For The Following Exercises, Solve Each System By Elimination. 4x + 6y + 9z = 0 . -5x + 2y-6z = 3 7x-4y

Introduction to Solving Systems by Elimination

For The Following Exercises, Solve Each System By Elimination. 4x + 6y + 9z = 0 . -5x + 2y-6z = 3 7x-4y. Solving systems of equations is a fundamental skill in algebra that allows us to find the values of variables that satisfy multiple equations simultaneously. The elimination method is one of the most effective techniques for solving systems, especially when the equations are linear.

In this article, we will explore step-by-step how to approach and solve systems of three equations with three variables using the elimination method. We will illustrate the process with the given system and provide tips and strategies to make the process efficient and accurate. Whether you're a student preparing for exams or someone looking to strengthen your algebra skills, understanding the elimination method is essential for tackling complex systems.

Understanding the System of Equations

The Given System

The system of equations provided is:

```
4x + 6y + 9z = 0
-5x + 2y - 6z = 3
7x - 4y + ? = ?
```

Note: The third equation appears incomplete in the provided prompt. For the purpose of this tutorial, we will assume the third equation is:

```
7x - 4y + 0z = ?  (or a specified constant)
```

If the original third equation was incomplete, please replace it with the correct one. For now, let's proceed with a complete system:

```
1) 4x + 6y + 9z = 0
2) -5x + 2y - 6z = 3
3) 7x - 4y + 0z = k  (where k is a constant)
```

For demonstration purposes, let's assume the third equation is:

```
7x - 4y + 0z = 5
```

Step-by-Step Solution Using Elimination

Step 1: Arrange the equations

- Write each equation clearly with variables aligned:
1) 4x + 6y + 9z = 0
2) -5x + 2y - 6z = 3
3) 7x - 4y = 5

Step 2: Choose two equations to eliminate one variable

- Our goal is to eliminate one variable by combining equations.
- Let's first eliminate z since the coefficients of z in equations 1 and 2 are 9 and -6, respectively.

Step 3: Make the coefficients of z opposites

- Find the least common multiple (LCM) of 9 and 6, which is 18.
- Multiply equation 1 by 2 to get the coefficient of z as 18:
- 2 (4x + 6y + 9z) = 2 0
- Result: 8x + 12y + 18z = 0
- Multiply equation 2 by 3:
- 3 (-5x + 2y - 6z) = 3 3
- Result: -15x + 6y - 18z = 9

Step 4: Add the new equations to eliminate z

- Add the two equations:
- (8x + 12y + 18z) + (-15x + 6y - 18z) = 0 + 9
- Simplify:
- (8x - 15x) + (12y + 6y) + (18z - 18z) = 9
- -7x + 18y = 9

Step 5: Obtain a new equation with two variables

- The resulting equation:
- -7x + 18y = 9

Step 6: Repeat elimination to eliminate another variable

- Now, choose equations 2 and 3 to eliminate x or y.
- Let's eliminate x between equations 2 and 3.

- Equations:
- 2) $-5x + 2y - 6z = 3$
- 3) $7x - 4y = 5$

- To eliminate x, find LCM of 5 and 7, which is 35.
- Multiply equation 2 by 7:
- $7 (-5x + 2y - 6z) = 7 3$
- Result: $-35x + 14y - 42z = 21$
- Multiply equation 3 by 5:
- $5 (7x - 4y) = 5 5$
- Result: $35x - 20y = 25$

- Add the two:
- $(-35x + 14y - 42z) + (35x - 20y) = 21 + 25$
- Simplify:
- $0x + (14y - 20y) + (-42z) = 46$
- $-6y - 42z = 46$

Step 7: Solve for remaining variables

- Now, we have two equations:
1) $-7x + 18y = 9$ (from earlier)
2) $-6y - 42z = 46$

- From the second equation:
- $-6y = 46 + 42z$
- $y = -(46 + 42z)/6$
- Simplify:
- $y = -(23 + 21z)/3$

- Substitute y into the first equation to find x:
- $-7x + 18 [-(23 + 21z)/3] = 9$
- Simplify:
- $-7x - 6 (23 + 21z) = 9$
- Expand:
- $-7x - 623 - 621z = 9$
- $-7x - 138 - 126z = 9$
- Solve for x:
- $-7x = 9 + 138 + 126z$
- $-7x = 147 + 126z$
- $x = - (147 + 126z)/7$
- $x = - (21 + 18z)$

Step 8: Express all variables in terms of a parameter

- x:
- $x = -(21 + 18z)$
- y:
- $y = -(23 + 21z)/3$
- z:
- $z = z$ (free parameter)

This means the solution is expressed in terms of z, which can be any real number. The solutions form a line in 3D space.

Interpreting the Solution

Parametric Form of the Solution

- Let $z = t$, where t is any real number.
- Then:
- $x = -(21 + 18t)$
- $y = -(23 + 21t)/3$
- $z = t$

Final General Solution

The solution set can be written as:

```
x = -21 - 18t
y = - (23 + 21t)/3
z = t
```

where $t \in \mathbb{R}$.

Tips for Solving Systems by Elimination

- **Always align your equations:** Write them with variables and constants in columns for clarity.
- **Choose variables to eliminate:** Pick the variables with coefficients that are easy to manipulate.
- **Multiply equations appropriately:** To eliminate a variable, make the coefficients equal in magnitude but opposite in sign.
- **Keep track of signs and coefficients:** Carefully perform multiplication and addition/subtraction to avoid errors.

- **Express remaining variables in terms of parameters:** When the system has infinitely many solutions, parametrize the solutions.
- **Always check your solutions:** Substitute back into original equations to verify correctness.

Conclusion

Solving systems of equations by elimination is a powerful technique that simplifies the process of finding solutions, especially in systems with multiple variables. By systematically eliminating variables through multiplication and addition, you can reduce complex systems to manageable equations. Remember to always organize your work, verify each step, and consider parametric solutions when the system has infinitely many solutions.

Practice with different systems to master the elimination method, and you'll find that even complex systems become manageable with a clear strategy. Whether dealing with three variables or more, the elimination technique

Frequently Asked Questions

What is the first step to solve the system using elimination for the equations 4x + 6y + 9z = 0, -5x + 2y - 6z = 3, and 7x - 4y = ?

First, identify the equations and align them properly. Since the third equation is incomplete, ensure it is fully written, for example, 7x - 4y + 0z = ?, to proceed with elimination. Then, choose variables to eliminate first, such as eliminating x or y.

How do you eliminate a variable between the first two equations: 4x + 6y + 9z = 0 and -5x + 2y - 6z = 3?

To eliminate a variable, multiply the equations by suitable coefficients so the coefficients of that variable are opposites. For example, multiply the first equation by 5 and the second by 4 to make the x coefficients 20 and -20, then add the equations to eliminate x.

What is the solution after eliminating x from the first two equations?

After multiplying and adding, you get an equation in terms of y and z, which you can then solve for one variable and substitute back into the original equations to find the remaining variables.

How do you incorporate the third equation 7x - 4y = ? into the elimination process?

Express the third equation fully, including the constant term. Then, use it along with the other equations to eliminate one variable at a time, typically starting with the easiest, such as y or z, depending on the coefficients.

What should you do if the third equation is incomplete, like 7x - 4y?

Identify or obtain the missing constant term in the third equation. Without this, the system cannot be fully solved. Once complete, proceed with elimination as usual.

Can you solve the system if the third equation is missing the constant term? Why or why not?

No, because the constant term is necessary to form a proper system of equations. Without it, the equations are incomplete, making it impossible to find a unique solution.

After eliminating variables, how do you find the values of x, y, and z?

Solve the resulting two-variable equations step by step, substituting back into previous equations to find the remaining variables, until all are determined.

What are common mistakes to avoid when solving systems by elimination?

Common mistakes include incorrect multiplication factors, sign errors, not aligning equations properly, and missing constants or parts of equations.

How can you check if your solution to the system is correct?

Substitute the found values of x, y, and z back into all original equations to verify if they satisfy each one.

What tools or methods can help solve this system more efficiently?

Using substitution, matrix methods like Gaussian elimination, or graphing tools can help visualize and verify solutions more efficiently, especially for complex systems.

Additional Resources

Mastering the Elimination Method for Solving Systems of Equations: A Comprehensive Guide

Solving systems of equations is a fundamental skill in algebra, essential for understanding how multiple variables interact within a mathematical framework. Among the various methods available—substitution, graphing, matrix methods—the elimination method stands out for its efficiency, especially when dealing with systems involving three or more equations. This detailed review delves into the process of solving such systems through elimination, illustrating the approach with the specific system:

$$
\begin{cases}
4x + 6y + 9z = 0 \quad \quad (1) \\
-5x + 2y - 6z = 3 \quad (2) \\
7x - 4y + ? \quad \quad (3)
\end{cases}
$$

(Note: The third equation appears incomplete; for the purpose of this guide, we'll assume a typical third equation to complete the system, such as $7x - 4y + 8z = k$, where k is a constant. If the original problem is incomplete, please clarify. Here, we will focus on the process with the first two equations, and demonstrate how to extend to the third once the full system is known.)

Understanding the Elimination Method: An Overview

The elimination method involves systematically adding or subtracting equations to eliminate one variable, thereby reducing the system's complexity step by step. This approach relies on manipulating equations to align coefficients of a variable, making it straightforward to cancel that variable when the equations are combined.

Key steps include:

1. Align coefficients of a chosen variable: Multiply equations by suitable constants to obtain matching coefficients for one variable.
2. Add or subtract equations: Eliminate the chosen variable by combining the equations.
3. Solve the resulting simpler equations: After eliminating one variable, solve the remaining two-variable system.
4. Back-substitution: Use the solved variables to find remaining unknowns.

Step 1: Analyzing the System

Given the equations:

$$
\begin{cases}
4x + 6y + 9z = 0 \quad (1) \\
-5x + 2y - 6z = 3 \quad (2) \\
7x - 4y + ? \quad ? \quad (3)
\end{cases}
$$

Assuming the third equation is:

\[
7x - 4y + 8z = k
\]

for some constant \(k\), the full system becomes:

\[
\begin{cases}
4x + 6y + 9z = 0 \quad (1) \\
-5x + 2y - 6z = 3 \quad (2) \\
7x - 4y + 8z = k \quad (3)
\end{cases}
\]

This assumption allows us to demonstrate the elimination process thoroughly. If the original problem has a different third equation, please specify.

Step 2: Choosing a Variable to Eliminate

The most strategic variable to eliminate first depends on the coefficients' compatibilities. Typically, we look for coefficients with common factors or values that can be easily manipulated.

In equations (1) and (2):

- Coefficients of \(x\): 4 and -5
- Coefficients of \(y\): 6 and 2
- Coefficients of \(z\): 9 and -6

Let's consider eliminating \(x\) initially, as the coefficients 4 and -5 are manageable.

Step 3: Making Coefficients of \(x\) Equal in Magnitude

To eliminate \(x\), we need to align the coefficients:

- Multiply equation (1) by 5: \(5 \times (1)\)

\[
5 \times (1): \quad 20x + 30y + 45z = 0
\]

- Multiply equation (2) by 4: \(4 \times (2)\)

\[
4 \times (2): \quad -20x + 8y - 24z = 12
\]

Now, adding these two equations cancels \(x\):

\[
(20x - 20x) + (30y + 8y) + (45z - 24z) = 0 + 12
\]

Simplifies to:

\[
0x + 38y + 21z = 12
\]

This yields a new, two-variable equation:

\[
38y + 21z = 12 \quad (4)
\]

Step 4: Eliminating \(x\) Using Other Equations

Next, to involve the third equation, multiply (1) and (3) appropriately to eliminate \(x\):

Suppose the third equation is:

\[
7x - 4y + 8z = k
\]

- Multiply equation (1) by 7:

\[
7 \times (1): \quad 28x + 42y + 63z = 0
\]

- Multiply equation (3) by -4:

\[
-4 \times (3): \quad -28x + 16y - 32z = -4k
\]

Adding:

\[
(28x - 28x) + (42y + 16y) + (63z - 32z) = 0 - 4k
\]

Simplifies to:

\[

0x + 58y + 31z = -4k
\]

This is another equation involving only \(y\) and \(z\):

\[
58y + 31z = -4k \quad (5)
\]

Now, our system with equations (4) and (5):

\[
\begin{cases}
38y + 21z = 12 \quad (4) \\
58y + 31z = -4k \quad (5)
\end{cases}
\]

Step 5: Solving for \(y\) and \(z\)

The next step is to solve the two equations (4) and (5). To do so, we can use elimination or substitution.

Let's choose elimination:

- To eliminate \(z\):

Multiply (4) by 31:

\[
31 \times (4): \quad 38 \times 31 y + 21 \times 31 z = 12 \times 31
\]

Calculate:

\[
38 \times 31 = 1178 \\
21 \times 31 = 651 \\
12 \times 31 = 372
\]

So:

\[
1178 y + 651 z = 372 \quad (6)
\]

Multiply (5) by -21:

\[
-21 \times (5): \quad -58 \times 21 y - 31 \times 21 z = 4k \times 21
\]

Calculate:

\[
58 \times 21 = 1218 \\
31 \times 21 = 651 \\
4k \times 21 = 84k
\]

Thus:

\[
-1218 y - 651 z = 84k \quad (7)
\]

Adding equations (6) and (7):

\[
(1178 y - 1218 y) + (651 z - 651 z) = 372 + 84k
\]

Simplifies to:

\[
-40 y = 372 + 84k
\]

Solving for \(y\):

\[
y = - \frac{372 + 84k}{40} = - \frac{93 + 21k}{10}
\]

Once \(y\) is known, substitute back into either (4) or (5) to find \(z\).

Step 6: Finding \(z\)

Using equation (4):

\[
38 y + 21 z = 12
\]

Substitute \(y\):

\[

38 \times \left(- \frac{93 + 21k}{10}\right) + 21 z = 12
\]

Calculate the first term:

\[
38 \times - \frac{93 + 21k}{10} = - \frac{38 \times (93 + 21k)}{10}
\]

Compute numerator:

\[
38 \times 93 = 3534 \\
38 \times 21k = 798k
\]

So:

\[
- \frac{3534 + 798k}{10} + 21 z = 12
\]

Multiply both sides by 10 to clear denominator:

\[
- (3534 + 798k) + 210 z = 120
\]

Rearranged:

\[
210 z = 120 + 3534 + 798k
\]

\[
210 z = 3654 + 798k
\]

Finally,

\[
z = \frac{3654 + 798k}{210}
\]

Simplify numerator and denominator:

Divide numerator and denominator by 6:

\[
z = \frac{609 + 133k}{35}
\]

Step 7: Back-Substituting to Find \(x\)

Now, with \(y\) and \(z\) expressed in terms of \(k\), we can determine \(x\) by substituting into one of the original equations, say equation (1):

\[
4x + 6y +

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