Find The Half-life Of An Element Which Decays By 3.403% Each Day. The Half-life Is Days, Help (numbers)

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Understanding radioactive decay and calculating half-life are fundamental concepts in physics and chemistry. When an element decays at a constant percentage rate per day, it can be modeled mathematically to determine how long it takes for half of the substance to decay, known as the half-life. In this article, we will explore how to find the half-life of an element that decreases by 3.403% daily, providing step-by-step explanations, formulas, and practical examples to help you grasp the concept thoroughly.

What Is Radioactive Decay and Half-life?

Radioactive decay is a spontaneous process where an unstable atomic nucleus loses energy by emitting radiation. This decay process is random at the level of individual atoms but exhibits predictable statistical behavior when considering large quantities.

Half-life is the duration it takes for half of the original amount of a radioactive substance to decay. It is a characteristic property of each radioactive isotope and remains constant regardless of the initial amount.

Key points:
- Decay rate is often expressed as a percentage per unit time.
- The half-life provides insight into the stability of an isotope.
- Understanding decay rates helps in applications like radiometric dating, nuclear medicine, and safety analysis.

Mathematical Model of Decay at a Percentage Rate

When an element decays by a fixed percentage each day, the process can be modeled using exponential decay formulas.

Decay formula:

$$N(t) = N_0 \times (1 - r)^t$$

where:
- \(N(t) \) = remaining amount after time \(t \),
- \(N_0 \) = initial amount,
- \(r \) = decay rate per day (as a decimal),
- \(t \) = time in days.

In this context:
- \(r = 3.403\% = 0.03403 \).

This formula signifies that each day, the remaining amount is multiplied by \((1 - 0.03403) \).

Calculating the Half-life

To find the half-life (\(T_{1/2} \)), we need to determine the number of days it takes for the initial amount \(N_0 \) to reduce to \(\frac{N_0}{2} \).

Mathematically, set:

\[\frac{N_0}{2} = N_0 \times (1 - r)^{T_{1/2}} \]

Dividing both sides by \(N_0 \):

\[\frac{1}{2} = (1 - r)^{T_{1/2}} \]

Taking natural logarithms on both sides:

\[\ln \left(\frac{1}{2} \right) = T_{1/2} \times \ln (1 - r) \]

Solving for \(T_{1/2} \):

\[T_{1/2} = \frac{\ln (1/2)}{\ln (1 - r)} \]

Since \(\ln (1/2) = -0.6931 \), the formula simplifies to:

\[T_{1/2} = \frac{-0.6931}{\ln (1 - r)} \]

Now, substitute \(r = 0.03403 \):

\[T_{1/2} = \frac{-0.6931}{\ln (1 - 0.03403)} \]

Calculate \(1 - 0.03403 = 0.96597 \).

Next, compute \(\ln(0.96597) \):

\[\ln(0.96597) \approx -0.03459 \]

Finally, compute \(T_{1/2} \):

$$ T_{1/2} = \frac{-0.6931}{-0.03459} \approx 20.02 \ \text{days} $$

Result: The half-life of the element is approximately 20 days.

Step-by-Step Summary of the Calculation

1. Identify the decay rate per day:
- $r = 3.403\% = 0.03403$.

2. Set up the decay formula for half-life:
- $\frac{1}{2} = (1 - r)^{T_{1/2}}$.

3. Use natural logarithms to solve for $T_{1/2}$:
- $T_{1/2} = \frac{\ln(1/2)}{\ln(1 - r)}$.

4. Calculate logs:
- $\ln(1/2) = -0.6931$,
- $\ln(0.96597) \approx -0.03459$.

5. Compute $T_{1/2}$:
- $T_{1/2} \approx \frac{-0.6931}{-0.03459} \approx 20.02$ days.

Practical Applications and Significance

Knowing the half-life of an element that decays at a specific percentage rate each day is crucial in various scientific and industrial fields.

Applications include:
- Radiometric Dating: Estimating the age of archaeological finds or geological samples.
- Nuclear Medicine: Calculating dosages and timing for radioactive tracers.
- Nuclear Power: Managing fuel cycles and waste decay.
- Safety Protocols: Planning for decay of radioactive hazards.

Why is this important?
Accurate half-life calculations help scientists predict how long a radioactive substance remains active or hazardous, ensuring safety and efficacy in applications.

Additional Considerations and Tips

- Understanding decay percentages:
When decay is expressed as a percentage, convert it to a decimal for calculations.

- Logarithm base:
Use natural logs (ln) unless specified otherwise; the formulas are based on natural logarithms.

- Approximate vs. Exact:
Small rounding errors can occur; for precise work, use high-precision calculators or software.

- Generalization:
The same approach applies to any percentage decay rate by adjusting \(r \).

Summary

Calculating the half-life of an element that decays by a fixed percentage each day involves understanding exponential decay and logarithmic functions. Given a daily decay rate of 3.403%, the element's half-life is approximately 20 days. This process underscores the power of mathematical modeling in predicting natural phenomena and has broad applications across science and industry. By mastering these calculations, scientists and engineers can make informed decisions about material stability, safety measures, and scientific research.

Final Notes

Always verify your calculations with multiple methods or tools when precision is critical. Remember that the decay rate remains constant, and the exponential decay model provides a reliable estimate of the half-life for any percentage-based decay process.

If you're applying this knowledge to real-world scenarios, consider factors like measurement uncertainties and environmental influences that might affect decay rates.

Disclaimer: The calculations and interpretations provided here are based on idealized models. Actual decay processes may involve additional complexities depending on the specific isotope and environmental factors.

Frequently Asked Questions

What is the formula to find the half-life of an element that decays exponentially?

The half-life $T_{1/2}$ can be found using the decay constant λ with the formula $T_{1/2} = \ln(2) / \lambda$.

If an element decays by 3.403% each day, what is its decay factor per day?

The decay factor per day is $1 - 0.03403 = 0.96597$.

How do you calculate the decay constant λ from the daily decay percentage?

$\lambda = -\ln(\text{decay factor}) = -\ln(0.96597)$.

What is the value of λ when the element decays by 3.403% daily?

$\lambda = -\ln(0.96597) \approx 0.0346$ per day.

How do you compute the half-life from the decay constant λ?

Half-life $T_{1/2} = \ln(2) / \lambda \approx 0.693 / 0.0346 \approx 20$ days.

What is the approximate half-life of the element in days, given it decays by 3.403% daily?

Approximately 20 days.

If an element's half-life is 20 days, what percentage does it decay each day?

About 3.465%, which is close to the given 3.403%.

Why is the natural logarithm used in calculating the half-life of a decaying element?

Because decay follows exponential laws, and the natural logarithm helps relate the decay constant to percentage decay.

Can the half-life be directly calculated from the daily decay percentage without involving λ?

Yes, using the formula $T_{1/2} = \ln(2) / (-\ln(1 - \text{decay rate}))$, but calculating λ simplifies the process.

What is the significance of knowing the half-life of a radioactive element?

It helps determine how long it takes for half of the substance to decay, which is crucial in fields like nuclear physics, medicine, and radiometric dating.

Additional Resources

Find The Half-life Of An Element Which Decays By 3.403% Each Day: A Detailed Investigation

Understanding radioactive decay and half-life calculations is fundamental in nuclear physics, chemistry, and related scientific disciplines. The process by which unstable isotopes diminish over time has profound implications, from radiometric dating to medical applications. In this comprehensive analysis, we explore the problem: Find the half-life of an element which decays by 3.403% each day. This exploration will involve dissecting the decay process, deriving relevant formulas, and applying mathematical principles to arrive at an accurate answer.

Introduction: The Significance of Radioactive Decay and Half-life

Radioactive decay is a stochastic process where an unstable atomic nucleus loses energy by emitting radiation. A key characteristic of this process is the half-life, the time required for half of a given quantity of radioactive material to decay. The half-life is a constant for a specific isotope, independent of the amount of substance, and provides a vital metric for scientists to understand the longevity and stability of radioactive materials.

The decay rate often follows an exponential law, which is mathematically expressed as:

$$N(t) = N_0 e^{-\lambda t}$$

where:

- $N(t)$ is the quantity remaining at time t,
- N_0 is the initial quantity,
- λ is the decay constant (per unit time),
- t is time.

The half-life ($T_{1/2}$) relates to λ via:

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

Given a decay rate per day, we can determine λ, and subsequently, the half-life.

Deciphering the Decay Rate: From Percentage Decay to Decay Constant

The problem states that the element decays by 3.403% each day. Interpreting this, after one day, the remaining amount $N(1)$ is:

$$N(1) = N_0 \times (1 - 0.03403) = N_0 \times 0.96597$$

In exponential terms:

$$N(1) = N_0 e^{-\lambda \times 1}$$

Which simplifies to:

$$0.96597 = e^{-\lambda}$$

By taking the natural logarithm of both sides, we find:

$$\ln(0.96597) = -\lambda$$

$$\lambda = -\ln(0.96597)$$

Calculating λ:

$$\lambda \approx -\ln(0.96597)$$

Using a calculator:

$$ \ln(0.96597) \approx -0.03466 $$

So,

$$ \lambda \approx 0.03466 \ \text{per day} $$

This decay constant indicates the probability per day that a single nucleus will decay.

Calculating the Half-life: From Decay Constant to Half-life

As noted earlier, the half-life relates to the decay constant via:

$$ T_{1/2} = \frac{\ln 2}{\lambda} $$

Substituting the calculated λ:

$$ T_{1/2} = \frac{\ln 2}{0.03466} $$

Since $\ln 2 \approx 0.6931$, we compute:

$$ T_{1/2} \approx \frac{0.6931}{0.03466} $$

Using a calculator:

$$ T_{1/2} \approx 20.0 \ \text{days} $$

Result: The half-life of the element is approximately 20 days.

Deep Dive: Verifying and Interpreting the Result

While the straightforward calculation suggests a half-life of approximately 20 days, it’s essential to understand the underlying assumptions and verify the accuracy.

1. Logarithmic Basis of Decay

The exponential decay law inherently assumes a constant decay probability per unit time. Our calculation hinges on the natural logarithm transformation:

$$ \lambda = -\ln(1 - r) $$

where r is the fractional decay per day (here, 0.03403). This approach is valid for small decay fractions, as the exponential model approximates real decay behavior effectively.

2. Relationship Between Decay Rate and Half-life

The formula:

$$ T_{1/2} = \frac{\ln 2}{\lambda} $$

is universally applicable for exponential decay processes, confirming our method's validity.

3. Alternative Approach: Direct Calculation

Alternatively, one can directly relate the percentage decay per day to the half-life:

$$ \text{Remaining after } T_{1/2} \text{ days} = \frac{1}{2} $$

Expressed mathematically:

$$ (1 - r)^{T_{1/2}} = \frac{1}{2} $$

Taking natural logarithms:

$$ T_{1/2} \times \ln(1 - r) = \ln \frac{1}{2} = -\ln 2 $$

Solving for $T_{1/2}$:

$$ T_{1/2} = \frac{-\ln 2}{\ln(1 - r)} $$

Plugging in $r = 0.03403$:

$$ T_{1/2} = \frac{-0.6931}{\ln(0.96597)} $$

Recall that $\ln(0.96597) \approx -0.03466$, so:

$$ T_{1/2} \approx \frac{-0.6931}{-0.03466} \approx 20.0 \ \text{days} $$

This confirms the previous result.

Implications and Practical Significance

Determining a half-life of approximately 20 days indicates that the element decays relatively quickly compared to isotopes with half-lives spanning years or millennia. This information is crucial for applications such as:

- Radiometric dating: Not suitable for dating geological samples over long periods.
- Medical uses: Potentially useful in radiotherapy or diagnostic imaging where rapid decay is advantageous.
- Nuclear waste management: Understanding decay rates helps design storage solutions for radioactive materials.

Furthermore, knowing the precise half-life allows scientists to model decay behavior over time, predict remaining quantities, and plan accordingly.

Conclusion: Summarizing the Findings

The problem posed was to find the half-life of an element that decays by 3.403% daily. Through mathematical derivation, we established:

- The decay constant $\lambda \approx 0.03466$ per day.
- The corresponding half-life $T_{1/2} \approx 20$ days.

This analysis demonstrates the utility of exponential decay laws and logarithmic transformations in solving real-world decay problems. The approximate 20-day half-life underscores the element's relatively rapid decay, with significant implications across scientific and practical domains.

Final answer: The half-life of the element is approximately 20 days.

References

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Note: Precise calculations may vary slightly depending on rounding, but the
core methodology remains valid and robust.
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