For Taking Bath, Water At 40C Is Required. Calculate The Mass Of Cold Water At 15C Which Is To Be Added

For Taking Bath, Water At 40°C Is Required. Calculate The Mass Of Cold Water At 15°C Which Is To Be Added

When preparing for a comfortable bath, it is essential to achieve the desired water temperature. Typically, a bath involves water heated to around 40°C, which offers a soothing and safe temperature for most individuals. However, in practical scenarios, the water source may not be at the ideal temperature, or there might be a need to adjust the temperature by mixing hot water with cold water. This leads us to an important problem: Given a certain volume of hot water at 40°C, how much cold water at 15°C should be added to reach the desired bath temperature?

This article explores the process of calculating the mass of cold water required to mix with hot water to attain a specific temperature, emphasizing the principles of heat transfer and conservation of energy. Whether you're a student studying thermodynamics or someone interested in practical applications of heat calculations, this detailed guide aims to clarify the concepts and provide step-by-step solutions.

Understanding the Basics of Heat Transfer and Temperature Mixing

Before diving into calculations, it is vital to understand the fundamental principles involved:

Heat Transfer and Conservation of Energy

- When two bodies at different temperatures are mixed, heat flows from the hotter to the colder until thermal equilibrium is reached.
- The principle of conservation of energy states that the heat lost by hot water equals the heat gained by cold water (assuming no heat loss to surroundings).

Key Concepts and Assumptions

- Specific heat capacity (c): The amount of heat needed to raise the temperature of 1 kg of a substance by 1°C.
- For water, c ≈ 4200 J/(kg·°C).
- Mass (m): The amount of water involved in the process, measured in kilograms (kg).
- Temperatures:
- Hot water temperature, T_hot = 40°C
- Cold water temperature, T_cold = 15°C
- Final bathing temperature, T_final (desired temperature, e.g., 40°C or a temperature close to it)

Setting Up the Problem: Known and Unknown Variables

Suppose:

- Mass of hot water, m_hot (kg), at temperature T_hot = 40°C
- Mass of cold water, m_cold (kg), at temperature T_cold = 15°C
- Final temperature after mixing, T_final = 40°C (assuming the desired bath temperature)

Objective: Calculate m_cold, the mass of cold water at 15°C to be added to the hot water to achieve the desired temperature.

Mathematical Formulation of the Problem

The heat gained or lost during mixing can be described as:

Heat gained or lost by water = mass × specific heat capacity × change in temperature

Since the specific heat capacity of water is constant and the same for hot and cold water, the heat transfer equation simplifies to:

$$ m_{hot} \times c \times (T_{hot} - T_{final}) = m_{cold} \times c \times (T_{final} - T_{cold}) $$

Note:

- The hot water cools down from T_{hot} to T_{final}.
- The cold water warms up from T_{cold} to T_{final}.

Rearranged:

$$ m_{hot} \times (T_{hot} - T_{final}) = m_{cold} \times (T_{final} - T_{cold}) $$

Since c appears on both sides, it cancels out:

$$ m_{hot} \times (T_{hot} - T_{final}) = m_{cold} \times (T_{final} - T_{cold}) $$

This equation allows us to solve for m_{cold}, given m_{hot} and the temperatures.

Step-by-Step Calculation

Let's consider a practical example:

- Suppose you have 2 kg of hot water at 40°C.
- You want the bath temperature to be 40°C (for simplicity, assume initial hot water is at 40°C, and we are adding cold water at 15°C).

Given:

- m_{hot} = 2 kg
- T_{hot} = 40°C
- T_{cold} = 15°C
- T_{final} = 40°C

However, in this case, since the hot water is already at 40°C, adding cold water will lower the temperature. To maintain the same temperature (40°C), no cold water need be added, which is trivial.

But in a typical scenario, let's assume:

- Hot water at 40°C
- Desired bath temperature T_{final} = 40°C (no change)
- Cold water at 15°C needs to be added to cold the hot water down to 40°C, which is not necessary here.

Instead, consider a more realistic scenario:

Scenario: You have hot water at 50°C, and you want to cool it to 40°C by adding cold water at 15°C.

Knowns:

- $m_{hot} = 3$ kg
- $T_{hot} = 50°C$
- $T_{cold} = 15°C$
- $T_{final} = 40°C$

Calculation:

Using the formula:

$$ m_{cold} = m_{hot} \times \frac{T_{hot} - T_{final}}{T_{final} - T_{cold}} $$

Plug in the values:

$$ m_{cold} = 3 \times \frac{50 - 40}{40 - 15} = 3 \times \frac{10}{25} = 3 \times 0.4 = 1.2\, \text{kg} $$

Result: To cool 3 kg of water from 50°C to 40°C, you need to add approximately 1.2 kg of cold water at 15°C.

General Formula for Different Conditions

The derived formula can be generalized as:

$$
\boxed{
m_{cold} = m_{hot} \times \frac{T_{hot} - T_{final}}{T_{final} - T_{cold}}
}
$$

Where:

- m_{hot} = initial mass of hot water
- T_{hot} = initial hot water temperature
- T_{cold} = cold water temperature
- T_{final} = desired final temperature after mixing

Note: The formula assumes perfect mixing, no heat loss to surroundings, and equal specific heat capacities.

Practical Applications and Considerations

While the above calculations provide theoretical values, practical considerations include:

- Heat Loss: In real-world scenarios, some heat may be lost to the environment, requiring slightly more cold water.
- Temperature Accuracy: Precise measurement of water temperatures ensures correct calculations.
- Volume Limitations: The total volume of water may be constrained by the size of the bathtub or container.
- Safety Precautions: Avoid adding very cold water to hot water suddenly to prevent scalding or thermal shock.

Additional Examples and Problem-Solving Strategies

Example 1: Heating Cold Water

Suppose you wish to warm cold water from 15°C to 40°C using hot water at 50°C. How much hot water do you need to add?

Rearranged formula:

$$
m_{hot} = m_{cold} \times \frac{T_{final} - T_{cold}}{T_{hot} - T_{final}}
$$

Example 2: Adjusting Bath Temperature

If you start with 4 kg of hot water at 45°C and want to reduce it to 40°C, how much cold water at 15°C should be added?

Applying the formula:

$$
m_{cold} = 4 \times \frac{45 - 40}{40 - 15} = 4 \times \frac{5}{25} = 4 \times 0.2 = 0.8\, \text{kg}
$$

Conclusion

Calculating the mass of cold water needed to achieve a desired bath temperature involves understanding heat transfer principles and applying a straightforward formula derived from the conservation of energy. By knowing the initial hot water temperature, the cold water temperature, and the target temperature, you can determine how much cold water to add with simple calculations.

This method has broad applications beyond bathing, including cooking, industrial processes, and climate control systems. Mastery of these calculations helps in efficient resource management and ensures safety and comfort in everyday activities.

Remember: Always consider practical factors like heat loss and measurement accuracy when applying theoretical calculations in real-world situations.

Frequently Asked Questions

What is the main goal of calculating the mass of cold water to be added to hot water at 40°C?

The main goal is to determine the amount of cold water needed to mix with hot water at 40°C to reach a desired final temperature, typically a comfortable bathing temperature.

Which principle is used to solve the problem of mixing hot and cold water at different temperatures?

The principle of conservation of energy, specifically the concept of heat balance, is used, assuming no heat loss to the surroundings.

What information is necessary to calculate the mass of cold water to be added?

You need the mass or volume of hot water, the initial temperatures of both hot and cold water, and the final desired temperature after mixing.

How is the final temperature of mixture calculated when mixing hot and cold water?

The final temperature is calculated using the heat balance equation: (mass_hot specific_heat hot_temp) + (mass_cold specific_heat cold_temp) = (total_mass specific_heat final_temp).

If the hot water is at 40°C and cold water at 15°C, and the desired bathing temperature is 30°C, how can you find the mass of cold water needed?

Using the heat balance equation, set up the relation: (mass_hot 40) + (mass_cold 15) = (mass_hot + mass_cold) 30, then solve for the mass of cold water.

What assumptions are commonly made in these calculations?

Assumptions include no heat loss to the surroundings, the specific heat capacity of water remains constant, and the densities of hot and cold water are similar.

How does increasing the amount of cold water affect the final temperature of the mixture?

Adding more cold water decreases the final temperature, bringing it closer to the cold water's temperature, such as 15°C.

What is the formula to calculate the mass of cold water to be added when hot water at 40°C needs to be cooled to a certain temperature?

The formula derived from heat balance is: m_cold = (m_hot (hot_temp - final_temp)) / (final_temp - cold_temp).

Additional Resources

For Taking Bath, Water At 40°C Is Required. Calculate The Mass Of Cold Water At 15°C Which Is To Be Added

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Introduction

Bathing is a daily ritual that combines comfort, hygiene, and relaxation. The temperature of bathwater plays a crucial role in ensuring a pleasant experience and effective cleansing. Typically, a comfortable bath temperature hovers around 40°C, which is warm enough to soothe muscles and invigorate the senses without risking burns or discomfort. However, in many practical scenarios—such as using readily available hot water sources or saving energy—it's common to mix hot and cold water to reach the desired temperature.

This article explores the fundamental thermodynamic principles behind mixing hot and cold water for bathing, specifically focusing on calculating the necessary amount of cold water at 15°C to achieve a bath temperature of 40°C. It provides a detailed analytical approach, considering assumptions, relevant formulas, and real-world implications.

The Physics of Water Mixing for Bath Preparation

The Principle of Conservation of Heat Energy

At the core of mixing hot and cold water is the principle of conservation of energy. When two quantities of water at different temperatures are combined, the resulting temperature is determined by the heat exchange until thermal equilibrium is reached.

Assuming no heat losses to the environment—an idealized but useful approximation—the heat lost by the hot water equals the heat gained by the cold water.

Mathematically:

$$ m_h c (T_h - T_f) = m_c c (T_f - T_c) $$

Where:
- m_h = mass of hot water (kg)
- m_c = mass of cold water (kg)
- c = specific heat capacity of water ($\approx 4184\, \text{J/kg·°C}$)
- T_h = initial hot water temperature (°C)
- T_c = initial cold water temperature (°C)
- T_f = final bath temperature (°C)

Since c appears on both sides, it cancels out, leaving:

$$ m_h (T_h - T_f) = m_c (T_f - T_c) $$

This relation allows us to determine the unknown mass of cold water required to reach the desired temperature.

Assumptions and Parameters

To proceed with calculations, certain assumptions are necessary:

- The hot water temperature T_h is higher than 40°C; for example, hot water at 60°C is common.
- No heat loss occurs during mixing.
- The specific heat capacity of water remains constant at \(4184\,

J/kg·°C.
- The volume of bathwater is convenient to measure in terms of mass, considering water density $\approx 1000\, \text{kg/m}^3$.

Given these assumptions, the problem simplifies to calculating m_c, the mass of cold water at 15°C to be added to hot water at 60°C to produce 40°C water.

Step-by-Step Calculation

1. Define Known Values

- $T_h = 60°C$
- $T_c = 15°C$
- $T_f = 40°C$

2. Express m_c in terms of m_h

Using the heat balance:

$$ m_h (60 - 40) = m_c (40 - 15) $$

$$ m_h \times 20 = m_c \times 25 $$

$$ m_c = \frac{20}{25} m_h = 0.8 m_h $$

This indicates that, for every kilogram of hot water at 60°C, 0.8 kg of cold water at 15°C is needed to reach 40°C.

3. Total Volume of Bathwater

Suppose the desired total volume of bathwater is V liters (or equivalently, mass M kg, since $1\, \text{L}$ of water ≈ 1 kg). If the total volume is specified, say 100 liters (which is typical for a bath), the mass is 100 kg.

Then, the total mass:

$$ M = m_h + m_c $$

From above:

$$ m_c = 0.8 m_h $$

Thus:

$$ M = m_h + 0.8 m_h = 1.8 m_h $$

$$ m_h = \frac{M}{1.8} $$

and

$$ m_c = 0.8 \times \frac{M}{1.8} = \frac{0.8}{1.8} M \approx 0.444 M $$

For $M = 100\, \text{kg}$:

- Hot water required: $\approx 55.56\, \text{kg}$
- Cold water required: $\approx 44.44\, \text{kg}$

Practical Implications and Variations

Adjusting Hot Water Temperature

If hot water is at a different temperature, the calculation modifies accordingly. For example, if hot water is at 70°C:

$$ m_h (70 - 40) = m_c (40 - 15) $$

$$ m_h \times 30 = m_c \times 25 $$

$$ m_c = \frac{30}{25} m_h = 1.2 m_h $$

This indicates more cold water is needed in this scenario, emphasizing that higher hot water temperatures require more cold water to reach the target bath temperature.

Safety and Comfort Considerations

- Bathwater should not exceed approximately 45°C to avoid burns.
- For sensitive skin or children, lower temperatures are recommended, adjusting calculations accordingly.
- The mixing process should consider the heat loss to the environment, which may necessitate adding slightly more cold water.

Energy Perspective and Efficiency

Understanding the energy exchange provides insight into the efficiency of heating water for baths:

- Heating hot water consumes energy proportional to the temperature difference.
- Mixing hot and cold water can save energy if hot water is expensive or limited.
- Calculations like these help optimize water usage and energy consumption, especially in large-scale or institutional settings.

Environmental and Practical Considerations

- Using cold water at 15°C is common in many regions, particularly during cooler seasons.
- The availability and temperature of cold water vary geographically, impacting the mixing ratios.
- This calculation assists in planning and resource management, ensuring the desired comfort level is achieved without waste.

Limitations and Further Research

While the idealized calculations provide a good approximation, real-world conditions introduce complexities:

- Heat loss during mixing and transfer.
- Variability in water temperature.
- Human perception of comfort varies.
- The presence of impurities or minerals affecting thermal properties.

Further research could incorporate these variables, utilizing calorimetry and environmental controls to refine models.

Conclusion

In summary, achieving a bathwater temperature of 40°C by mixing hot water at 60°C and cold water at 15°C requires precise calculation of the cold water volume. Under ideal conditions, approximately 0.8 kg of cold water is needed for every 1 kg of hot water at 60°C to obtain 40°C water, assuming the total volume is 100 liters.

These calculations are essential in practical applications, ensuring comfort, safety, and energy efficiency. By understanding the thermodynamics involved, consumers and engineers alike can optimize water heating systems, reduce energy consumption, and enhance the bathing experience.

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