Given Function F(x) = (1 - 5x)/(1 + 5x) Find The Following:f' * (4) =f^ Prime Prime (4) =f^ Prime Prime

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Introduction

Understanding derivatives and second derivatives of functions is fundamental in calculus, especially when analyzing the behavior of functions such as slopes, concavity, and points of inflection. The function $F(x) = \frac{1 - 5x}{1 + 5x}$ presents an interesting case for differentiation because it is a rational function involving a quotient. This article provides a comprehensive guide to calculating the first and second derivatives of $F(x)$, focusing specifically on evaluating these derivatives at $x = 4$. We will explore the concepts step-by-step, ensuring clarity and thorough understanding for students and enthusiasts.

Overview of the Function $F(x) = \frac{1 - 5x}{1 + 5x}$

The function in question is a rational function, where both numerator and denominator are linear expressions:

- Numerator: $1 - 5x$
- Denominator: $1 + 5x$

This type of function often arises in calculus problems involving rates of change and can be differentiated using the quotient rule. The goal is to:

- Find $F'(x)$, the first derivative of $F(x)$.
- Evaluate $F'(4)$.
- Find $F''(x)$, the second derivative of $F(x)$.
- Evaluate $F''(4)$.

Step 1: Deriving the First Derivative $F'(x)$

1.1 Understanding the Quotient Rule

The quotient rule states that if:

$$
f(x) = \frac{u(x)}{v(x)}
$$

then its derivative is:

$$
f'(x) = \frac{u'(x) v(x) - u(x) v'(x)}{[v(x)]^2}
$$

where:

- $u(x)$ is the numerator,
- $v(x)$ is the denominator.

1.2 Applying the Quotient Rule to $F(x)$

Let:

- $u(x) = 1 - 5x$
- $v(x) = 1 + 5x$

Compute derivatives:

- $u'(x) = -5$
- $v'(x) = 5$

Applying the quotient rule:

$$
F'(x) = \frac{(-5)(1 + 5x) - (1 - 5x)(5)}{(1 + 5x)^2}
$$

1.3 Simplifying $F'(x)$

Calculate numerator:

$$
-5(1 + 5x) - 5(1 - 5x) = -5 - 25x - 5 + 25x
$$

Combine like terms:

$$
(-5 - 5) + (-25x + 25x) = -10 + 0 = -10
$$

Thus:

$$
F'(x) = \frac{-10}{(1 + 5x)^2}
$$

\]

Step 2: Evaluating \(F'(4) \)

Substitute \(x = 4 \):

\[
F'(4) = \frac{-10}{(1 + 5 \times 4)^2} = \frac{-10}{(1 + 20)^2} = \frac{-10}{21^2} =
\frac{-10}{441}
\]

Result:

\[
\boxed{
F'(4) = -\frac{10}{441}
}
\]

This value indicates the slope of the tangent line to the function at \(x = 4 \).

Step 3: Deriving the Second Derivative \(F''(x) \)

3.1 Differentiating \(F'(x) \)

Recall:

\[
F'(x) = \frac{-10}{(1 + 5x)^2}
\]

Rewrite as:

\[
F'(x) = -10 \times (1 + 5x)^{-2}
\]

Apply the chain rule:

\[
F''(x) = -10 \times \frac{d}{dx} \left[(1 + 5x)^{-2} \right]
\]

Derivative of \((1 + 5x)^{-2} \):

\[
\frac{d}{dx} \left[(1 + 5x)^{-2} \right] = -2 (1 + 5x)^{-3} \times 5 = -10 (1 + 5x)^{-3}

\]

Therefore:

\[
F''(x) = -10 \times (-10) (1 + 5x)^{-3} = 100 (1 + 5x)^{-3}
\]

3.2 Simplified Expression for \(F''(x) \)

\[
\boxed{
F''(x) = \frac{100}{(1 + 5x)^3}
}
\]

Step 4: Evaluating \(F''(4) \)

Substitute \(x = 4 \):

\[
F''(4) = \frac{100}{(1 + 5 \times 4)^3} = \frac{100}{(1 + 20)^3} = \frac{100}{21^3}
\]

Calculate \(21^3 \):

\[
21^3 = 21 \times 21 \times 21 = 441 \times 21 = 9261
\]

Thus:

\[
F''(4) = \frac{100}{9261}
\]

Summary of Results

Quantity	Expression	Numeric Value
First derivative at 4	\(F'(4) \)	\(-\frac{10}{441} \)
Second derivative at 4	\(F''(4) \)	\(\frac{100}{9261} \)

Additional Insights and Applications

1. Interpreting the Derivatives

- The negative value of \(F'(4) \) indicates that at \(x=4 \), the function \(F(x) \) is decreasing.
- The positive second derivative \(F''(4) \) suggests the function is concave upward at \(x=4 \), implying a local minimum or the start of an upward curvature in the graph.

2. Graphical Behavior

Understanding derivatives helps in sketching the graph:

- The slope at \(x=4 \) is approximately \(-0.0227 \).
- The concavity at \(x=4 \) is positive, indicating the graph curves upward.

3. Practical Applications

- Rational functions such as \(F(x) \) are common in physics, economics, and engineering to model rates, proportions, or ratios.
- Calculating derivatives allows for optimization, rate analysis, and understanding the behavior of such functions.

Conclusion

Calculating derivatives of rational functions like \(F(x) = \frac{1 - 5x}{1 + 5x} \) involves applying the quotient rule followed by chain rule for higher derivatives. This comprehensive guide demonstrated each step, from deriving the first derivative to evaluating the second derivative at \(x=4 \). Mastery of these techniques is essential for analyzing complex functions and understanding their behavior in various scientific and mathematical contexts.

References

- Stewart, James. Calculus: Early Transcendentals. Cengage Learning.
- Thomas, George B., and Ross L. Finney. Calculus and Analytic Geometry. Addison Wesley.
- Khan Academy. Derivative Rules. https://www.khanacademy.org/math/ap-calculus-ab

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Note: Always verify your derivatives and evaluations to ensure accuracy, especially when applying to real-world problems.

Frequently Asked Questions

What is the function F(x) given as (1 - 5x)/(1 + 5x)?

The function F(x) is a rational function defined as F(x) = (1 - 5x) divided by (1 + 5x).

How do you find the first derivative F'(x) of the function F(x)?

To find F'(x), apply the quotient rule: if F(x) = u(x)/v(x), then F'(x) = (u'v - uv')/v^2. Here, u(x) = 1 - 5x and v(x) = 1 + 5x.

What is the value of F(4) for the given function?

F(4) = (1 - 54)/(1 + 54) = (1 - 20)/(1 + 20) = (-19)/21.

How do you find the second derivative F''(x) of the function F(x)?

First, compute F'(x) using the quotient rule, then differentiate F'(x) to get F''(x). This involves applying the quotient rule again or differentiating the simplified first derivative.

What is the value of F''(4), the second derivative at x=4?

After computing F''(x), substitute x=4 into the expression to find F''(4). The exact value depends on the derived second derivative formula.

Are there any symmetries or special properties of the function F(x)?

Yes, the function exhibits symmetry related to its form; it is a Möbius transformation and often has properties like being its own inverse under certain conditions.

What is the significance of finding F(4) and F''(4)?

Finding F(4) gives the function's value at x=4, while F''(4) provides information about the concavity or convexity of the function at that point, indicating the nature of its curvature.

How can the derivatives of F(x) be used to analyze the function’s behavior near x=4?

The first derivative F'(4) indicates the slope or rate of change at x=4, while the second derivative F''(4) reveals whether the function is concave up or down at that point, helping understand local extrema and curvature.

Additional Resources

Given Function F(x) = (1 - 5x)/(1 + 5x) Find The Following: f'(4) = f''(4) = f'''(4)

Understanding the intricacies of calculus, particularly derivatives, is fundamental for students and professionals working with functions and their behaviors. The function F(x) = (1 - 5x) / (1 + 5x) presents an interesting case for derivative calculation due to its rational structure. This article aims to thoroughly analyze the derivative and second derivative at a specific point, x = 4, providing a detailed breakdown of the process, the mathematical reasoning involved, and the implications of the results obtained.

Overview of the Function F(x) = (1 - 5x) / (1 + 5x)

Before diving into derivatives, it is crucial to understand the nature of the function itself. The function is a rational function, which typically exhibits interesting behaviors such as asymptotes and possible points of discontinuity.

Features of F(x):

- Type: Rational function
- Domain: All real numbers except where the denominator equals zero, i.e., $x \neq -1/5$
- Range: Dependent on the numerator and denominator, with potential for vertical asymptotes at $x = -1/5$
- Symmetry: The function is neither even nor odd; analysis of symmetry can be performed but is not the primary focus here.

Calculating the First Derivative f'(x)

The first derivative of a rational function is obtained through the quotient rule, which states:

$$
f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{[h(x)]^2}
$$

where $g(x)$ and $h(x)$ are the numerator and denominator functions respectively.

Step-by-step calculation:

- Let $g(x) = 1 - 5x$
- Let $h(x) = 1 + 5x$

Compute derivatives:

- $g'(x) = -5$
- $h'(x) = 5$

Applying quotient rule:

$$
f'(x) = \frac{(-5)(1 + 5x) - (1 - 5x)(5)}{(1 + 5x)^2}
$$

Simplify numerator:

$$
f'(x) = \frac{-5(1 + 5x) - 5(1 - 5x)}{(1 + 5x)^2}
$$

$$
f'(x) = \frac{-5 - 25x - 5 + 25x}{(1 + 5x)^2}
$$

Observe that $-25x + 25x = 0$, so numerator simplifies to:

$$
-5 - 5 = -10
$$

Thus,

$$
f'(x) = \frac{-10}{(1 + 5x)^2}
$$

Result:

$$
f'(x) = -\frac{10}{(1 + 5x)^2}
$$

Evaluating f'(4)

Substitute $x=4$:

$$
f'(4) = -\frac{10}{(1 + 5 \times 4)^2} = -\frac{10}{(1 + 20)^2} = -\frac{10}{21^2} = -\frac{10}{441}
$$

Final value:

$$
f'(4) = -\frac{10}{441}
$$

This negative value indicates that the function is decreasing at $x=4$.

Calculating the Second Derivative f''(x)

The second derivative provides insight into the concavity and the rate of change of the slope.

Given:

$$
f'(x) = -\frac{10}{(1 + 5x)^2}
$$

We can write:

$$
f'(x) = -10 \times (1 + 5x)^{-2}
$$

Differentiate using the chain rule:

$$
f''(x) = -10 \times d/dx \left[(1 + 5x)^{-2}\right]
$$

Applying the chain rule:

$$
d/dx \left[(1 + 5x)^{-2}\right] = -2 (1 + 5x)^{-3} \times 5 = -10 (1 + 5x)^{-3}
$$

Therefore,

$$
f''(x) = -10 \times [-10 (1 + 5x)^{-3}] = 100 (1 + 5x)^{-3}
$$

Simplified:

$$
f''(x) = \frac{100}{(1 + 5x)^3}
$$

Evaluating f''(4)

Substitute $x=4$:

$$
f''(4) = \frac{100}{(1 + 20)^3} = \frac{100}{21^3} = \frac{100}{9261}
$$

\]

Final value:

\[
f''(4) = \frac{100}{9261}
\]

Since this is positive, the function is concave upward at \(x=4 \).

Calculating the Third Derivative f'''(x)

While the problem asks for \(f'''(4) \), the third derivative involves differentiating \(f''(x) \):

\[
f''(x) = 100 (1 + 5x)^{-3}
\]

Differentiating again:

\[
f'''(x) = 100 \times d/dx \left[(1 + 5x)^{-3}\right]
\]

Applying the chain rule:

\[
d/dx \left[(1 + 5x)^{-3}\right] = -3 (1 + 5x)^{-4} \times 5 = -15 (1 + 5x)^{-4}
\]

Thus,

\[
f'''(x) = 100 \times [-15 (1 + 5x)^{-4}] = -1500 (1 + 5x)^{-4}
\]

Evaluating f'''(4)

Substitute \(x=4 \):

\[
f'''(4) = -1500 \times (1 + 20)^{-4} = -1500 \times 21^{-4}
\]

Expressed explicitly:

$$
f'''(4) = -1500 / 21^4
$$

Calculate 21^4:

$$
21^2 = 441
$$

$$
21^4 = (21^2)^2 = 441^2 = 194,481
$$

Therefore:

$$
f'''(4) = -1500 / 194,481
$$

Final value:

$$
f'''(4) \approx -0.00771
$$

The negative sign indicates the rate at which the concavity is decreasing at $x=4$.

Summary of Results at x=4

Derivative	Expression	Numerical Value	Interpretation
First derivative $f'(4)$	$-\frac{10}{441}$	approximately -0.0227	Function decreasing at $x=4$
Second derivative $f''(4)$	$\frac{100}{9261}$	approximately 0.0108	Concave upward at $x=4$
Third derivative $f'''(4)$	$-\frac{1500}{21^4}$	approximately -0.00771	Rate of change of concavity is negative

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Implications and Insights

The derivative calculations reveal several key features:

- The negative first derivative at \(x=4\) indicates the function is decreasing at this point.
- The positive second derivative suggests the function is concave upward, meaning the slope is increasing despite the function decreasing.
- The negative third derivative implies that the concavity is decreasing; the function's curvature is becoming less pronounced as \(x\) increases.

Understanding these derivatives aids in analyzing the behavior of the function, especially in contexts like optimization, curve sketching, and understanding the dynamics around specific points.

Pros and Cons of the Derivative Approach

Pros:

- The quotient rule simplifies the process for rational functions.
- Analytical derivatives provide detailed insights into the function's behavior.
- The process is systematic, allowing for straightforward computation of higher-order derivatives.

Cons:

- For more complex functions, derivatives can become cumbersome and error-prone.
- Rational functions with higher degrees may require more advanced techniques.
- Numerical approximations at specific points can introduce minor inaccuracies.

Conclusion

The detailed analysis of the function \(F(x) = \frac{1 - 5x}{1 + 5x}\) and its derivatives at \(x=4\) highlights the power of calculus tools like the quotient rule, chain rule, and higher-order derivatives. These calculations enable a comprehensive understanding of the function's behavior at a specific point, revealing insights into its decreasing nature, conc

Given Function F X 1 5x 1 5x Find The Following F 4 F Prime Prime 4 F Prime Prime

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