

Find The P-value For The Following Hypothesis Test. $H_0: \mu = 21$, $H_1: \mu < 21$, $N = 81$, $\bar{X} = 19.25$, $\sigma = 7$ Round

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Understanding how to find the p-value for a hypothesis test is a fundamental skill in statistical analysis. In this article, we will walk through the process of calculating the p-value step-by-step for a specific hypothesis test scenario. The hypothesis test involves testing whether the population mean is less than 21 based on sample data with known population standard deviation. By the end of this guide, you will know exactly how to determine the p-value for similar tests and interpret the results accurately.

Understanding the Components of the Hypothesis Test

Before diving into calculations, it's essential to understand the given data and the structure of the hypothesis test.

Given Data:

- Null hypothesis (H_0): $\mu = 21$
- Alternative hypothesis (H_1): $\mu < 21$
- Sample size (N): 81
- Sample mean ($\bar{X}$): 19.25
- Population standard deviation (σ): 7

Type of Test:

- Since the population standard deviation is known, and the sample size is large ($N=81$), a z-test is appropriate.
- The hypothesis is one-sided (less than), so we will compute a one-tailed p-value.

Step-by-Step Calculation of the P-value

Calculating the p-value involves several steps: computing the test statistic, then finding the probability associated with that statistic under the null hypothesis.

Step 1: State the Null and Alternative Hypotheses

- $H_0: \mu = 21$
- $H_1: \mu < 21$

Step 2: Calculate the Standard Error (SE)

The standard error measures the variability of the sample mean:

$$SE = \frac{\sigma}{\sqrt{N}} = \frac{7}{\sqrt{81}} = \frac{7}{9} \approx 0.7778$$

Step 3: Compute the Z-Score

The z-score indicates how many standard errors the sample mean is from the hypothesized population mean:

$$z = \frac{\bar{X} - \mu_0}{SE} = \frac{19.25 - 21}{0.7778} \approx \frac{-1.75}{0.7778} \approx -2.252$$

Step 4: Find the P-value

Since the alternative hypothesis is $\mu < 21$, we're interested in the probability that Z is less than -2.252.

Using standard normal distribution tables or statistical software:

- The p-value is:

$$p = P(Z < -2.252)$$

Consulting a Z-table or calculator:

- $P(Z < -2.252) \approx 0.0122$

Thus, the p-value for this hypothesis test is approximately 0.012 when rounded to three decimal places.

Interpreting the P-value

Once the p-value is calculated, it needs to be interpreted in the context of the significance level (α), which is typically 0.05.

Decision Rule:

- If $p\text{-value} \leq \alpha$: reject the null hypothesis.
- If $p\text{-value} > \alpha$: fail to reject the null hypothesis.

In this case:

- $p\text{-value} \approx 0.012$
- $\alpha = 0.05$ (common significance level)

Since $0.012 < 0.05$, we reject the null hypothesis, suggesting that there is significant evidence to support the claim that the population mean is less than 21.

Summary of the Process

To summarize, here are the key steps to find the p-value for the hypothesis test:

1. Identify the hypotheses and given data.
2. Calculate the standard error (SE).
3. Compute the z-score using the sample mean, hypothesized mean, and SE.
4. Find the p-value by calculating the probability that Z is less than the observed z-score.
5. Interpret the p-value in relation to the significance level.

Additional Tips for Hypothesis Testing

- Always verify whether to use a z-test or t-test based on whether the population standard deviation is known.
- Remember that for one-sided tests, the p-value is the probability in the tail where the alternative hypothesis indicates.
- Use statistical software or z-tables for accurate p-value calculations, especially when the z-score is not on the table directly.
- When the p-value is small (typically less than 0.05), it provides strong evidence against the null hypothesis.

Conclusion

Calculating the p-value for a hypothesis test is a critical step in statistical inference. In our example, with a sample size of 81, a sample mean of 19.25, and a known population standard deviation of 7, we calculated a z-score of approximately -2.252 and a p-value of about 0.012. This indicates strong evidence to reject the null hypothesis that the population mean is 21 in favor of the alternative that it is less than 21.

By mastering this process, you can confidently analyze similar hypothesis tests and interpret their results, enhancing your statistical analysis skills for research, data analysis, and decision-making.

Remember: Always double-check your calculations and consider the context of your data and hypotheses before drawing conclusions.

Frequently Asked Questions

What is the null hypothesis (H_0) in the given hypothesis test?

$H_0: \mu = 21$

What is the alternative hypothesis (H_1) in the given hypothesis test?

$H_1: \mu < 21$

What is the sample size (N) used in this hypothesis test?

$N = 81$

What is the sample mean ($\bar{X}$) provided in the problem?

$\bar{X} = 19.25$

What is the population standard deviation (σ) given in the problem?

$\sigma = 7$

How do you calculate the test statistic (z) for this hypothesis test?

$$z = (\bar{X} - \mu_0) / (\sigma / \sqrt{N}) = (19.25 - 21) / (7 / \sqrt{81}) = (-1.75) / (7 / 9) = (-1.75) / (0.7778) \approx -2.25$$

What is the approximate p-value for the calculated z-score?

The p-value for $z \approx -2.25$ in a left-tailed test is approximately 0.0122

Based on the p-value, what conclusion can be made about the hypothesis test at $\alpha = 0.05$?

Since the p-value (≈ 0.0122) is less than 0.05, we reject the null hypothesis H_0 in favor of H_1 .

Additional Resources

Find The P-value For The Following Hypothesis Test: A Comprehensive Analytical Review

In the realm of statistical hypothesis testing, the p-value serves as a crucial metric that helps researchers determine the strength of evidence against a null hypothesis. When analyzing a specific scenario—such as testing whether a population mean is less than a hypothesized value—the calculation of the p-value provides an essential basis for decision-making. This article offers a detailed, step-by-step exploration of how to find the p-value for a hypothesis test with the following parameters:

- Null hypothesis, $(H_0: \mu = 21)$
- Alternative hypothesis, $(H_1: \mu < 21)$
- Sample size, $(N = 81)$
- Sample mean, $(\bar{X} = 19.25)$
- Population standard deviation (or known standard deviation), $(\sigma = 7)$

This scenario exemplifies a one-sample z-test for the mean, and the process involves understanding the test's structure, calculating the test statistic, and then deriving the p-value. Let's delve into each component systematically.

Understanding the Hypotheses and Context

The Null and Alternative Hypotheses

In hypothesis testing, the null hypothesis (H_0) represents a statement of no effect or

status quo, while the alternative hypothesis (H_1) reflects the research question or the effect one seeks to detect.

- Null hypothesis ($H_0: \mu = 21$): This states that the population mean is exactly 21.
- Alternative hypothesis ($H_1: \mu < 21$): This indicates an interest in testing whether the mean is less than 21, suggesting a one-tailed test.

This is a left-tailed test because the alternative hypothesis specifies a direction (less than).

Sample Data and Known Parameters

- Sample size (N): 81 — indicating that data were collected from 81 independent observations.
- Sample mean ($\bar{X}$): 19.25 — the average observed in the sample.
- Population standard deviation (σ): 7 — assumed known, which justifies using the z-test.

Knowing the sample size and the population standard deviation allows for precise calculation of the test statistic.

Theoretical Foundations: Z-test for the Mean

When to Use a Z-test

A z-test for the mean is appropriate under the following conditions:

- The population standard deviation (σ) is known.
- The sample size (N) is sufficiently large (typically $N \geq 30$) to invoke the Central Limit Theorem, which ensures the sampling distribution of $\bar{X}$ is approximately normal.

In this case, both conditions are satisfied: $N=81$ (large sample size), and $\sigma=7$ is known.

Test Statistic Formula

The z-test statistic is calculated as:

$$z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{N}}$$

where:

- $\bar{X}$: sample mean
- μ_0 : hypothesized population mean under H_0
- σ : population standard deviation
- N : sample size

This formula measures how many standard errors the sample mean is away from the hypothesized mean.

Calculating the Test Statistic

Plugging in the Values

Using the provided data:

$$z = \frac{19.25 - 21}{7 / \sqrt{81}}$$

Calculate the denominator first:

$$\sigma / \sqrt{N} = 7 / 9 = 0.7778$$

Now, compute z :

$$z = \frac{-1.75}{0.7778} \approx -2.253$$

This z-score indicates that the sample mean is approximately 2.253 standard errors below the hypothesized mean of 21.

Interpreting the Z-Score

A negative z-score aligns with the alternative hypothesis $\mu < 21$. The magnitude suggests the extent of deviation, which will be used to find the p-value.

Finding the P-value: Step-by-Step

Understanding the P-value Concept

The p-value quantifies the probability of observing a test statistic as extreme as, or more extreme than, the one calculated, assuming the null hypothesis is true.

- For a left-tailed test, the p-value is $P(Z \leq z)$.
- It provides a measure of evidence against H_0 . Smaller p-values indicate stronger evidence to reject H_0 .

Using Standard Normal Distribution Tables or Software

To find the p-value corresponding to $(z \approx -2.253)$, one can use:

- Standard normal distribution tables
- Statistical software (e.g., R, Python)
- Scientific calculators

Example using a standard normal table:

Locate $(z = -2.25)$. From the z-table, the cumulative probability $(P(Z \leq -2.25))$ is approximately 0.0122.

Alternatively, using software:

```
```python
import scipy.stats as stats
p_value = stats.norm.cdf(-2.253)
print(p_value)
```
```

which yields approximately 0.0122.

Therefore, the p-value is approximately 0.012.

Interpreting the Results in Context

Significance Level and Decision Making

Suppose the researcher adopts a significance level (α) of 0.05. Comparing the p-value:

- p-value $\approx 0.012 < 0.05$: The evidence against H_0 is strong enough to reject it at the 5% significance level.
- Implication: There is statistically significant evidence to conclude that the true population mean is less than 21.

If the significance level were more stringent, such as 0.01, the p-value of 0.012 would not be sufficient for rejection, and the null hypothesis would not be rejected at that level.

Practical Significance and Limitations

While statistical significance indicates that the observed difference is unlikely due to random sampling variability, practical significance considers whether the magnitude of the difference is meaningful in real-world terms. A mean of 19.25 versus 21 could have different implications depending on the context (e.g., quality control, medical testing).

Limitations to consider include:

- The assumption that σ is known; in many real-world scenarios, σ is estimated from the sample, leading to a t-test.
- The normality assumption for the sample mean distribution, though justified here due to the large sample size.

Summary and Final Remarks

This comprehensive review underscores the importance of understanding each step involved in calculating the p-value for a hypothesis test. Starting from the hypotheses, through the calculation of the z-statistic, and finally to the derivation of the p-value, each component plays a vital role in statistical inference.

To recap:

1. Define hypotheses: $H_0: \mu=21$ vs. $H_1: \mu<21$.
2. Calculate the z-statistic:

$$z = \frac{19.25 - 21}{7 / \sqrt{81}} \approx -2.253$$

3. Find the p-value:

$$p \approx P(Z \leq -2.253) \approx 0.012$$

\]

4. Interpretation: At $(\alpha = 0.05)$, the p-value indicates strong evidence against (H_0) , leading to rejection of the null hypothesis in favor of the alternative.

This process exemplifies how statistical tools enable researchers to make informed decisions based on sample data, with the p-value serving as a pivotal metric in hypothesis testing.

In conclusion, the p-value for the given hypothesis test is approximately 0.012, providing compelling evidence that the true population mean is less than 21 at commonly used significance levels. This analytical approach highlights the essential steps and considerations when conducting hypothesis tests, fostering a deeper understanding of statistical inference.

Note: Always ensure data meets the assumptions of the test used and interpret p-values within the broader context of the research question and real-world implications.

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