Find The Quadratic Function Y = F(x) That Has The Given Vertex And Whose Graph Passes Through The Given

Find The Quadratic Function Y = F(x) That Has The Given Vertex And Whose Graph Passes Through The Given

Understanding how to determine a quadratic function given specific properties such as a vertex and a point through which its graph passes is fundamental in algebra and coordinate geometry. This process involves applying the vertex form of a quadratic equation, which provides a clear pathway to derive the function based on the given information. Whether you're a student preparing for exams, a teacher designing lesson plans, or a math enthusiast seeking to deepen your understanding, mastering this technique is essential for solving various quadratic problems efficiently.

In this comprehensive guide, we will explore the step-by-step process to find the quadratic function $y = f(x)$ with a specified vertex and a point through which the parabola passes. We will also discuss common mistakes, practical tips, and real-world applications to ensure a well-rounded understanding.

Understanding the Basics of Quadratic Functions

Before diving into the step-by-step process, it's important to revisit some fundamental concepts related to quadratic functions.

The Standard Form of a Quadratic Equation

The most common form of a quadratic function is:

$$y = ax^2 + bx + c$$

where:
- $a \neq 0$,
- b and c are real numbers.

While this form is useful, it doesn't directly provide the vertex of the parabola, making it less convenient when the vertex is known.

The Vertex Form of a Quadratic Equation

The vertex form is particularly useful when the vertex (h, k) is known:

$$y = a(x - h)^2 + k$$

Here:
- (h, k) is the vertex of the parabola,
- a determines the opening and the 'width' of the parabola.

This form makes it straightforward to write the quadratic function once the vertex and another point are known.

Step-by-Step Process to Find the Quadratic Function

The key steps involve using the vertex form and substituting known values to determine the parameter a.

1. Write the General Form Using the Vertex

Begin by expressing the quadratic in vertex form:

$$y = a(x - h)^2 + k$$

where:
- (h, k) is the given vertex.

2. Substitute the Known Vertex

Insert the coordinates of the vertex (h, k) into the equation. Since these are the coordinates of the vertex, the equation simplifies as:

$$y = a(x - h)^2 + k$$

with h and k known.

3. Use the Given Point to Solve for a

Suppose the parabola passes through a point (x_1, y_1). Substitute this point into the equation:

$$y_1 = a(x_1 - h)^2 + k$$

Now, solve for a:

$$ a = \frac{y_1 - k}{(x_1 - h)^2} $$

This calculation yields the value of a, which determines the parabola's width and direction.

4. Write the Final Equation

With a known, the quadratic function becomes:

$$ y = a(x - h)^2 + k $$

This is the desired quadratic function passing through the given point and having the specified vertex.

Worked Example: Finding the Quadratic Function

Let's apply this process with a concrete example for clarity.

Given:

- Vertex: $(3, 2)$
- Passes through: $(5, 6)$

Step 1: Write the Vertex Form

$$ y = a(x - 3)^2 + 2 $$

Step 2: Substitute the Point $(5, 6)$

$$ 6 = a(5 - 3)^2 + 2 $$
$$ 6 = a(2)^2 + 2 $$
$$ 6 = 4a + 2 $$

Step 3: Solve for a

$$ 4a = 6 - 2 $$
$$ 4a = 4 $$
$$ a = 1 $$

Step 4: Write the Final Equation

$$ y = (x - 3)^2 + 2 $$

This quadratic function has the vertex at $(3, 2)$ and passes through $(5, 6)$.

Additional Considerations and Tips

While the above method is straightforward, several additional factors can influence how you approach similar problems.

1. Multiple Points and Different Quadratic Forms

- If multiple points are given, verify the consistency of the data.
- Sometimes, the parabola may be wider or narrower depending on the value of a.

2. When the Vertex Is Not Known

- You may need to find the vertex first using other methods such as completing the square or analyzing the symmetry of the parabola.

3. Parabolas Opening Up or Down

- The sign of a determines the opening:
- $a > 0$: parabola opens upward.
- $a < 0$: parabola opens downward.

4. Verifying the Solution

- Always substitute the given point back into your derived equation to ensure correctness.

Common Mistakes to Avoid

- Incorrectly substituting the vertex: Remember that the vertex form is $y = a(x - h)^2 + k$ and not $y = a(x + h)^2 + k$.
- Miscalculating a: Be cautious with the algebraic steps, especially

when squaring or simplifying.
- Using the wrong point: Ensure the point used to solve for a actually lies on the parabola.
- Ignoring the sign of a: The direction of the parabola depends on the sign of a.

Practical Applications of Finding Quadratic Functions

Understanding how to find quadratic functions from given points and vertices has numerous real-world applications, such as:

- Projectile motion: Modeling the path of an object under gravity.
- Engineering design: Designing parabolic antennas or bridges.
- Economics: Analyzing profit functions that are quadratic in nature.
- Physics: Describing the trajectory of particles or projectiles.

Conclusion

Finding the quadratic function $y = f(x)$ that has a given vertex and passes through a specific point is a fundamental skill in algebra that combines understanding of the vertex form with algebraic manipulation. By following the structured steps—writing the function in vertex form, substituting known values, solving for the parameter a, and verifying your results—you can accurately determine the quadratic equation for a wide range of problems.

Mastering this technique enhances problem-solving skills, deepens understanding of quadratic functions, and provides a solid foundation for tackling more complex mathematical concepts or real-world challenges involving parabolas.

Keywords for SEO
- Find quadratic function given vertex and point
- Quadratic function from vertex and passing point
- How to derive quadratic equation from given data
- Vertex form of quadratic function
- Solving quadratic functions with known vertex
- Quadratic equations passing through specific points
- Step-by-step guide to quadratic functions

- Algebraic methods for quadratic functions

Frequently Asked Questions

How do I find the quadratic function y = f(x) given a vertex and a point it passes through?

To find the quadratic function, start with the vertex form $y = a(x - h)^2 + k$, where (h, k) is the vertex. Plug in the coordinates of the passing point to solve for 'a', then write the complete quadratic function.

What is the vertex form of a quadratic function, and why is it useful for this problem?

The vertex form is $y = a(x - h)^2 + k$, which directly incorporates the vertex (h, k). It simplifies finding the quadratic equation when the vertex and a passing point are known because you only need to determine the value of 'a'.

If the vertex is at (3, -2) and the graph passes through (5, 2), how do I determine the quadratic function?

Start with $y = a(x - 3)^2 - 2$. Substitute the point (5, 2): $2 = a(5 - 3)^2 - 2$. Simplify: $2 = a(2)^2 - 2$, so $2 = 4a - 2$. Add 2 to both sides: $4 = 4a$, then divide both sides by 4: $a = 1$. The quadratic is $y = (x - 3)^2 - 2$.

Can I use the standard form y = ax^2 + bx + c instead of vertex form when given the vertex and passing point?

Yes, but it requires converting the vertex form to standard form after finding the value of 'a'. It's often easier to work in vertex form initially, then expand to standard form if needed.

Why is it important to verify the quadratic function after finding it using the given vertex and point?

Verifying ensures that the found quadratic correctly passes through the given point and has the specified vertex. Substitute the point into your equation to confirm accuracy and avoid errors in calculation.

Additional Resources

Quadratic Function: Unlocking the Secrets of Parabolic Equations with Prescribed Vertices and Points

---

Introduction: The Significance of Quadratic Functions in Mathematics and Real-World Applications

Quadratic functions are foundational elements in algebra, offering a powerful way to model a wide array of phenomena—from projectile trajectories and satellite orbits to economic cost functions and biological growth patterns. Their characteristic parabolic shape makes them both visually intuitive and mathematically rich, capable of describing complex relationships with elegance and precision.

One of the most intriguing challenges in working with quadratic functions is determining the specific quadratic equation that fits a particular set of criteria. Often, this involves finding a quadratic function that possesses a predefined vertex and passes through a specific point. Whether you're a student tackling homework, an educator designing problems, or a professional modeling data, mastering this process is essential.

In this comprehensive article, we explore how to find the quadratic function $y = f(x)$ given its vertex and a point it passes through. We will break down the conceptual underpinnings, provide step-by-step methods, and discuss practical tips, making this a definitive guide for learners and practitioners alike.

Understanding the Core Concepts

What is a Quadratic Function?

A quadratic function has the general form:

$$
y = ax^2 + bx + c
$$

where:
- $a \neq 0$,
- b and c are real numbers.

This form produces a parabola when graphed on the Cartesian plane, with the shape and position influenced by the coefficients a, b, and c.

The Vertex of a Parabola

The vertex is the highest or lowest point on the parabola, depending on whether it opens upward ($a > 0$) or downward ($a < 0$). The vertex's coordinates are given by:

$$
V = (h, k)
$$

where:

$$
h = -\frac{b}{2a}
$$

and

$$
k = f(h) = c - \frac{b^2}{4a}
$$

Alternatively, the vertex form of a quadratic function emphasizes the vertex explicitly:

$$
y = a(x - h)^2 + k
$$

This form simplifies the process of incorporating known vertex information.

The Problem: Given a Vertex and a Point, Find the Quadratic Function

Suppose you're provided with:
- The vertex $V = (h, k)$,
- A point $P = (x_0, y_0)$ through which the parabola passes.

Your task is to determine the specific quadratic function $y = f(x)$ that:

1. Has the vertex at (h, k),
2. Passes through (x_0, y_0).

This problem appears straightforward but requires careful application of algebraic principles and understanding of parabola properties.

Methodology: Step-by-Step Approach

Step 1: Write the Function in Vertex Form

Since the vertex is known, the most direct route is to express the quadratic in vertex form:

$$
y = a(x - h)^2 + k
$$

This form is advantageous because the vertex directly informs the structure, and the only unknown is the coefficient a.

Step 2: Use the Known Point to Find a

Substitute the coordinates of the point (x_0, y_0) into the vertex form:

$$
y_0 = a(x_0 - h)^2 + k
$$

Solve for a:

$$
a = \frac{y_0 - k}{(x_0 - h)^2}
$$

Important considerations:
- Ensure $x_0 \neq h$ to avoid division by zero.
- The sign of a determines the parabola's opening direction and whether the point lies above or below the vertex.

Step 3: Write the Complete Equation

With a determined, the quadratic function is:

$$
\boxed{
y = \frac{y_0 - k}{(x_0 - h)^2} (x - h)^2 + k
}
$$

This explicit formula provides the quadratic function that satisfies both conditions.

Practical Example: Applying the Method

Suppose:
- Vertex $V = (2, 3)$,
- Point $P = (4, 7)$.

Step 1: Write in vertex form:

$$
y = a(x - 2)^2 + 3
$$

Step 2: Substitute point $(4, 7)$:

$$
7 = a(4 - 2)^2 + 3
$$

$$
7 = a(2)^2 + 3
$$

$$
7 = 4a + 3
$$

Step 3: Solve for a:

$$
4a = 7 - 3 = 4
$$

$$
a = 1
$$

Final Equation:

$$
\boxed{
y = (x - 2)^2 + 3
}
$$

This quadratic opens upward, has a vertex at $(2, 3)$, and passes through $(4, 7)$.

Extending the Approach: Converting to Standard Form

While the vertex form is most intuitive when the vertex is known, sometimes the standard quadratic form $y = ax^2 + bx + c$ is required.

Conversion process:

1. Expand the vertex form:

$$
y = a(x - h)^2 + k

$$
= a(x^2 - 2hx + h^2) + k
$$

$$
= ax^2 - 2ahx + a h^2 + k
$$

2. Identify coefficients:

$$
b = -2a h
$$

$$
c = a h^2 + k
$$

3. Substitute the value of a:

$$
b = -2a h
$$

$$
c = a h^2 + k
$$

In the previous example:

$$
a = 1, \quad h = 2, \quad k = 3
$$

then:

$$
b = -2 \times 1 \times 2 = -4
$$

$$
c = 1 \times 2^2 + 3 = 4 + 3 = 7
$$

Standard form:

$$
\boxed{
y = x^2 - 4x + 7
}
$$

Additional Considerations and Tips

Handling Special Cases

- When $x_0 = h$:
The point coincides with the vertex's vertical line. In this case, the point is at the vertex itself, and the quadratic function's equation is fully determined by the vertex alone, with a remaining as a free parameter. For example, any parabola with vertex (h, k) passing exactly through (h, y) satisfies:

$$
y = a(0)^2 + k = k
$$

So, if the point is exactly at the vertex, the quadratic is:

$$
y = a(x - h)^2 + k
$$

with a arbitrary, indicating infinitely many solutions unless additional constraints are provided.

- Ensuring the parabola passes through the point:
Always verify the computed a makes sense in context, especially when dealing with real-world data or constraints.

Using Graphing Tools and Software

Modern graphing calculators and software like Desmos, GeoGebra, or WolframAlpha can visualize the parabola once the vertex and a point are inputted. This can serve as a valuable validation step.

Applications of This Method

- Trajectory modeling: Calculating the path of a projectile given its highest point and a point it reaches.
- Economic modeling: Finding cost functions that reach a minimum or maximum at a specific point.
- Design and engineering: Creating curves with prescribed peaks and points for structural integrity or aesthetic purposes.

Conclusion: Mastering the Art of Quadratic Reconstruction

Finding the quadratic function $y = f(x)$ with a known vertex and passing through a specific point is a fundamental skill, blending algebraic manipulation with geometric intuition. The process hinges on recognizing the utility of the vertex form, which makes the known vertex explicit, and then leveraging substitution to find the unknown coefficient \(a \).

By following these steps—formulating the parabola in vertex form, substituting the known point, solving for \(a \), and optionally converting to standard form—you can confidently determine the precise quadratic equation fitting your criteria. Whether for academic exercises, real-world modeling, or creative design, this method equips you with a robust tool to decode and craft parabolic relationships with clarity and precision.

Remember, mastery comes with practice. Experiment with different vertices and points, verify your solutions graphically, and explore the rich geometry underpinning quadratic functions. With these skills, you'll navigate the world of parabolas with confidence, turning abstract equations into concrete visualizations and practical solutions.

Find The Quadratic Function Y F X That Has The Given Vertex And Whose Graph Passes Through The Given

Find The Quadratic Function Y F X That Has The Given Vertex And Whose Graph Passes Through The Given

Related Articles

- Consider The Following Sample Of Five Measurements. 5,1,4,0,6 A. Calculate The Range, S 2, And S. Range
- Consider The Standard Solow Model Of Economic Growth.Production Is Described By The Following Function:
- Holtzman Clothiers's Stock Currently Sells For $40.00 A Share. It Just Paid A Dividend Of $3.25 A Share

Back to Home