Give The General Solution For The Following Trigonometric Equation. 1 Sin(4x) Cos(3x) - Sin(32) Cos(4x)

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Understanding how to solve trigonometric equations is a fundamental aspect of advanced mathematics and essential for fields like engineering, physics, and computer science. Today, we will explore the detailed process of finding the general solution for the equation:

$$ \sin(4x) \cos(3x) - \sin(32) \cos(4x) = 0 $$

This article will guide you step-by-step through the derivation, simplifying the equation using various trigonometric identities, and ultimately expressing the solution in a comprehensive, general form suitable for all values of x.

Understanding the Given Trigonometric Equation

Before delving into the solution, it's important to analyze the structure of the equation and identify useful identities that can simplify the problem.

Equation Breakdown

The equation:

$$ \sin(4x) \cos(3x) - \sin(32) \cos(4x) = 0 $$

contains multiple trigonometric functions with different arguments, as well as a constant $\sin(32)$. Our goal is to manipulate this expression into a form where we can solve for x.

Initial Observations

- The presence of products like $\sin(4x) \cos(3x)$ suggests the use of product-to-sum identities.
- The term $\sin(32)$ is a constant, likely representing an angle measure in degrees, so we must confirm whether the angles are in degrees or radians. For this article, we assume degrees, but the method applies similarly if in radians.
- The equation is set equal to zero, indicating that solutions occur when either factor equals zero or the

combined expression simplifies to a form that makes the entire expression zero.

Applying Trigonometric Identities to Simplify the Equation

To solve the equation efficiently, we utilize common identities such as the product-to-sum formulas.

Product-to-Sum Formulas

The relevant identities are:

- $\sin A \cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$
- $\cos A \sin B = \frac{1}{2}[\sin(A + B) - \sin(A - B)]$

Using these identities, we can rewrite the products in our equation.

Rewriting $\sin(4x) \cos(3x)$

Applying the first identity:

$$ \sin(4x) \cos(3x) = \frac{1}{2} [\sin(4x + 3x) + \sin(4x - 3x)] = \frac{1}{2} [\sin(7x) + \sin(x)] $$

Rewriting $\sin(32) \cos(4x)$

Similarly, since $\sin(32)$ is a constant, the term remains as is for now, but if necessary, we can express $\sin(32) \cos(4x)$ as:

$$ \sin(32) \cos(4x) $$

which can be considered as a constant times a cosine function.

Rewriting the Entire Equation

Substituting the simplified forms back into the original equation:

$$ \frac{1}{2} [\sin(7x) + \sin(x)] - \sin(32) \cos(4x) = 0 $$

Multiplying through by 2 to clear the fraction:

$$ \sin(7x) + \sin(x) - 2 \sin(32) \cos(4x) = 0 $$

This is a more manageable form for solving.

Expressing $\sin(7x) + \sin(x)$ as a Sum

Recall that:

$$ \sin A + \sin B = 2 \sin \left(\frac{A + B}{2} \right) \cos \left(\frac{A - B}{2} \right) $$

Applying this to $\sin(7x) + \sin(x)$:

$$ 2 \sin \left(\frac{7x + x}{2} \right) \cos \left(\frac{7x - x}{2} \right) = 2 \sin(4x) \cos(3x) $$

Interestingly, this is the same as our original product, indicating a consistent pattern.

So, the equation becomes:

$$ 2 \sin(4x) \cos(3x) - 2 \sin(32) \cos(4x) = 0 $$

which simplifies to:

$$ 2 \left[\sin(4x) \cos(3x) - \sin(32) \cos(4x) \right] = 0 $$

Dividing both sides by 2:

$$ \sin(4x) \cos(3x) - \sin(32) \cos(4x) = 0 $$

which reaffirms our original equation, confirming the consistency of the identities used.

Factoring the Equation to Find Solutions

Now, observe that the original equation can be viewed as:

$$ \sin(4x) \cos(3x) = \sin(32) \cos(4x) $$

This suggests that solutions occur when either side is equal, leading us to consider possible factorizations or substitution methods.

Expressing as a Single Equation

Let's write the equation as:

$$\sin(4x) \cos(3x) - \sin(32) \cos(4x) = 0$$

which can be rearranged:

$$\sin(4x) \cos(3x) = \sin(32) \cos(4x)$$

Dividing through both sides by $\cos(4x)$, provided $\cos(4x) \neq 0$:

$$\frac{\sin(4x) \cos(3x)}{\cos(4x)} = \sin(32)$$

which simplifies to:

$$\tan(4x) \cos(3x) = \sin(32)$$

Alternatively, because dividing by $\cos(4x)$ introduces potential for undefined values, it's safer to analyze the original equation in cases where $\cos(4x) = 0$.

Case 1: $\cos(4x) \neq 0$

In this case, the previous step applies, and the equation reduces to:

$$\tan(4x) \cos(3x) = \sin(32)$$

This relation can be complex to solve directly, so an alternative approach is to consider the original form and solve for x using known identities.

Case 2: $\cos(4x) = 0$

When:

$$\cos(4x) = 0$$

the original equation becomes:

$$ \sin(4x) \cos(3x) - \sin(32) \times 0 = 0 $$

which simplifies to:

$$ \sin(4x) \cos(3x) = 0 $$

Since $\sin(4x) \cos(3x) = 0$, the solutions are when either $\sin(4x) = 0$ or $\cos(3x) = 0$.

Solutions when $\cos(4x) = 0$

$$\cos(4x) = 0$$

$$\Rightarrow 4x = 90^\circ + 180^\circ n, \quad n \in \mathbb{Z}$$

$$\Rightarrow x = \frac{90^\circ + 180^\circ n}{4} = \frac{90^\circ}{4} + 45^\circ n = 22.5^\circ + 45^\circ n$$

Thus, the solutions are:

$$ x = 22.5^\circ + 45^\circ n, \quad n \in \mathbb{Z} $$

Solutions when $\sin(4x) = 0$

$$\sin(4x) = 0$$

$$\Rightarrow 4x = 0^\circ + 180^\circ m, \quad m \in \mathbb{Z}$$

$$\Rightarrow x = \frac{180^\circ m}{4} = 45^\circ m$$

Now, check whether these solutions satisfy the original equation.

At these x, the original equation becomes:

$$ \sin(4x) \cos(3x) - \sin(32) \cos(4x) = 0 $$

Since $\sin(4x) = 0$, the first term is zero, and the equation reduces to:

$$ - \sin(32) \cos(4x) = 0 $$

which implies:

\[\cos(4x) = 0 \quad \text{or} \

Frequently Asked Questions

What is the general solution for the trigonometric equation Sin(4x) Cos(3x) - Sin(32) Cos(4x) = 0?

The general solution is $x = 5\pi/4 + n\pi$, where n is any integer.

How can the given equation Sin(4x) Cos(3x) - Sin(32) Cos(4x) be simplified?

Using product-to-sum identities, the equation simplifies to (1/2) [Sin(7x) - Sin(32)] = 0.

What identities are useful for solving Sin(4x) Cos(3x) - Sin(32) Cos(4x) = 0?

The product-to-sum identities, such as Sin A Cos B = (1/2)[Sin(A+B) + Sin(A−B)], are helpful.

After simplification, what equation do we need to solve to find x?

We need to solve Sin(7x) = Sin(32), which leads to solutions of the form $7x = 32 + 2n\pi$ or $7x = \pi - 32 + 2n\pi$.

What are the specific solutions for x after solving Sin(7x) = Sin(32)?

The solutions are $x = (32 + 2n\pi)/7$ and $x = (\pi - 32 + 2n\pi)/7$, for all integers n.

Why is it important to consider the periodicity of sine when solving the equation?

Because sine is periodic with period 2π, solutions repeat every $2\pi/n$, so all solutions must account for this periodicity.

Can you provide a brief step-by-step approach to solving the given equation?

Yes. First, apply product-to-sum identities to simplify the equation, then set the resulting sine expressions equal to each other, solve for x, and incorporate the periodicity to find the general solution.

Additional Resources

Give The General Solution For The Following Trigonometric Equation: Sin(4x) Cos(3x) - Sin(32) Cos(4x)

When tackling trigonometric equations, finding the general solution is often the ultimate goal, especially when the equations involve multiple angles and products of sine and cosine functions. In this comprehensive guide, we will analyze and solve the equation Sin(4x) Cos(3x) - Sin(32) Cos(4x) step-by-step, employing fundamental identities, algebraic manipulations, and strategic substitutions to arrive at the full general solution. Whether you are a student preparing for exams or a mathematics enthusiast seeking a deeper understanding, this detailed walkthrough aims to clarify the process and reinforce key concepts in solving complex trigonometric equations.

Understanding the Equation

The given equation is:

$\sin(4x)\cos(3x) - \sin(32)\cos(4x) = 0$

At first glance, it contains a mixture of sine and cosine functions with different angles, as well as a constant term, Sin(32). The goal is to find all values of x that satisfy this equation, considering the periodic nature of sine and cosine functions.

Step 1: Recognizing Patterns and Strategic Approach

Before diving into algebraic manipulations, it's essential to identify potential identities or formulas that can simplify the equation:

- The product-to-sum identities for sine and cosine:
- $\sin A \cos B = \frac{1}{2} [\sin(A + B) + \sin(A - B)]$
- $\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$
- $\cos A \cos B = \frac{1}{2} [\cos(A + B) + \cos(A - B)]$

Given the structure of our equation, the product-to-sum identities seem most relevant because the equation involves products of sine and cosine functions.

Step 2: Applying Product-to-Sum Identities

Let's rewrite each product term separately:

Rewrite $\sin(4x) \cos(3x)$:

Using the identity:

$$\sin A \cos B = \tfrac{1}{2} [\sin(A + B) + \sin(A - B)]$$

we have:

$$\sin(4x) \cos(3x) = \tfrac{1}{2} [\sin(4x + 3x) + \sin(4x - 3x)]$$
$$= \tfrac{1}{2} [\sin(7x) + \sin(x)]$$

Rewrite $\sin(32) \cos(4x)$:

Since $\sin(32)$ is a constant (a fixed numerical value), and $\cos(4x)$ depends on x, the product remains as is:

$$\sin(32) \cos(4x)$$

But to facilitate solving, it's convenient to express $\cos(4x)$ in terms of sine or cosine of multiple angles or recognize that it's a common factor if we manipulate the equation accordingly.

Step 3: Rewriting the Entire Equation

Substituting the identities into the original equation:

$$(\tfrac{1}{2} [\sin(7x) + \sin(x)]) - \sin(32) \cos(4x) = 0$$

Multiply through by 2 to clear fractions:

$$\sin(7x) + \sin(x) - 2 \sin(32) \cos(4x) = 0$$

Rearranged as:

$$\sin(7x) + \sin(x) = 2 \sin(32) \cos(4x)$$

Step 4: Expressing $\cos(4x)$ in Terms of Sine or Cosine

Since the right side involves $\cos(4x)$, and the left side involves sinusoids with different angles, it's beneficial to express $\cos(4x)$ in terms of sine or cosine of multiple angles.

Recall that:

$\cos(4x) = 1 - 2 \sin^2(2x)$ (from the double angle formula)

or

$\cos(4x) = 2 \cos^2(2x) - 1$

Alternatively, to keep things manageable, we might consider expressing both sides in terms of sine or cosine functions that can be combined or equated directly.

However, because the right side involves a constant multiplied by $\cos(4x)$, it might be more straightforward to treat $\cos(4x)$ as a variable and express the entire equation in terms of x.

Step 5: Using Sum-to-Product or Sum-to-Linear Formulas

The left side involves $\sin(7x) + \sin(x)$. Recall the sum-to-product identity:

$\sin A + \sin B = 2 \sin[(A + B)/2] \cos[(A - B)/2]$

Applying this:

$\sin(7x) + \sin(x) = 2 \sin[(7x + x)/2] \cos[(7x - x)/2]$
$= 2 \sin(4x) \cos(3x)$

But notice this is the same as our original product $\sin(4x) \cos(3x)$, which we initially expanded using product-to-sum identities.

This suggests a pattern: the sum $\sin(7x) + \sin(x)$ simplifies to $2 \sin(4x) \cos(3x)$, which is consistent with our initial expansion.

Step 6: Expressing $\cos(4x)$ in Terms of Sin and Cosine

Recall the double angle formula:

$\cos(4x) = 1 - 2 \sin^2(2x)$

Alternatively,

$\mathrm{Cos}(4x) = 2\ \mathrm{Cos}^2(2x) - 1$

Expressing in terms of Cos(2x):

- $\mathrm{Cos}(2x) = 2\ \mathrm{Cos}^2 x - 1$

- $\mathrm{Sin}(2x) = 2\ \mathrm{Sin}\ x\ \mathrm{Cos}\ x$

But this can become complex; perhaps a more straightforward approach is to express Cos(4x) in terms of $\mathrm{Cos}^2(2x)$:

$\mathrm{Cos}(4x) = 2\ \mathrm{Cos}^2(2x) - 1$

Similarly, Cos(2x) can be written as:

$\mathrm{Cos}(2x) = 2\ \mathrm{Cos}^2 x - 1$

However, to avoid overcomplicating, perhaps we should revisit the original structure and consider alternative methods.

Step 7: Re-arranged Equation for Direct Solution

Recall the simplified form:

$\mathrm{Sin}(7x) + \mathrm{Sin}(x) = 2\ \mathrm{Sin}(32)\ \mathrm{Cos}(4x)$

Let's denote:

- $\mathrm{LHS} = \mathrm{Sin}(7x) + \mathrm{Sin}(x)$

- $\mathrm{RHS} = 2\ \mathrm{Sin}(32)\ \mathrm{Cos}(4x)$

We can express LHS using the sum-to-product:

$\mathrm{LHS} = 2\ \mathrm{Sin}(4x)\ \mathrm{Cos}(3x)$

So, the equation becomes:

$2\ \mathrm{Sin}(4x)\ \mathrm{Cos}(3x) = 2\ \mathrm{Sin}(32)\ \mathrm{Cos}(4x)$

Divide both sides by 2:

$\text{Sin}(4x)\ \text{Cos}(3x) = \text{Sin}(32)\ \text{Cos}(4x)$

Now, observe that both sides involve products of sine and cosine functions, which suggests rewriting it as:

$\text{Sin}(4x)\ \text{Cos}(3x) - \text{Sin}(32)\ \text{Cos}(4x) = 0$

which matches the original equation.

Step 8: Factoring the Equation

Expressed as:

$\text{Sin}(4x)\ \text{Cos}(3x) - \text{Sin}(32)\ \text{Cos}(4x) = 0$

Factor out Cos(4x):

$\text{Cos}(4x)\ [\text{Sin}(4x)\ (\text{Cos}(3x)/\text{Cos}(4x)) - \text{Sin}(32)] = 0$

But dividing by Cos(4x) requires caution: $\text{Cos}(4x) \neq 0$, and we need to consider that case separately.

Alternatively, rewrite the original as:

$\text{Cos}(4x)\ [\text{Sin}(4x)\ (\text{Cos}(3x)/\text{Cos}(4x)) - \text{Sin}(32)] = 0$

But dividing by Cos(4x) may be complicated; perhaps a better approach is to write the original as a sum and factor accordingly.

Step 9: Setting Up for the Zero Product Property

The original equation:

$\text{Sin}(4x)\ \text{Cos}(3x) - \text{Sin}(32)\ \text{Cos}(4x) = 0$

can be viewed as:

$A\ B - C\ D = 0$

which suggests:

A B = C D

or, rearranged as:

$\text{Sin}(4x)\ \text{Cos}(3x) = \text{Sin}(32)\ \text{Cos}(4x)$

Now, express the right side as the product of sine and cosine functions, and consider the potential for the zero product rule.

Step 10: Solving for x

Now, we recognize that the equation can be rearranged into two cases:

Case 1: $\text{Sin}(4x)\ \text{Cos}(3x) = 0$

- Either $\text{Sin}(4x) = 0$

- Or $\text{Cos}(3x) = 0$

Case 2: $\text{Sin}(32)\ \text{Cos}(4x) = 0$

- Since Sin(32) is a constant (~0.5299), it's not zero, so the only way for the product to be zero is $\text{Cos}(4x) = 0$.

Let's analyze each case:

Case 1: $\text{Sin}(4x) = 0$

Solve:

$\text{Sin}(4x) = 0$

General solution:

$4x = n\pi$, where $n \in Z$

Thus:

$x = n\pi/4$

Case 1: $\cos(3x) = 0$

Solve:

$\cos(3x) = 0$

General solution:

$3x = \pi/2 + m\pi$, where $m \in \mathbb{Z}$

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