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Given That One Of The Roots Of X 2 + hx10=0 Is 2 . (a) Find The Value Of Second Root. (b) Find The Value

Understanding quadratic equations is fundamental in algebra, and they frequently appear in various mathematical and real-world contexts. In this article, we will analyze the quadratic equation $X^2 + hX + 10 = 0$, given that one of its roots is 2. We will explore how to find the second root and determine the specific value of the parameter h involved in the equation. This comprehensive guide aims to clarify these concepts with step-by-step solutions, explanations, and relevant formulas to enhance your understanding.

Introduction to Quadratic Equations

A quadratic equation is a second-degree polynomial equation of the form:

$$
ax^2 + bx + c = 0
$$

where:

- $a \neq 0$,
- b and c are constants.

The solutions, or roots, of the quadratic are the values of x that satisfy the equation. These roots can be real or complex, and their nature is determined by the discriminant:

$$
D = b^2 - 4ac
$$

- If $D > 0$, there are two distinct real roots.
- If $D = 0$, there is one real root (a repeated root).
- If $D < 0$, the roots are complex conjugates.

Quadratic equations can be solved using various methods, including factoring, completing the square, and applying the quadratic formula:

$$
x = \frac{-b \pm \sqrt{D}}{2a}

\]

Given Data and Problem Statement

The quadratic equation under consideration is:

\[
X^2 + hX + 10 = 0
\]

It is given that one of its roots is 2. The goal is to:

- (a) Find the second root.
- (b) Find the value of \(h\).

This problem involves understanding the relationships between roots and coefficients of quadratic equations, particularly Vieta's formulas.

Understanding Vieta's Formulas

Vieta's formulas relate the roots of a quadratic equation to its coefficients:

- Sum of roots:

\[
\alpha + \beta = -\frac{b}{a}
\]

- Product of roots:

\[
\alpha \beta = \frac{c}{a}
\]

In our case:

\[
a = 1, \quad b = h, \quad c = 10
\]

Thus:

$$
\alpha + \beta = -h
$$

$$
\alpha \beta = 10
$$

Given that one root $\alpha = 2$, we can leverage these relationships to find the second root β and the value of h.

Part (a): Find the Second Root

Since one root $\alpha = 2$, and using the product of roots:

$$
\alpha \beta = 10
$$

Substitute $\alpha = 2$:

$$
2 \times \beta = 10
$$

Solve for β:

$$
\beta = \frac{10}{2} = 5
$$

Therefore, the second root is $\boxed{5}$.

Part (b): Find the Value of h

Recall from Vieta's formulas:

$$
\alpha + \beta = -h
$$

Substitute the known roots:

$$
2 + 5 = -h
$$

$$
7 = -h
$$

Solve for h:

$$
h = -7
$$

Thus, the value of h is $\boxed{-7}$.

Summary of Results

Parameter	Value
Second root β	5
Coefficient h	-7

The quadratic equation with these parameters is:

$$
X^2 - 7X + 10 = 0
$$

which indeed has roots 2 and 5, satisfying the initial condition.

Verification of Solutions

To ensure our solutions are correct, let's verify the roots by substituting them back into the quadratic equation:

$$
X^2 - 7X + 10 = 0
$$

- For $X = 2$:

$$
(2)^2 - 7 \times 2 + 10 = 4 - 14 + 10 = 0
$$

- For $X = 5$:

$$
(5)^2 - 7 \times 5 + 10 = 25 - 35 + 10 = 0
$$

Both roots satisfy the equation, confirming the correctness of our calculations.

Additional Insights and Applications

Understanding how to find roots and coefficients in quadratic equations has numerous applications:

- Physics: Calculating projectile trajectories involves quadratic equations.
- Engineering: Structural analysis often requires solving quadratic equations for stress and strain.
- Economics: Profit maximization models sometimes involve quadratic functions.

Moreover, mastering Vieta's formulas provides an efficient way to analyze quadratics without fully solving them, especially when roots are partially known.

Conclusion

In this detailed analysis, we've established a method to determine the second root and the parameter h for the quadratic equation $X^2 + hX + 10 = 0$, given that one root is 2. By leveraging Vieta's formulas, we deduced:

- The second root is 5.
- The value of h is -7.

These results not only solve the specific problem but also reinforce fundamental algebraic principles applicable across various mathematical and scientific disciplines.

Further Practice Problems

To solidify your understanding, consider practicing these problems:

1. If one root of \(x^2 + 4x + k = 0\) is \(-3\), find the other root and the value of \(k\).
2. For the quadratic equation \(2x^2 + h x + 8 = 0\), given that one root is 4, determine the other root and the value of \(h\).
3. Given the quadratic \(x^2 + 6x + 9 = 0\), verify its roots and discuss their nature (real or complex).

Engaging with such problems enhances problem-solving skills and deepens understanding of quadratic equations.

In summary, understanding the relationship between roots and coefficients using Vieta's formulas enables efficient solving of quadratic equations, especially when some roots are known. The methodology demonstrated here can be applied broadly to similar algebraic problems, reinforcing the importance of foundational algebra in various fields.

Frequently Asked Questions

Given that one of the roots of the equation $x^2 + h x + 10 = 0$ is 2, how do we find the value of h?

Since 2 is a root, substitute $x = 2$ into the equation: $(2)^2 + h(2) + 10 = 0 \rightarrow 4 + 2h + 10 = 0 \rightarrow 14 + 2h = 0 \rightarrow 2h = -14 \rightarrow h = -7$.

What is the second root of the quadratic equation $x^2 - 7x + 10 = 0$, given that one root is 2?

Using the sum of roots, which is -h (here 7), and knowing one root is 2, the second root is 7 - 2 = 5.

How do you verify the second root once you know one root is 2 and h is -7 in the quadratic $x^2 - 7x + 10 = 0$?

Substitute the second root, 5, into the equation: $5^2 - 7(5) + 10 = 25 - 35 + 10 = 0$, confirming 5 is a root.

If one root of $x^2 + h x + 10 = 0$ is 2, what is the product of the roots?

The product of roots, given by c/a, is 10. Since one root is 2, the other root is 5, and their product: $2 \times 5 = 10$.

Can the quadratic $x^2 + h x + 10 = 0$ have real roots if one root is 2? How do you determine this?

Yes, because the roots are real if the discriminant $D = h^2 - 4 \times 1 \times 10 \geq 0$. With $h = -7$, $D = 49 - 40 = 9 \geq 0$, so roots are real.

What is the significance of knowing one root to find the second root in a quadratic equation?

Knowing one root allows you to use the sum and product of roots formulas to find the other root directly, simplifying the solving process.

How does Vieta's formula relate to the roots of the quadratic when one root is known?

Vieta's formulas state that the sum of roots = -h and the product = 10. With one root known, you can find the other root using these relationships.

What steps are involved in solving for both roots of $x^2 + h x + 10 = 0$ when one root is given as 2?

First, find h using the known root: substitute x = 2 to find h. Then, use the sum or product of roots to find the second root. Alternatively, factor or use quadratic formula if needed.

Additional Resources

Given That One Of The Roots Of $X^2 + h x + 10 = 0$ Is 2: An Investigative Approach to Roots and Coefficients

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Introduction

Mathematics, at its core, is a pursuit of patterns and relationships. Quadratic equations, in particular, serve as foundational tools not only in academic contexts but also in real-world applications spanning engineering, physics, economics, and beyond. The quadratic equation of the form:

\[ax^2 + bx + c = 0 \]

presents a rich landscape for exploration, especially when parameters within the equation are variable or unknown. A recurring theme in algebra involves understanding how roots—the solutions of the quadratic—relate to the coefficients, and vice versa.

This article investigates a specific problem:

> Given that one of the roots of the quadratic \(x^2 + h x + 10 = 0 \) is 2, find the second root and the value of \(h \).

This problem exemplifies a classic algebraic technique: leveraging the relationships between roots and coefficients, known as Vieta's formulas. We will dissect the problem thoroughly, explore underlying principles, and offer a comprehensive solution, ultimately serving as a detailed reference for students, educators, and enthusiasts alike.

Setting the Scene: The Quadratic Equation and Its Roots

Before delving into the specifics of the problem, it’s essential to understand the general properties of quadratic equations.

Roots and Coefficients Relationship: Vieta’s Formulas

For a quadratic equation:

$$ ax^2 + bx + c = 0 $$

- The sum of roots:

$$ r_1 + r_2 = - \frac{b}{a} $$

- The product of roots:

$$ r_1 r_2 = \frac{c}{a} $$

In our case, the quadratic is:

$$ x^2 + h x + 10 = 0 $$

with $a = 1$, $b = h$, and $c = 10$.

Applying Vieta’s formulas simplifies the process of relating roots to coefficients:

- Sum of roots:

$$ r_1 + r_2 = - h $$

- Product of roots:

$$ r_1 r_2 = 10 $$

Part 1: Establishing the Known Root and Its Consequences

Given that one of the roots is 2, let's denote:

$$ r_1 = 2 $$

Our goal is to find:

- The second root r_2
- The value of h

This problem is straightforward yet rich in algebraic implications. We will approach it systematically.

Part 2: Deriving the Second Root

Given $r_1 = 2$, and knowing the product of roots:

$$r_1 r_2 = 10$$

we have:

$$2 \times r_2 = 10$$

which simplifies to:

$$r_2 = \frac{10}{2} = 5$$

Thus, the second root is 5.

This result aligns with expectations: since roots of quadratics are interconnected via their coefficients, knowing one root directly yields the other via the product relation.

Part 3: Computing the Coefficient h

Recall that:

$$r_1 + r_2 = - h$$

Substituting the roots:

$$2 + 5 = - h$$

which simplifies to:

$$7 = - h$$

Therefore:

$$h = -7$$

The value of h is -7.

This completes the primary objective: identifying the second root and the coefficient h based on the given root.

Part 4: Verifying the Solution

It’s essential to verify the solution within the context of the original quadratic:

$$x^2 + h x + 10 = 0$$

with $h = -7$. The quadratic becomes:

$$x^2 - 7 x + 10 = 0$$

Factoring:

$$x^2 - 7 x + 10 = (x - 2)(x - 5)$$

which confirms roots at 2 and 5, matching our earlier deductions.

Deep Dive: Underlying Principles and Broader Implications

While the problem appears straightforward, its solution exemplifies fundamental algebraic concepts with broader significance.

The Power of Vieta’s Formulas

Vieta's formulas not only streamline solutions but also provide insight into the intrinsic symmetry of quadratic roots and coefficients. They reveal that:

- The sum of roots directly relates to the linear coefficient.
- The product of roots relates to the constant term.

This duality facilitates problem-solving where one root is known, or certain coefficients are specified.

Significance in Mathematical Modeling

In applied mathematics, such relationships underpin models where parameters are unknown but related through observable solutions. For example:

- Engineering systems where roots represent physical quantities.
- Economics models where roots correspond to equilibrium points.

Understanding these fundamental relationships enhances analytical capabilities in diverse fields.

Part 5: Extending the Analysis

Although the initial problem is specific, it opens avenues for further exploration.

1. Impact of Changing the Constant Term

Suppose the constant term were different. How would this alter roots and coefficient relationships?

2. Multiple Known Roots

What if both roots are known? How would that determine the coefficients?

3. Complex Roots

If roots are complex, how do the relationships adapt? For example, when roots are conjugates, what implications follow?

Part 6: Educational Insights and Pedagogical Value

This problem serves as an excellent teaching tool, illustrating:

- The practical application of algebraic identities.
- The importance of Vieta’s formulas in problem-solving.
- The process of logical deduction based on known information.

By engaging with such problems, students develop critical thinking and deepen their understanding of quadratic relationships.

Conclusion

The problem:

> Given that one of the roots of $x^2 + h x + 10 = 0$ is 2, find the second root and the value of h.

encapsulates the elegant interconnectedness of algebraic elements. Through systematic application of Vieta’s formulas, we found:

- The second root: 5
- The coefficient h: -7

This solution not only addresses the immediate question but also underscores the broader significance of root-coefficient relationships in algebra. Mastery of these concepts empowers mathematicians, students, and professionals to analyze, model, and solve a variety of problems with confidence and clarity.

References

- Stewart, J. (2015). Precalculus: Mathematics for Calculus. Cengage Learning.
- Vieta, F. (1594). Introduction to Algebra.
- Larson, R., & Edwards, B. H. (2017). Precalculus with Limits. Cengage Learning.

End of Article

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