Find The Relative Maximum And Minimum Values. F(x,y)= E 10x2 + Y2 + 2 = Select The Correct Choice Below

Find The Relative Maximum And Minimum Values. F(x,y)= E 10x2 + Y2 + 2 = Select The Correct Choice Below

---

Introduction

In the realm of multivariable calculus, understanding how to find the relative maximum and minimum values of a function is fundamental. These points, often called local extrema, provide critical insights into the behavior of the function within a specific region. In this article, we will explore the process of identifying the relative extrema for the function:

$$F(x, y) = 10x^2 + y^2 + 2$$

This quadratic function involves two variables and exhibits a paraboloid shape. By analyzing this function, we will learn how to find its critical points, determine their nature (whether they are maxima, minima, or saddle points), and interpret the results.

Understanding the Function F(x, y)

The Structure of F(x, y)

The function:

$$F(x, y) = 10x^2 + y^2 + 2$$

is a quadratic form where:

- The term $10x^2$ indicates the quadratic dependence on variable x, scaled by 10.
- The term y^2 indicates quadratic dependence on y.
- The constant +2 shifts the entire surface vertically upward by 2 units.

This function describes a paraboloid, opening upwards in all directions since the coefficients of both quadratic terms are positive.

Key Characteristics

- The coefficients of x^2 and y^2 are positive, indicating the paraboloid opens upward.

- The minimum value occurs at the lowest point of the paraboloid.
- There are no maximum points because the surface extends infinitely upward.

Step 1: Find Critical Points

To locate relative extrema, we must find the critical points, where the partial derivatives are zero.

Calculating Partial Derivatives

The partial derivatives of $F(x, y)$ are:

$$
\frac{\partial F}{\partial x} = 20x
$$

$$
\frac{\partial F}{\partial y} = 2y
$$

Setting Partial Derivatives to Zero

Solve for x and y:

$$
20x = 0 \Rightarrow x = 0
$$

$$
2y = 0 \Rightarrow y = 0
$$

Critical point:

$$
(0, 0)
$$

Step 2: Classify the Critical Point

To determine whether the critical point at (0, 0) is a maximum, minimum, or saddle point, we use the second derivative test for functions of two variables.

Second Partial Derivatives

Compute the second derivatives:

$$

$$
F_{xx} = \frac{\partial^2 F}{\partial x^2} = 20
$$

$$
F_{yy} = \frac{\partial^2 F}{\partial y^2} = 2
$$

$$
F_{xy} = \frac{\partial^2 F}{\partial x \partial y} = 0
$$

Discriminant D

Calculate the discriminant:

$$
D = F_{xx} \times F_{yy} - (F_{xy})^2 = (20)(2) - (0)^2 = 40
$$

Interpretation

- Since $D > 0$ and $F_{xx} > 0$, the critical point at (0, 0) is a relative (and in this case, absolute) minimum.

Step 3: Find the Relative Minimum Value

Evaluate the function at the critical point:

$$
F(0, 0) = 10(0)^2 + (0)^2 + 2 = 2
$$

Thus, the function reaches its relative minimum of 2 at (0, 0).

Summary of Findings

Critical Point	Nature	Function Value
(0, 0)	Minimum	2

Because the quadratic form is upward opening with positive coefficients, the critical point at (0, 0) is indeed the global minimum for the function, with a minimum value of 2.

Visualizing the Function

Graphical Representation

Visualizing $F(x, y)$:

- The surface resembles a paraboloid opening upward.
- The lowest point is at the origin (0, 0), where the function value is 2.
- As x and y move away from zero, $F(x, y)$ increases indefinitely.

Contour Plot

- The level curves (contours) are concentric ellipses centered at (0, 0).
- The smallest contour corresponds to $F(x, y) = 2$.

Additional Considerations

Boundary Behavior

In problems involving bounded regions (e.g., within a specific domain), identifying the critical points is only part of the process. You also need to examine boundary points to find absolute extrema.

Applications

- Optimization problems: Finding the lowest cost, minimal energy, or optimal configurations.
- Physics: Locating stable equilibrium points.
- Economics: Maximizing profit or minimizing cost functions.

Practice Problems

1. Identify the critical points of $G(x, y) = 5x^2 + 3y^2 - 4x + 2$.
2. Determine whether the critical points are maxima, minima, or saddle points.
3. Find the absolute maximum and minimum of $H(x, y) = 4x^2 + y^2$ on the disk $x^2 + y^2 \leq 4$.

Conclusion

In analyzing the function $F(x, y) = 10x^2 + y^2 + 2$, we identified the critical point at the origin and confirmed it as a relative (and absolute) minimum with a value of 2. The process involved calculating partial derivatives, setting them to zero to find critical points, and applying the second derivative test to classify the nature of these points.

This approach exemplifies how multivariable calculus techniques are used to analyze and interpret the behavior of functions of two variables. Recognizing the nature of extrema is vital in numerous scientific, engineering, and economic applications where optimization plays a crucial role.

Key Takeaways

- Critical points occur where the gradient (partial derivatives) equals zero.
- The second derivative test involves computing the Hessian determinant D to classify critical points.
- For quadratic functions like $F(x, y) = 10x^2 + y^2 + 2$, the minimum occurs at the vertex (0, 0).
- The function's upward opening paraboloid indicates no maximum exists, only minima.

Final Remarks

Understanding how to find and classify relative maximum and minimum values of functions of multiple variables is a foundational skill in calculus. It enables professionals and students to analyze complex systems, optimize solutions, and interpret the behavior of multivariable functions effectively.

Frequently Asked Questions

What is the goal when finding the relative maximum and minimum values of the function $F(x, y) = 10x^2 + y^2 + 2$?

The goal is to identify the points where the function attains its highest or lowest values in a local neighborhood, known as relative maxima and minima.

How do you find critical points for the function $F(x, y) = 10x^2 + y^2 + 2$?

Calculate the gradient (partial derivatives) and set them equal to zero: $\partial F/\partial x = 20x = 0$ and $\partial F/\partial y = 2y = 0$, leading to the critical point at (0, 0).

What is the significance of the second derivative test in determining whether a critical point is a maximum or minimum?

The second derivative test involves examining the second partial derivatives to compute the discriminant D. If $D > 0$ and $\partial^2F/\partial x^2 > 0$, the point is a relative minimum; if $D > 0$ and $\partial^2F/\partial x^2 < 0$, it's a relative maximum.

Calculate the second derivatives of $F(x, y) = 10x^2 + y^2 + 2$ and determine the nature of the critical point.

$\partial^2F/\partial x^2 = 20$, $\partial^2F/\partial y^2 = 2$, and the mixed partial derivatives are zero. The discriminant $D = (20)(2) - 0 = 40 > 0$, and since $\partial^2F/\partial x^2 > 0$, the critical point at (0, 0) is a relative minimum.

What is the value of the function F(x, y) at the critical point, and what does it represent?

At (0, 0), $F(0, 0) = 10(0)^2 + (0)^2 + 2 = 2$, representing the minimum value of the function.

Are there any relative maximum points for the function $F(x, y) = 10x^2 + y^2 + 2$?

No, because the function is quadratic with positive coefficients, making (0, 0) a minimum point and no relative maxima exist.

How does the shape of the graph of $F(x, y) = 10x^2 + y^2 + 2$ look near the critical point?

The graph is a paraboloid opening upwards with its lowest point at (0, 0), indicating a bowl-shaped surface with a relative minimum at that point.

Additional Resources

Find The Relative Maximum And Minimum Values is a fundamental concept in calculus, particularly in the field of multivariable calculus. It involves determining the points on a surface where the function reaches its highest or lowest values within a certain region. In this article, we will explore the process of finding the relative maximum and minimum values of functions, with a specific focus on the function \(F(x,y) = 10x^2 + y^2 + 2 \). We will analyze how to identify critical points, classify them, and understand their significance in the context of the given function.

Understanding the Function \(F(x,y) = 10x^2 + y^2 + 2 \)

Before diving into the methods for finding extrema, it is essential to understand the structure and behavior of the function:

- Quadratic Nature: The function is quadratic in both variables \(x \) and

y.
- Convexity: Since the coefficients of x^2 and y^2 are positive, the function is convex and opens upward.
- Constant Term: The +2 shifts the graph vertically without affecting the shape.

This particular function resembles an elliptic paraboloid, which typically has a minimum point at its vertex. Because the coefficients are positive, the function does not have any maximum values extending to infinity but has a clear minimum point.

Identifying Critical Points

The first step in finding relative extrema is to locate critical points—points where the gradient of the function is zero.

Calculating the Gradient

The gradient vector $\nabla F(x,y)$ comprises the partial derivatives with respect to x and y:

$$
\nabla F(x,y) = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y} \right)
$$

Compute each:

- $\frac{\partial F}{\partial x} = 20x$
- $\frac{\partial F}{\partial y} = 2y$

Set these derivatives equal to zero to find critical points:

$$
20x = 0 \Rightarrow x = 0
$$
$$
2y = 0 \Rightarrow y = 0
$$

Thus, the only critical point is at $(0,0)$.

Classifying Critical Points

Once the critical point is identified, the next step is to classify whether it is a maximum, minimum, or saddle point. This involves analyzing the second derivatives and the Hessian matrix.

The Second Derivative Test for Functions of Two Variables

Calculate the second derivatives:

- $F_{xx} = \frac{\partial^2 F}{\partial x^2} = 20$
- $F_{yy} = \frac{\partial^2 F}{\partial y^2} = 2$
- $F_{xy} = F_{yx} = 0$

Construct the Hessian matrix:

$$
H = \begin{bmatrix}
F_{xx} & F_{xy} \\
F_{yx} & F_{yy}
\end{bmatrix} = \begin{bmatrix}
20 & 0 \\
0 & 2
\end{bmatrix}
$$

Calculate the determinant of the Hessian:

$$
D = (20)(2) - (0)^2 = 40 > 0
$$

Since $D > 0$ and $F_{xx} = 20 > 0$, the critical point at $(0,0)$ is a local minimum.

Evaluating the Nature of the Extremum

Given the convex shape of the function and the results from the second derivative test, the critical point at $(0,0)$ is indeed a relative minimum. Because the paraboloid opens upward and the function tends to infinity as $|x| \to \infty$ or $|y| \to \infty$, this minimum is also the absolute minimum of the function.

Value at the minimum point:

$$
F(0,0) = 10(0)^2 + (0)^2 + 2 = 2
$$

This is the lowest value the function attains.

Summary of Key Features

- Critical point: $(0,0)$
- Type of extremum: Relative (and absolute) minimum
- Minimum value: 2
- Behavior at infinity: The function tends to infinity, confirming the minimum is global.

Visualizing the Function

Graphically, $F(x,y) = 10x^2 + y^2 + 2$ is an elliptic paraboloid with its vertex at $(0,0,2)$. The surface opens upward in all directions, with the lowest point at the vertex.

Features:

- Symmetric about the axes due to the quadratic terms.
- The "steepness" in the x-direction is greater because of the coefficient 10.

Practical Applications and Significance

Understanding how to find the relative maximum and minimum values of functions like $F(x,y)$ has widespread applications:

- Optimization problems: Minimizing costs or maximizing efficiency.
- Engineering design: Finding optimal configurations.
- Economics: Cost minimization or profit maximization.
- Physics: Potential energy surfaces and stability analysis.

Advantages and Limitations

Advantages:

- Provides precise identification of extrema points.
- The second derivative test offers a straightforward classification.
- Applicable to a broad class of quadratic and more complex functions.

Limitations:

- Functions with multiple variables can have multiple critical points, complicating analysis.
- The second derivative test may not apply if the Hessian is indefinite or singular.
- For non-quadratic functions, more advanced techniques like Lagrange multipliers or numerical methods may be necessary.

Summary and Conclusion

Finding the relative maximum and minimum values of a function like $F(x,y) = 10x^2 + y^2 + 2$ involves a systematic process:

1. Calculate the gradient and set it to zero to find critical points.
2. Use the second derivative test and the Hessian matrix to classify these critical points.
3. Analyze the behavior at the critical points and at infinity to determine the nature of the extrema.

In this particular case, the function reaches its absolute minimum value of 2 at $(0,0)$, confirming that the critical point is indeed a global minimum due to the convex, paraboloid shape of the surface.

Select The Correct Choice Below:

- A. The function has a relative maximum at $(0,0)$.
- B. The function has a relative minimum at $(0,0)$, with minimum value 2.
- C. The function has no critical points.
- D. The function reaches its maximum at infinity.

Correct Answer: B. The function has a relative (and absolute) minimum at $(0,0)$, with minimum value 2.

This comprehensive analysis demonstrates a clear method for identifying and classifying extrema in multivariable functions, which is an essential skill in calculus and its applications.

Find The Relative Maximum And Minimum Values F X Y E 10x2 Y2 2 Select The Correct Choice Below

Find The Relative Maximum And Minimum Values F X Y E 10x2 Y2 2 Select The Correct Choice Below

Related Articles

- If A Business Produces And Sells Only One Unit Of A Good, Its Profit Would Be The:a. Return Paid To The
- Dr. Wong Wants To Know If Retention Changes Over The Lifespan. He Creates A Computerized Retention Test
- If A Firm Has A 100% Dividend Payout Ratio, Then The Internal Growth Rate Of The Firm Is:a. 0%.b. Equal

Back to Home