Find The Limit, If It Exists. (If An Answer Does Not Exist, Enter DNE.) Lim (x, Y)(0, 0) X2yey X4 + 9y2

Find The Limit, If It Exists. (If An Answer Does Not Exist, Enter DNE.) Lim (x, y) ⍰ (0, 0) of (x^2 y e^y) / (x^4 + 9 y^2)

Introduction to Multivariable Limits

Understanding the behavior of functions as variables approach specific points is fundamental in calculus. When dealing with functions of multiple variables, such as $f(x, y)$, the concept of a limit extends beyond single-variable cases. Specifically, evaluating the limit:

$$
\lim_{(x, y) \to (0, 0)} \frac{x^2 y e^y}{x^4 + 9 y^2}
$$

requires analyzing the function's behavior as both x and y approach zero simultaneously. Determining whether this limit exists involves checking if the value approaches the same number regardless of the path taken toward $(0,0)$. If different paths lead to different limit values, then the limit does not exist, and we should report DNE (Does Not Exist).

Understanding the Function and Its Components

Function Components

The function under consideration is:

$$
f(x, y) = \frac{x^2 y e^y}{x^4 + 9 y^2}
$$

Breaking it down:

- Numerator: $x^2 y e^y$

- Contains the polynomial terms $x^2 y$ and the exponential e^y.

- Denominator: $x^4 + 9 y^2$

- Sum of a quartic in x and a quadratic in y.

Behavior Near the Origin

As $(x, y) \to (0, 0)$:

- Both numerator and denominator tend to zero, leading to an indeterminate form $0/0$.

- To evaluate the limit, we need to analyze the dominant behaviors and see if the ratio approaches a finite number or not.

Methodology for Evaluating the Limit

Evaluating multivariable limits involves:

1. Checking the limit along different paths to see if the results are consistent.
2. Applying algebraic manipulations and estimates to understand the function's behavior.
3. Using substitutions or path equations to simplify the analysis.

Step-by-Step Evaluation

1. Limit Along the Path $y = 0$

Let's analyze the limit along the line $y = 0$:

- Substitute $y = 0$:

$$
f(x, 0) = \frac{x^2 \cdot 0 \cdot e^0}{x^4 + 9 \cdot 0^2} = \frac{0}{x^4} = 0
$$

- As $x \to 0$, $f(x, 0) \to 0$.

Result along $y = 0$: Limit is 0.

---

2. Limit Along the Path $x = 0$

Now, analyze along $x = 0$:

- Substitute $x = 0$:

$$
f(0, y) = \frac{0 \cdot y \cdot e^y}{0 + 9 y^2} = 0
$$

- As $y \to 0$, $f(0, y) \to 0$.

Result along $x = 0$: Limit is 0.

---

3. Limit Along the Path $y = k x^2$

To check for path dependence, consider the path $y = k x^2$, where k is a constant.

- Substitute $y = k x^2$:

$$
f(x, k x^2) = \frac{x^2 \cdot (k x^2) \cdot e^{k x^2}}{x^4 + 9 (k x^2)^2} = \frac{k x^4 e^{k x^2}}{x^4 + 9 k^2 x^4} = \frac{k x^4 e^{k x^2}}{x^4 (1 + 9 k^2)} = \frac{k e^{k x^2}}{1 + 9 k^2}
$$

- As $x \to 0$, $e^{k x^2} \to e^{0} = 1$.

- Therefore,

$$
\lim_{x \to 0} f(x, k x^2) = \frac{k \cdot 1}{1 + 9 k^2} = \frac{k}{1 + 9 k^2}
$$

- This limit depends on k.

Implication: Since the limit varies with k, the limit along different quadratic paths depends on the chosen k, meaning the limit does not exist in a well-defined sense if these values differ.

Conclusion on the Existence of the Limit

From the calculations above:

- Along $y=0$, limit is 0.

- Along $x=0$, limit is 0.

- Along $y = k x^2$, the limit is $\frac{k}{1 + 9 k^2}$, which varies with k.

Because the limit depends on the path chosen (different paths yield different results), the overall limit as $(x, y) \to (0, 0)$ does not exist.

Final Answer

Given the path dependence shown above, the limit does not approach a single finite value. Therefore, the answer is:

DNE (Does Not Exist).

Summary and Key Takeaways

- When evaluating multivariable limits, always test multiple paths to check for path dependence.
- If different paths yield different limits, the overall limit does not exist.
- For the given function, various paths demonstrate different limit values, confirming the non-existence of a unique limit.
- Recognizing the behavior of exponential and polynomial terms near zero helps in precise analysis.

Additional Tips for Evaluating Multivariable Limits

- Use substitution paths such as $y = m x$ or $y = k x^2$ to simplify.
- Consider dominant terms in numerator and denominator to estimate the limit.
- If the limit along different paths differs, explicitly state that the limit does not exist.
- Remember that the existence of a limit requires the same value along every path toward the point.

Conclusion

The limit

$$
\lim_{(x, y) \to (0, 0)} \frac{x^2 y e^y}{x^4 + 9 y^2}
$$

does not exist because the value depends on the path taken to approach $(0, 0)$. As such, the correct final answer is DNE.

If you are preparing for calculus exams or working on multivariable calculus problems, understanding how to analyze limits from multiple paths is crucial. Practice with different functions and paths to develop intuition about when limits exist or do not.

Frequently Asked Questions

What is the limit of the function $f(x, y) = x^2 y e^y / (x^4 + 9 y^2)$ as (x, y) approaches $(0, 0)$?

DNE (Does Not Exist)

Does the limit of $f(x, y) = x^2 y e^y / (x^4 + 9 y^2)$ exist at (0, 0)?

No, the limit does not exist.

How can we test whether the limit of $f(x, y) = x^2 y e^y / (x^4 + 9 y^2)$ exists at (0, 0)?

By approaching (0,0) along different paths and checking if the limits are the same.

What is the limit of $f(x, y)$ along the path $y = 0$ as (x, y) approaches (0, 0)?

0

What is the limit of $f(x, y)$ along the path $y = k x^2$ as (x, y) approaches (0, 0)?

0

Why does the limit of $f(x, y) = x^2 y e^y / (x^4 + 9 y^2)$ not exist at (0, 0)?

Because approaching along different paths yields different limit values, indicating the overall limit does not exist.

Is the limit of $f(x, y) = x^2 y e^y / (x^4 + 9 y^2)$ at (0, 0) equal to 0?

No, because the limit depends on the path taken; thus, it does not exist.

Additional Resources

Find The Limit, If It Exists. (If An Answer Does Not Exist, Enter DNE.) Lim (x, y) → (0, 0) of $(x^2 y e^y) / (x^4 + 9 y^2)$

Introduction

In the realm of multivariable calculus, the concept of limits extends the familiar idea from single-variable functions into higher dimensions. Unlike their single-variable counterparts, multivariable limits require careful analysis to determine whether a function approaches a specific value as the variables approach a point from all directions. One common challenge is deciding if the limit exists at a particular point, especially when the function involves complex expressions like exponential functions, polynomials, and ratios.

Today, we examine the limit:

> Find the limit, if it exists, of the function:
>
> \[
> \lim_{(x, y) \to (0, 0)} \frac{x^2 y e^y}{x^4 + 9 y^2}
> \]

If this limit exists, it will provide insight into the behavior of the function near the origin. If it does not, we will conclude with "DNE" (Does Not Exist). This analysis demonstrates the importance of various methods in multivariable calculus, including substitution, path analysis, and comparison techniques.

Understanding the Function

Before delving into calculations, let's analyze the structure of the function:

$$
f(x, y) = \frac{x^2 y e^y}{x^4 + 9 y^2}
$$

Key observations:

- The numerator involves $x^2 y e^y$. Since e^y approaches 1 as $y \to 0$, near the origin, $e^y \approx 1 + y$, but for limit purposes, the exponential's behavior simplifies to its value at zero.
- The denominator is a sum of two terms: x^4 and $9 y^2$. Both tend to zero as $(x, y) \to (0, 0)$.

The challenge lies in the numerator and denominator tending to zero, which suggests an indeterminate form $0/0$. Therefore, standard substitution won't immediately yield the limit; we must analyze the behavior more carefully.

Approaching the Limit: Strategies and Methods

To determine if the limit exists, mathematicians often employ several techniques:

1. Path Analysis

Check the limit along different paths toward $(0, 0)$. If the limits differ along different paths, the overall limit does not exist.

2. Polar Coordinates

Express the variables in polar coordinates:

$$
x = r \cos \theta, \quad y = r \sin \theta
$$

and analyze the limit as $r \to 0$. If the limit depends on θ, then the two-dimensional limit does not exist.

3. Comparison and Bounding

Compare the numerator and denominator to find bounds or dominant terms to infer the behavior as variables tend to zero.

Path Analysis: Testing Specific Directions

Let's investigate the behavior along a few representative paths.

Path 1: $y = 0$

Substitute $y = 0$:

$$
f(x, 0) = \frac{x^2 \cdot 0 \cdot e^0}{x^4 + 0} = 0

\]

Because numerator becomes zero, regardless of \(x\), the limit along this path is:

\[
\lim_{x \to 0} 0 = 0
\]

Path 2: \(x = 0 \)

Substitute \(x = 0 \):

\[
f(0, y) = \frac{0 \cdot y \cdot e^y}{0 + 9 y^2} = 0
\]

Similarly, the limit along this path is zero.

Path 3: \(y = k x^2 \)

Let's choose \(y = k x^2 \), where \(k\) is a constant, to analyze the behavior along a quadratic path:

\[
f(x, k x^2) = \frac{x^2 \cdot k x^2 \cdot e^{k x^2}}{x^4 + 9 (k x^2)^2} = \frac{k x^4 e^{k x^2}}{x^4 + 9 k^2 x^4}
\]

Simplify numerator and denominator:

\[
= \frac{k x^4 e^{k x^2}}{x^4 (1 + 9 k^2)} = \frac{k e^{k x^2}}{1 + 9 k^2}
\]

As \(x \to 0 \), \(e^{k x^2} \to 1 \), so the limit becomes:

\[
\lim_{x \to 0} \frac{k}{1 + 9 k^2}
\]

This value depends on \(k\). For example:

- If \(k = 1\):

\[
\frac{1}{1 + 9} = \frac{1}{10}
\]

- If \(k = 2\):

\[
\frac{2}{1 + 36} = \frac{2}{37}
\]

- If \(k = -1\):

\[
\frac{-1}{1 + 9} = -\frac{1}{10}
\]

Thus, along different quadratic paths, the limit varies depending on k. Since the limit depends on the path (choice of k), the overall two-variable limit does not exist.

Polar Coordinates: Confirming the Path-Dependence

To further verify, express the function in polar coordinates:

$$
x = r \cos \theta, \quad y = r \sin \theta
$$

Substituting into $f(x, y)$:

$$
f(r, \theta) = \frac{(r \cos \theta)^2 (r \sin \theta) e^{r \sin \theta}}{(r \cos \theta)^4 + 9 (r \sin \theta)^2}
$$

Simplify numerator:

$$
r^3 \cos^2 \theta \sin \theta e^{r \sin \theta}
$$

Simplify denominator:

$$
r^4 \cos^4 \theta + 9 r^2 \sin^2 \theta
$$

Expressed as:

$$
f(r, \theta) = \frac{r^3 \cos^2 \theta \sin \theta e^{r \sin \theta}}{r^4 \cos^4 \theta + 9 r^2 \sin^2 \theta}
$$

Divide numerator and denominator by r^2:

$$
f(r, \theta) = \frac{r \cos^2 \theta \sin \theta e^{r \sin \theta}}{r^2 \cos^4 \theta + 9 \sin^2 \theta}
$$

As $r \to 0$, $e^{r \sin \theta} \to 1$. So,

$$
f(r, \theta) \sim \frac{r \cos^2 \theta \sin \theta}{r^2 \cos^4 \theta + 9 \sin^2 \theta}
$$

Now, analyze the behavior depending on θ:

- If $\sin \theta \neq 0$:

$$
f(r, \theta) \sim \frac{r \cos^2 \theta \sin \theta}{r^2 \cos^4 \theta + 9 \sin^2 \theta}
$$

As $r \to 0$, numerator behaves like r, denominator approaches $9 \sin^2 \theta$ (a non-zero constant). Therefore,

$$
f(r, \theta) \to 0
$$

- If $\sin \theta = 0$, i.e., along the x-axis, then:

$$
f(r, 0) = 0
$$

which matches previous path analysis.

Conclusion from polar coordinates:

- Along paths where $\sin \theta \neq 0$, the limit tends to zero.
- However, in the path $y = k x^2$, the limit varies with k.

Since the limits depend on the path taken (e.g., different quadratic paths yield different limits), the overall limit does not exist.

Final Verdict: DNE (Does Not Exist)

Given the thorough analysis, the key insight is that the limit depends on the path approaching $(0, 0)$. Along some paths, the limit tends to zero; along others, it takes on different values. This path dependence confirms that the two-variable limit:

$$
\lim_{(x, y) \to (0, 0)} \frac{x^2 y e^y}{x^4 + 9 y^2}
$$

does not exist.

Summary of Key Points

- The function presents an indeterminate form $0/0$ at $(0,0)$.
- Path analysis reveals the limit varies depending on the approach, especially along quadratic paths $y = k x^2$.
- Polar coordinate analysis supports the conclusion, showing the limit's

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