Find The Equation Of The Line That Contains The Point (-1,-7) And Is Parallel To The Line 6x + 5y = 11.

Find The Equation Of The Line That Contains The Point (-1,-7) And Is Parallel To The Line 6x + 5y = 11.

When working with lines in coordinate geometry, one of the most fundamental tasks is to find the equation of a line given certain conditions. In this case, the problem requires us to determine the equation of a line that passes through the point (-1, -7) and is parallel to another given line, 6x + 5y = 11. This process involves understanding the concepts of slope, parallel lines, and how to derive the equation of a line in various forms such as slope-intercept form or point-slope form. Whether you're a student brushing up on your algebra skills or someone preparing for exams, mastering this type of problem is essential for a solid foundation in coordinate geometry.

In this comprehensive guide, we will explore the steps involved in solving this problem, explain key concepts, and provide useful tips to help you approach similar questions with confidence.

Understanding the Problem

Before diving into the solution, it is crucial to clarify what the problem is asking:

- Given Point: (-1, -7)
- Given Line: 6x + 5y = 11 (which the new line must be parallel to)
- Goal: Find the equation of the line that passes through the given point and is parallel to the given

line.

Key Concepts Needed

To solve this problem effectively, we need to understand several important concepts:

1. Slope of a Line

The slope of a line indicates its steepness. It is often denoted as 'm' and can be calculated if the line's equation is in slope-intercept form ($y = mx + b$).

2. Parallel Lines

Parallel lines have the same slope but different y-intercepts. This means that if two lines are parallel, their slopes are equal.

3. Standard Form of a Line

Many lines are expressed in the standard form:

$$Ax + By = C$$

To find the slope of such a line, convert it to slope-intercept form.

4. Converting Standard Form to Slope-Intercept Form

Rearranged as:

$$ y = -\frac{A}{B}x + \frac{C}{B} $$

From this, the slope is $-\frac{A}{B}$.

Step-by-Step Solution

This section guides you through the process of finding the equation of the required line.

Step 1: Find the slope of the given line 6x + 5y = 11

Begin by rewriting the line in slope-intercept form:

$$
6x + 5y = 11
$$

Solve for y:

$$
5y = -6x + 11
$$

$$
y = -\frac{6}{5}x + \frac{11}{5}
$$

Therefore, the slope of the given line is $-\frac{6}{5}$.

Step 2: Determine the slope of the parallel line

Since the new line must be parallel to the given line, it will have the same slope:

$$
m = -\frac{6}{5}
$$

Step 3: Use point-slope form to find the equation of the new line

The point-slope form of a line's equation is:

$$
y - y_1 = m(x - x_1)
$$

Where:

- (x_1, y_1) is a point on the line, in this case, (-1, -7)

- m is the slope, $-\frac{6}{5}$

Plugging in the known values:

$$
y - (-7) = -\frac{6}{5}(x - (-1))
$$

Simplify:

$$
y + 7 = -\frac{6}{5}(x + 1)
$$

Step 4: Simplify the equation to slope-intercept form

Distribute the slope:

$$
y + 7 = -\frac{6}{5}x - \frac{6}{5}
$$

Subtract 7 from both sides:

$$
y = -\frac{6}{5}x - \frac{6}{5} - 7
$$

Express 7 as a fraction with denominator 5:

$$
7 = \frac{35}{5}
$$

So,

$$
y = -\frac{6}{5}x - \frac{6}{5} - \frac{35}{5}
$$

Combine the constants:

$$
y = -\frac{6}{5}x - \frac{41}{5}
$$

Final Equation of the Line

The equation of the line in slope-intercept form is:

$$
\boxed{
y = -\frac{6}{5}x - \frac{41}{5}
}
$$

Alternatively, if you prefer the standard form:

Multiply through by 5 to clear denominators:

$$
5y = -6x - 41
$$

Rearranged as:

$$
6x + 5y = -41
$$

This standard form confirms the line is parallel to the original, as the coefficients for x and y are proportional to those in $6x + 5y = 11$, but with a different constant term.

Additional Tips and Insights

Understanding the Relationship Between Lines and Slopes

- Parallel lines have identical slopes.
- Perpendicular lines have slopes that are negative reciprocals (e.g., slope 2 and slope -1/2).

Converting Between Forms

- To switch from standard form to slope-intercept form, isolate y.
- To write in standard form from slope-intercept, rearrange and clear fractions if necessary.

Handling Fractions in Calculations

- When dealing with fractions, it’s often easier to work with common denominators.
- Double-check algebraic steps to avoid errors when simplifying.

Verifying Your Solution

- Confirm the point (-1, -7) satisfies the final equation.
- Verify the slope of the line matches the original line's slope.

Summary

To find the equation of a line passing through a specific point and parallel to a given line:

1. Convert the given line to slope-intercept form to identify its slope.
2. Use the same slope for the new line.
3. Apply the point-slope form with the given point and the slope.
4. Simplify to your preferred form (slope-intercept or standard form).

In our case, we determined the line passing through (-1, -7) and parallel to 6x + 5y = 11 is:

\[

\boxed{

y = -\frac{6}{5}x - \frac{41}{5}

}

\]

or equivalently in standard form:

\[

6x + 5y = -41

\]

Practice Problems to Master the Concept

1. Find the equation of a line that passes through (2, 3) and is parallel to the line 4x - y = 7.
2. Determine the line passing through (0, 0) and parallel to 3x + 2y = 6.
3. Write the equation of the line in standard form that passes through (-2, 5) and is parallel to y = \(\frac{2}{3}x + 4\).

Practice makes perfect, and understanding these steps will help you confidently tackle similar problems involving parallel lines.

Conclusion

Finding the equation of a line parallel to a given line and passing through a specific point is a common problem in algebra and coordinate geometry. By understanding the relationship between slopes and line equations, converting between different forms, and applying the point-slope formula, you can efficiently solve these problems. With practice, you'll develop a strong intuition for the geometric relationships and algebraic manipulations necessary to succeed.

Remember, always verify your solutions by checking if the point lies on the line and if the slopes match for parallel lines. Keep practicing with different problems to strengthen your skills and become proficient in coordinate geometry!

Frequently Asked Questions

How do I find the equation of a line parallel to 6x + 5y = 11 passing through point (-1, -7)?

First, find the slope of the given line by rewriting it in slope-intercept form, then use that slope and the point (-1, -7) to write the new line's equation.

What is the slope of the line 6x + 5y = 11?

Rewrite the line as 5y = -6x + 11, so y = (-6/5)x + 11/5. Therefore, the slope is -6/5.

How do I write the equation of a line parallel to 6x + 5y = 11 that passes through (-1, -7)?

Use the same slope, -6/5, and plug in the point (-1, -7) into the point-slope form: $y - y_1 = m(x - x_1)$.

What is the point-slope form of the line passing through (-1, -7) with

slope -6/5?

The equation is y + 7 = (-6/5)(x + 1).

How do I convert the point-slope form to slope-intercept form?

Distribute the slope and simplify: y + 7 = (-6/5)x - 6/5, then subtract 7 to get y = (-6/5)x - 6/5 - 7.

What is the final equation of the line in slope-intercept form?

Simplify the constants: y = (-6/5)x - (6/5 + 35/5) = (-6/5)x - 41/5.

Can I verify if the point (-1, -7) lies on the line y = (-6/5)x - 41/5?

Yes, substitute x = -1: y = (-6/5)(-1) - 41/5 = 6/5 - 41/5 = -35/5 = -7, confirming the point lies on the line.

Why is the slope of the parallel line the same as the original line?

Parallel lines have identical slopes; since the original line's slope is -6/5, the parallel line must also have slope -6/5.

What are common mistakes to avoid when finding the equation of a parallel line?

Avoid mixing up the slopes, forgetting to convert the original line to slope form, or making arithmetic errors when simplifying the equation.

Additional Resources

Parallel Lines and Their Equations: An Expert Breakdown

Understanding the Fundamentals: The Concept of Parallel Lines and Their Equations

Before we delve into deriving the specific equation of a line passing through a particular point and parallel to a given line, it's essential to establish a solid understanding of the core concepts involved. This includes the nature of linear equations, the significance of slope, and what it fundamentally means for lines to be parallel.

What Is a Linear Equation in Two Variables?

A linear equation in two variables (x and y) typically takes the form:

$$Ax + By + C = 0$$

or, more commonly,

$$y = mx + b$$

where:

- m: the slope of the line,
- b: the y-intercept, the point where the line crosses the y-axis.

This form makes it straightforward to analyze the line's behavior, such as its steepness and position.

The Slope and Its Role in Parallelism

The slope (m) measures the rate at which y changes with respect to x. Two lines are parallel if and only if they have identical slopes. This is a fundamental property in coordinate geometry, meaning that:

$$ m_1 = m_2 $$

for lines L_1 and L_2 to be parallel.

Deciphering the Given Line: 6x + 5y = 11

The first step in our problem-solving journey involves converting the given line into the slope-intercept form $y = mx + b$.

Rearranging the Equation

Starting with:

$$ 6x + 5y = 11 $$

We isolate y:

$$ 5y = -6x + 11 $$

$$ y = -\frac{6}{5}x + \frac{11}{5} $$

Key takeaway: The slope of the given line is:

$$ m = -\frac{6}{5} $$

This information is vital because it directly informs us about the slope of any line parallel to this one.

Finding the Equation of the Parallel Line Through a Specific Point

Our goal is to find the equation of a line that:

- Passes through the point (-1, -7)
- Is parallel to the line $6x + 5y = 11$

Step 1: Use the Slope of the Given Line

Since parallel lines share the same slope, the line we seek must have:

$$m_{parallel} = -\frac{6}{5}$$

Step 2: Apply the Point-Slope Form

The point-slope form of a line is:

$$y - y_1 = m(x - x_1)$$

where (x_1, y_1) is a point on the line, and m is the slope.

Plugging in our point $(-1, -7)$ and the slope $-\frac{6}{5}$:

$$ y - (-7) = -\frac{6}{5}(x - (-1)) $$

which simplifies to:

$$ y + 7 = -\frac{6}{5}(x + 1) $$

Step 3: Expand and Simplify to Slope-Intercept Form

Distribute the slope:

$$ y + 7 = -\frac{6}{5}x - \frac{6}{5} $$

Subtract 7 from both sides:

$$ y = -\frac{6}{5}x - \frac{6}{5} - 7 $$

Express 7 as a fraction with denominator 5:

$$ 7 = \frac{35}{5} $$

Now, combine the constants:

$$ y = -\frac{6}{5}x - \frac{6}{5} - \frac{35}{5} $$

$$ y = -\frac{6}{5}x - \frac{41}{5} $$

Thus, the equation of the line in slope-intercept form is:

$$ y = -\frac{6}{5}x - \frac{41}{5} $$

Expressing the Final Equation: Multiple Formats for Clarity

Depending on the context or the preference, the line’s equation can be expressed in various forms:

1. Slope-Intercept Form

$$ y = -\frac{6}{5}x - \frac{41}{5} $$

This form clearly shows the slope and y-intercept, making it easy to graph and analyze.

2. Standard Form

Multiply through by 5 to clear fractions:

$$ 5y = -6x - 41 $$

Rearranged:

$$ 6x + 5y = -41 $$

This is a clean, integer-based standard form, directly comparable to the original line.

3. General Form

$$6x + 5y + 41 = 0$$

which can be useful in certain algebraic contexts, such as setting the equation equal to zero for various calculations.

Summary of the Process and Key Takeaways

- Identify the slope of the given line by converting it to slope-intercept form.
- Use the same slope for the new line, since parallel lines share identical slopes.
- Apply the point-slope formula with the given point to find the specific line.
- Simplify the resulting equation into the desired form.

Practical Applications and Broader Context

Understanding how to find the equation of a line parallel to a given line through a specific point isn’t just an academic exercise; it has real-world applications:

- Engineering and Design: Ensuring components or structures are parallel.
- Navigation and Mapping: Drawing parallel routes or boundaries.
- Data Visualization: Drawing parallel trend lines in graphs to compare datasets.
- Computer Graphics: Rendering parallel lines and shapes.

Mastering this process enhances spatial reasoning and algebraic manipulation skills, essential for advanced mathematics, physics, engineering, and computer science.

Final Thoughts and Recommendations

This detailed exploration demonstrates that deriving the equation of a line through a specific point and parallel to a given line is a systematic process grounded in fundamental principles of algebra and geometry. By understanding the properties of slopes and line equations, students and professionals can confidently approach similar problems with clarity and precision.

Pro tip: Always verify your final equation by substituting the given point to ensure it satisfies the line, reinforcing your understanding and accuracy.

In essence: The equation of the line passing through (-1, -7) and parallel to $6x + 5y = 11$ is:

$$\boxed{6x + 5y = -41}$$

or equivalently,

$$y = -\frac{6}{5}x - \frac{41}{5}$$

This comprehensive approach not only provides the answer but also consolidates your understanding of lines, slopes, and algebraic transformations, empowering you with skills applicable across numerous mathematical and real-world scenarios.

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