Find The Inverse Laplace Transform Of A) F(s)= 10/s(s+2)(s+3) B) F(s)= S/s+4s+5 C) F(s)=e^-3s S/(s-2)^2

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Understanding the inverse Laplace transform is fundamental in solving differential equations, analyzing control systems, and modeling physical phenomena. This comprehensive guide will walk you through the process of finding the inverse Laplace transform for three different functions:

- A) $F(s) = \frac{10}{s(s+2)(s+3)}$
- B) $F(s) = \frac{s}{s+4s+5}$ (which appears to have a typo, but we will interpret it as $F(s) = \frac{s}{(s+4)(s+5)}$)
- C) $F(s) = e^{-3s} \frac{s}{(s-2)^2}$

By the end of this article, you'll understand the techniques involved, including partial fraction decomposition, shifting theorems, and recognizing standard Laplace transform pairs.

Introduction to the Inverse Laplace Transform

The Laplace transform is a powerful integral transform that converts a time-domain function $f(t)$ into a complex frequency domain function $F(s)$:

\[

F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st}f(t) dt

\]

The inverse Laplace transform, denoted as \(\mathcal{L}^{-1} \), allows us to recover the original time-domain function from its Laplace domain counterpart:

\[

f(t) = \mathcal{L}^{-1}\{F(s)\}

\]

Finding the inverse Laplace transform often involves:

- Recognizing standard Laplace transform pairs
- Decomposing complicated functions into simpler partial fractions
- Applying shifts and derivatives in the \(s \)-domain
- Utilizing theorems like the shifting theorem and convolution theorem

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Part A: Inverse Laplace Transform of \(F(s) = \frac{10}{s(s+2)(s+3)} \)

Step 1: Recognize the form and prepare for partial fractions

Given:

\[

F(s) = \frac{10}{s(s+2)(s+3)}

\]

The goal is to decompose \(F(s) \) into simpler fractions whose inverse Laplace transforms are known.

Step 2: Partial Fraction Decomposition

Express \(F(s) \) as:

\[
\frac{10}{s(s+2)(s+3)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3}
\]

Multiply both sides by the denominator:

\[
10 = A(s+2)(s+3) + B s(s+3) + C s(s+2)
\]

To find \(A, B, C \), plug in suitable values of \(s \):

- For \(s=0 \):

\[
10 = A(2)(3) \Rightarrow 10 = 6A \Rightarrow A = \frac{10}{6} = \frac{5}{3}
\]

- For \(s=-2 \):

\[
10 = B(-2)(1) \Rightarrow 10 = -2B \Rightarrow B = -5

\]

- For \(s=-3 \):

\[
10 = C(-3)(-1) \Rightarrow 10 = 3C \Rightarrow C = \frac{10}{3}
\]

Thus:

\[
F(s) = \frac{5/3}{s} - \frac{5}{s+2} + \frac{10/3}{s+3}
\]

Step 3: Find the inverse Laplace transforms of each term

Recall standard pairs:

\[
\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = 1, \quad \mathcal{L}^{-1}\left\{\frac{1}{s+a}\right\} = e^{-a t}
\]

Applying linearity:

\[
f(t) = \frac{5}{3} \cdot 1 - 5 e^{-2t} + \frac{10}{3} e^{-3t}
\]

Final result:

\[

\boxed{

f(t) = \frac{5}{3} - 5 e^{-2t} + \frac{10}{3} e^{-3t}

}

\]

This is the inverse Laplace transform of \(F(s) \).

Part B: Inverse Laplace Transform of \(F(s) = \frac{s}{(s+4)(s+5)} \)

Step 1: Recognize the structure

The function:

\[

F(s) = \frac{s}{(s+4)(s+5)}

\]

This resembles the form suitable for partial fractions, but note the numerator is \(s \), which can be expressed as a combination of \((s + 4) \) and \((s + 5) \).

Step 2: Partial Fraction Decomposition

Express \(F(s) \) as:

$$
\frac{s}{(s+4)(s+5)} = \frac{A}{s+4} + \frac{B}{s+5}
$$

Multiply both sides by $(s+4)(s+5)$:

$$
s = A(s+5) + B(s+4)
$$

Set s as a variable:

$$
s = A s + 5A + B s + 4B = (A + B)s + (5A + 4B)
$$

Matching coefficients:

- Coefficient of s:

$$
1 = A + B
$$

- Constant term:

$$
0 = 5A + 4B
$$

From the second:

$$
5A = -4B \Rightarrow A = -\frac{4}{5} B
$$

Substitute into the first:

$$
1 = -\frac{4}{5} B + B = B \left(1 - \frac{4}{5}\right) = B \left(\frac{1}{5}\right) \Rightarrow B = 5
$$

Then:

$$
A = -\frac{4}{5} \times 5 = -4
$$

Partial fraction form:

$$
F(s) = -\frac{4}{s+4} + \frac{5}{s+5}
$$

Step 3: Inverse Laplace Transform

Using standard pairs:

$$
\mathcal{L}^{-1}\left\{\frac{1}{s+a}\right\} = e^{-a t}
$$

Therefore:

\[

f(t) = -4 e^{-4 t} + 5 e^{-5 t}

\]

Final result:

\[

\boxed{

f(t) = -4 e^{-4 t} + 5 e^{-5 t}

}

\]

Part C: Inverse Laplace Transform of \(F(s) = e^{-3s} \frac{s}{(s-2)^2} \)

Step 1: Recognize the shifting property

The presence of \(e^{-3s} \) indicates a time shift in the inverse transform, based on the second shifting theorem:

\[

\mathcal{L}^{-1}\left\{ e^{-a s} F(s) \right\} = u(t - a) f(t - a)

\]

where $u(t - a)$ is the unit step function, and $f(t)$ is the inverse Laplace transform of $F(s)$ without the exponential term.

Set:

$$
F_1(s) = \frac{s}{(s-2)^2}
$$

and

$$
f(t) = \mathcal{L}^{-1}\left\{ F_1(s) \right\}
$$

so that:

$$
\mathcal{L}^{-1}\left\{ F(s) \right\} = u(t - 3) \cdot f(t - 3)
$$

Step 2: Find $f(t) = \mathcal{L}^{-1}\left\{ \frac{s}{(s-2)^2} \right\}$

Note that:

$$
F_1(s) = \frac{s}{(s-2)^2}
$$

Recall the standard Laplace pairs:

- \(\mathcal{L}\{ t e^{a t} \} = \frac{1}{(s - a)^2} \)
- \(\mathcal{L}\{ e^{a t} \} = \frac{1}{s - a} \)

But here, numerator is \(s \), which can be written as:

\[
s = (s - 2) + 2

Frequently Asked Questions

How do you find the inverse Laplace Transform of F(s) = 10 / [s(s+2)(s+3)]?

To find the inverse Laplace Transform, first perform partial fraction decomposition on F(s). Then, take the inverse Laplace of each term separately using known transforms: L^{-1} {1/s} = 1, L^{-1} {1/(s+a)} = e^{-a t}, etc. The result will be a sum of exponentials and constants based on the decomposed terms.

What is the step-by-step method to invert F(s) = 10 / [s(s+2)(s+3)]?

Step 1: Decompose into partial fractions: 10 / [s(s+2)(s+3)] = A/s + B/(s+2) + C/(s+3). Step 2: Solve for A, B, and C. Step 3: Find the inverse Laplace of each term: A/s □ A, B/(s+2) □ B e^{-2t}, C/(s+3) □ C e^{-3t}. Step 4: Sum the results to get the inverse Laplace transform.

How do you handle the inverse Laplace of F(s) = s / (s + 4s + 5)?

First, clarify the expression, which is likely F(s) = s / [(s + 4)(s + 5)]. Then, perform partial fraction decomposition: s / [(s+4)(s+5)] = A/(s+4) + B/(s+5). Solve for A and B, then apply inverse Laplace transforms: L^{-1} {1/(s+a)} = e^{-a t}, and multiply by the numerator constants accordingly.

What is the inverse Laplace transform of $F(s) = s / [(s+4)(s+5)]$?

Partial fractions give: $s / [(s+4)(s+5)] = 3/(s+4) - 2/(s+5)$. Therefore, the inverse Laplace transform is: $3 e^{-4t} - 2 e^{-5t}$.

How do you invert $F(s) = e^{-3s} s / (s - 2)^2$?

Recognize that multiplication by e^{-as} in Laplace domain corresponds to a delay in the time domain: $L^{-1} \{ e^{-as} F(s) \} = u(t - a) f(t - a)$. First, find the inverse of $s / (s - 2)^2$, which is $(t) e^{2t}$. Then, multiply by the unit step $u(t - 3)$ to account for the e^{-3s} term. The final inverse is $u(t - 3) (t - 3) e^{2(t - 3)}$.

What is the inverse Laplace transform of $F(s) = e^{-3s} s / (s - 2)^2$?

The inverse Laplace transform is $u(t - 3) (t - 3) e^{2(t - 3)}$, where $u(t - 3)$ is the unit step function delaying the response by 3 units.

How can partial fractions be used to invert $F(s) = 10 / [s(s+2)(s+3)]$?

Express $F(s)$ as $A/s + B/(s+2) + C/(s+3)$, then solve for A, B, and C. Once obtained, apply known inverse transforms: A/s □ A, $B/(s+2)$ □ $B e^{-2t}$, $C/(s+3)$ □ $C e^{-3t}$. Summing these gives the inverse Laplace transform.

What are common techniques for finding inverse Laplace Transforms of complex functions like these?

Common techniques include partial fraction decomposition, recognizing standard Laplace transform pairs, handling exponential factors via shifting theorems, and applying convolution when necessary. Breaking down complex functions into simpler parts simplifies the inversion process.

What is the significance of the exponential term e^{-3s} in $F(s)$ when performing inverse Laplace transforms?

The term e^{-3s} in $F(s)$ indicates a time delay of 3 units in the inverse time domain. Using the shifting theorem, the inverse transform involves multiplying the corresponding function by a unit step $u(t - 3)$ and replacing t with $(t - 3)$.

Additional Resources

Understanding the Inverse Laplace Transform: A Comprehensive Analysis of Key Examples

The inverse Laplace transform is a fundamental concept in mathematical analysis, particularly within the fields of engineering, physics, and applied mathematics. It serves as a bridge between the frequency domain (represented by the complex variable s) and the time domain (represented by t). By understanding how to efficiently compute the inverse Laplace transform for various functions $F(s)$, practitioners can solve complex differential equations, analyze system behaviors, and model real-world phenomena. In this detailed review, we will explore the process of finding the inverse Laplace transform for three specific functions:

1. $F(s) = \frac{10}{s(s+2)(s+3)}$
2. $F(s) = \frac{s}{s+4s+5}$ (which appears to have a typo, but we'll clarify and interpret it accordingly)
3. $F(s) = e^{-3s} \frac{s}{(s - 2)^2}$

Each case presents unique challenges and techniques, from partial fraction decomposition to the application of shifting theorems and recognition of standard transforms. Let's delve into each one systematically.

Fundamentals of the Inverse Laplace Transform

Before analyzing specific examples, it is essential to revisit the core concepts:

What Is the Inverse Laplace Transform?

The inverse Laplace transform, denoted as $\mathcal{L}^{-1} \{F(s)\}$, retrieves the original time-domain function $f(t)$ from its Laplace transform $F(s)$. Formally,

$$
f(t) = \mathcal{L}^{-1} \{F(s)\} = \frac{1}{2\pi i} \int_{c - i\infty}^{c + i\infty} e^{st} F(s) ds
$$

where c is a real number such that the contour path lies in the region of convergence.

Common Techniques

- Partial Fraction Decomposition: Breaking down complex rational functions into simpler fractions whose inverse transforms are known.
- Standard Laplace Transform Pairs: Recognizing functions whose transforms are tabulated.
- Shifting Theorems: Handling exponential factors and shifts in the s-domain.
- Completing the Square: Rewriting quadratics to match standard forms.
- Convolution Theorem: Combining transforms to handle products that are difficult to invert directly.

Practical Significance

In engineering contexts, inverse Laplace transforms allow us to translate solutions from the frequency domain back into the time domain, enabling physical interpretation of system responses such as transient behaviors, steady-state responses, and damping effects.

Case A: Inverse Laplace Transform of $F(s) = \frac{10}{s(s+2)(s+3)}$

Step 1: Recognize the Structure

The function is a rational expression with simple poles at $s=0$, $s=-2$, and $s=-3$. The task is to find

$$
f(t) = \mathcal{L}^{-1} \left\{ \frac{10}{s(s+2)(s+3)} \right\}
$$

which involves decomposing the function into simpler fractions.

Step 2: Partial Fraction Decomposition

Express $F(s)$ as a sum of simpler fractions:

$$
\frac{10}{s(s+2)(s+3)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3}
$$

Multiply through by the denominator:

$$
10 = A (s+2)(s+3) + B s (s+3) + C s (s+2)
$$

Expand each term:

- $A (s+2)(s+3) = A (s^2 + 5s + 6)$
- $B s (s+3) = B (s^2 + 3s)$
- $C s (s+2) = C (s^2 + 2s)$

Combine:

$$
10 = (A + B + C) s^2 + (5A + 3B + 2C) s + 6A
$$

Matching coefficients:

- For s^2:

$$
A + B + C = 0
$$

- For s:

$$
5A + 3B + 2C = 0
$$

- Constant term:

$$
6A = 10 \Rightarrow A = \frac{10}{6} = \frac{5}{3}
$$

Now, substitute $A = \frac{5}{3}$ into the first two equations:

1. $\frac{5}{3} + B + C = 0 \Rightarrow B + C = -\frac{5}{3}$

2. $5 \times \frac{5}{3} + 3B + 2C = 0 \Rightarrow \frac{25}{3} + 3B + 2C = 0$

Express C from the first:

$$
C = -\frac{5}{3} - B
$$

Substitute into the second:

$$
\frac{25}{3} + 3B + 2 \left(-\frac{5}{3} - B \right) = 0
$$

Simplify:

$$
\frac{25}{3} + 3B - \frac{10}{3} - 2B = 0
$$

Combine like terms:

$$
\left(\frac{25}{3} - \frac{10}{3} \right) + (3B - 2B) = 0
$$

$$
\frac{15}{3} + B = 0 \Rightarrow 5 + B = 0 \Rightarrow B = -5
$$

Now, find C:

$$
C = -\frac{5}{3} - (-5) = -\frac{5}{3} + 5 = -\frac{5}{3} + \frac{15}{3} = \frac{10}{3}
$$

Final partial fractions:

$$
\frac{10}{s(s+2)(s+3)} = \frac{5/3}{s} - \frac{5}{s+2} + \frac{10/3}{s+3}
$$

Step 3: Inverse Laplace Transform of Each Term

Use standard pairs:

- $\mathcal{L}^{-1} \left\{ \frac{1}{s} \right\} = 1$
- $\mathcal{L}^{-1} \left\{ \frac{1}{s + a} \right\} = e^{-a t}$

Applying to each term:

$$
f(t) = \frac{5}{3} \times 1 - 5 e^{-2 t} + \frac{10}{3} e^{-3 t}
$$

Thus, the inverse Laplace transform is:

\[

\boxed{

f(t) = \frac{5}{3} - 5 e^{-2 t} + \frac{10}{3} e^{-3 t}

}

\]

Analysis of Case A: Significance and Applications

This example demonstrates how partial fraction decomposition simplifies the inversion process. The resulting sum of exponentials corresponds to a response in a linear system characterized by damping or decay rates \(2 \) and \(3 \) with specific weights. This form is common in solving differential equations modeling electrical circuits, mechanical vibrations, or thermal processes where multiple exponential decay modes interact.

Case B: Inverse Laplace Transform of \(F(s) = \frac{s}{s+4s+5} \)

Clarification of the Expression

At first glance, \(F(s) = \frac{s}{s + 4s + 5} \) appears to contain a typographical error or ambiguity. The denominator \(s + 4s + 5 \) simplifies to \(5s + 5 \):

\[

s + 4s + 5 = 5s + 5

\]

Hence, the function reduces to:

$$
F(s) = \frac{s}{5s + 5} = \frac{s}{5 (s + 1)} = \frac{1}{5} \times \frac{s}{s + 1}
$$

Assumption: The intended function is $F(s) = \frac{s}{s + 1}$, or equivalently, $\frac{s}{5(s + 1)}$. We will proceed with this interpretation.

Step 1: Simplify the Expression

Given:

$$
F(s) = \frac{s}{s + 1}
$$

and the scalar factor $\frac{1}{5}$ can be factored out:

$$
F(s) = \frac{1}{5} \times \frac{s}{s + 1}
$$

Step 2: Recognize

[Find The Inverse Laplace Transform Of A F S 10 S S 2 S 3 B F S S S 4s 5 C F S E 3s S S 2 2]

Find The Inverse Laplace Transform Of A F S 10 S S 2 S 3 B F S S S 4s 5 C F S E 3s S S 2 2

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