (L - 6) = 72 $$

This leads to a quadratic equation:

$$ L^2 - 6L = 72 $$

Bring all terms to one side:

$$ L^2 - 6L - 72 = 0 $$

This quadratic equation will allow us to solve for L (the length).

Step 3: Solve the Quadratic Equation

The quadratic equation:

$$ L^2 - 6L - 72 = 0 $$

can be solved using the quadratic formula:

$$ L = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} $$

where:

- $a = 1$
- $b = -6$
- $c = -72$

Plugging in these values:

$$ L = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 1 \times (-72)}}{2 \times 1} $$

Simplify:

$$ L = \frac{6 \pm \sqrt{36 + 288}}{2} $$

$$ L = \frac{6 \pm \sqrt{324}}{2} $$

Since $\sqrt{324} = 18$:

$$ L = \frac{6 \pm 18}{2} $$

Now, consider both solutions:

1. $L = \frac{6 + 18}{2} = \frac{24}{2} = 12$
2. $L = \frac{6 - 18}{2} = \frac{-12}{2} = -6$

Step 4: Interpret the Solutions

A rectangle's length cannot be negative, so:

- Length $L = 12$ miles (valid)
- Length $L = -6$ miles (invalid in this context)

Now, find the corresponding width:

$$ W = L - 6 $$

For $L = 12$:

$$W = 12 - 6 = 6 \text{ miles}$$

Thus, the dimensions are:

- Length = 12 miles
- Width = 6 miles

Step 5: Verify the Solution

Check if these dimensions satisfy the area condition:

$$\text{Area} = L \times W = 12 \times 6 = 72 \text{ square miles}$$

which matches the given area. The solution is consistent.

Additional Considerations

What if the problem had different parameters?

Suppose the area was different or the difference in width and length was specified differently. The same approach—defining variables, setting up equations, solving quadratics—would apply. It’s important to:

- Clearly interpret the problem statement.
- Translate words into algebraic expressions.
- Solve the resulting equations carefully.
- Verify solutions to ensure they make sense within the context.

Handling special cases

- If the quadratic yields negative solutions, discard them since physical dimensions cannot be negative.
- If multiple positive solutions exist, choose the one that makes sense in context.

Visual Representation and Graphical Insight

Visualizing the problem can reinforce understanding:

- Plot the quadratic function $y = L^2 - 6L - 72$.
- The roots of the quadratic are where the graph intersects the L-axis, corresponding to solutions.
- The positive root (L=12) indicates the feasible length.

Graphing helps illustrate the relationship between length and area, providing an intuitive understanding of the solutions.

Real-World Implications of the Findings

Knowing the exact dimensions of a plot of land or any rectangular space is crucial in planning and resource allocation. In this problem:

- A rectangle measuring 12 miles in length and 6 miles in width covers exactly 72 square miles.
- The aspect ratio (length to width) is 2:1, indicating a rectangular shape that is twice as long as it is wide, which might be relevant in designing land plots or architectural layouts.

Summary of Key Steps to Solve Similar Problems

1. Read the problem carefully and identify what is given and what needs to be found.
2. Define variables for unknown quantities.
3. Translate words into equations, paying attention to relationships like "less than" or "more than."
4. Set up algebraic equations based on geometric formulas (area, perimeter, etc.).
5. Solve the equations, often quadratic, using appropriate methods.
6. Interpret solutions in context, discarding non-physical results.
7. Verify that solutions satisfy original conditions.
8. Visualize the problem graphically if necessary for better understanding.

Conclusion

Determining the dimensions of a rectangle given specific constraints involves translating a word problem into an algebraic equation, solving that equation, and interpreting the results within a real-world context. In our example, the process revealed that a rectangle with a length of 12 miles and a width of 6 miles has an area of 72 square miles, with the width being 6 miles less than the length.

This method demonstrates the power of algebra in solving practical geometric problems. Whether you're tackling similar questions in academic settings or applying these skills in real-world scenarios, the principles remain consistent: define variables, set up equations, solve systematically, and interpret your solutions thoughtfully.

By mastering these steps, you'll be well-equipped to approach a broad range of problems involving dimensions, areas, and geometric relationships, enhancing both your mathematical proficiency and your ability to analyze and solve real-world challenges effectively.

Frequently Asked Questions

How do I find the dimensions of a rectangle if its width is 6 miles less than its length and its area is 72 square miles?

Let the length be L miles. Then, the width is L - 6 miles. The area is length times width: $L \times (L - 6) = 72$. Solve the quadratic equation $L^2 - 6L - 72 = 0$ to find L, then find the width as L - 6.

What is the quadratic equation derived from the problem of finding the rectangle's dimensions with area 72 and width 6 miles less than length?

The quadratic equation is $L^2 - 6L - 72 = 0$, where L is the length of the rectangle.

How do I solve the quadratic equation $L^2 - 6L - 72 = 0$ to find the rectangle's length?

Use the quadratic formula: $L = [6 \pm \sqrt{(36 + 288)}] / 2$. Simplify under the square root to get $L = [6 \pm \sqrt{324}] / 2 = [6 \pm 18] / 2$. So, $L = (6 + 18)/2 = 12$ or $(6 - 18)/2 = -6$. Discard the negative value, so L = 12 miles.

What are the dimensions of the rectangle if the length is 12 miles?

Since the width is 6 miles less than the length, the width is 12 - 6 = 6 miles. Therefore, the rectangle's dimensions are 12 miles by 6 miles.

How can I verify that the found dimensions satisfy the area condition?

Calculate the area: 12 miles × 6 miles = 72 square miles, which matches the given area. Thus, the dimensions are correct.

Are there any other possible dimensions for the rectangle based on the quadratic solution?

No, only the positive solution L = 12 miles is valid for the rectangle's dimensions, as the negative solution is not physically meaningful for length.

What is the importance of setting up the correct

equation in solving such problems?

Setting up the correct equation ensures an accurate representation of the problem constraints, allowing you to find the correct dimensions mathematically and verify their validity against the given area.

Additional Resources

Find the Dimension of a Rectangle Whose Width Is 6 Miles Less Than Its Length and Whose Area Is 72 Square Miles

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Introduction

In the realm of mathematical problem-solving, few challenges stimulate the analytical mind quite like geometric puzzles. Among them, word problems involving rectangles are particularly instructive, providing practical applications of algebraic reasoning to real-world scenarios. One such problem asks: Find the dimensions of a rectangle whose width is 6 miles less than its length and whose area is 72 square miles. This question not only tests one's understanding of basic algebra and geometry but also offers a window into how mathematical concepts are interconnected.

In this article, we will dissect this problem step-by-step, exploring the underlying principles, deriving the necessary equations, and analyzing the solutions with precision. Our goal is to present a comprehensive, accessible, and detailed explanation suitable for learners at various levels, while also highlighting the broader significance of such problems in mathematics education and practical applications.

Understanding the Problem

To approach this problem systematically, it's essential to understand and interpret the key components:

- Rectangle: A quadrilateral with four right angles. The area and dimensions are related by simple formulas.
- Width: One of the rectangle's two sides; in this problem, it's defined in relation to the length.
- Length: The longer side of the rectangle, which serves as the reference point.
- Relationship Between Width and Length: The width is 6 miles less than the length.
- Area: The measure of the surface enclosed by the rectangle, given as 72 square miles.

The core challenge is to determine the specific measurements of the rectangle's length and width based on these conditions.

Translating the Word Problem into Mathematical Equations

The first step in solving such a problem is converting verbal statements into algebraic expressions. This process involves defining variables and establishing relationships.

Defining Variables

Let:
- L = length of the rectangle (in miles)
- W = width of the rectangle (in miles)

Given the problem:
- The width is 6 miles less than the length, so:

$$
W = L - 6
$$

- The area of the rectangle is 72 square miles, which is calculated as:

$$
\text{Area} = L \times W = 72
$$

Substituting W into the area equation:

$$
L \times (L - 6) = 72
$$

This leads to a quadratic equation:

$$
L^2 - 6L = 72
$$

Rearranged:

$$
L^2 - 6L - 72 = 0
$$

This algebraic formulation serves as the foundation for finding the dimensions.

Solving the Equation: A Step-by-Step Approach

The quadratic equation:

$$
L^2 - 6L - 72 = 0
$$

can be solved using various methods, including factoring, completing the square, or quadratic formula. Given the coefficients, the quadratic formula provides a straightforward approach:

$$
L = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

where:
- $a = 1$
- $b = -6$
- $c = -72$

Calculating the discriminant:

$$
D = b^2 - 4ac = (-6)^2 - 4 \times 1 \times (-72) = 36 + 288 = 324
$$

Since D is positive, there are two real solutions:

$$
L = \frac{6 \pm \sqrt{324}}{2} = \frac{6 \pm 18}{2}
$$

Calculating both possibilities:

- When using the positive root:

$$
L = \frac{6 + 18}{2} = \frac{24}{2} = 12
$$

- When using the negative root:

$$
L = \frac{6 - 18}{2} = \frac{-12}{2} = -6
$$

Because length cannot be negative, the only physically meaningful solution is:

$$
L = 12 \text{ miles}
$$

Substituting back to find W:

$$
W = L - 6 = 12 - 6 = 6 \text{ miles}
$$

Interpreting the Solution

The calculations reveal the dimensions of the rectangle:

- Length: 12 miles
- Width: 6 miles

These dimensions satisfy the original conditions:

- The width is indeed 6 miles less than the length.
- The area is:

$$
12 \times 6 = 72 \text{ square miles}
$$

which matches the problem's given area.

Verifying the Solution

Verification is a critical step in mathematical problem-solving. It confirms the correctness and consistency of the solution.

- Check the relationship:

$$
W = L - 6 \Rightarrow 6 = 12 - 6
$$

Correct.

- Check the area:

$$
\text{Area} = L \times W = 12 \times 6 = 72
$$

Correct.

The solution aligns perfectly with the problem's conditions.

Broader Implications and Practical Applications

While this problem is presented in a purely mathematical context, its principles are widely applicable in various fields:

- Land Planning and Real Estate: Understanding how dimensions relate to area helps in designing plots, gardens, and building layouts.
- Engineering and Construction: Accurate calculations of dimensions ensure structures meet specifications.
- Environmental Management: Estimating land areas for conservation, agriculture, or urban development relies on similar mathematical reasoning.
- Educational Significance: Such problems foster critical thinking, algebraic skills, and the ability to translate verbal descriptions into equations.

Variations and Extensions

Mathematics is rich with variations that deepen understanding:

- Different relationships: What if the width is a multiple of the length, or differs by a percentage?
- Different areas: How do solutions change if the area varies?
- Three-dimensional analogs: Extending to volume calculations for rectangular prisms.

Exploring these variations enhances problem-solving flexibility and mathematical literacy.

Conclusion

The problem of finding the dimensions of a rectangle with a width 6 miles less than its length and an area of 72 square miles exemplifies the elegance and utility of algebraic reasoning. By carefully translating words into equations, applying the quadratic formula, and verifying solutions, we gain not only the specific dimensions—length of 12 miles and width of 6 miles—but also a deeper appreciation for the interconnectedness of mathematical concepts.

Such problems underscore the importance of analytical thinking and demonstrate how fundamental algebraic techniques serve as powerful tools across numerous practical and theoretical contexts. Whether planning land, designing structures, or simply sharpening mathematical skills, understanding how to approach and solve these problems remains an essential competency.

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