Given The Following Transfer Function And Input Signal : (i)G(s) S+3/ S^2 + 9s + 15 , = U(t)= 3 Sin(4t)

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Introduction

Understanding the behavior of dynamic systems is fundamental in control engineering, signal processing, and systems analysis. When analyzing such systems, one of the most vital tools at our disposal is the transfer function, which characterizes the system's response to various inputs. In this context, we are examining a specific transfer function coupled with a sinusoidal input signal, which is common in real-world applications such as electrical circuits, mechanical systems, and communication channels.

The given transfer function, $G(s) = \frac{s + 3}{s^2 + 9s + 15}$, describes the system's dynamics, including its poles and zeros, which influence stability and response characteristics. The input signal, $U(t) = 3 \sin(4t)$, is a sinusoidal excitation, and analyzing the system's response to such an input provides insight into the steady-state behavior, amplitude modulation, phase shift, and resonance phenomena.

This article aims to provide a comprehensive analysis of this system, exploring the transfer function's properties, the impact of the sinusoidal input, and the resulting output signal. We will delve into methods such as Laplace transforms, frequency response analysis, and steady-state sinusoidal response calculation to understand the system's behavior thoroughly.

Understanding the Transfer Function $G(s)$

Components of the Transfer Function

The transfer function $G(s) = \frac{s + 3}{s^2 + 9s + 15}$ is a rational function with a numerator (zero polynomial) and a denominator (pole polynomial). Let's analyze its components:

- Zero(s): The zero of $G(s)$ is at $s = -3$, since setting numerator to zero yields $s + 3 = 0$.
- Poles: The denominator $s^2 + 9s + 15$ can be factored or solved using quadratic formula:

$$
s = \frac{-9 \pm \sqrt{81 - 60}}{2} = \frac{-9 \pm \sqrt{21}}{2}
$$

Therefore, the poles are at approximately:

$$
s \approx \frac{-9 + 4.58}{2} \approx -2.21, \quad s \approx \frac{-9 - 4.58}{2} \approx -6.79
$$

The poles are located in the left half-plane, indicating a stable system, which is important when analyzing sinusoidal response.

Stability and Frequency Response

The stability of the system is confirmed by the poles' locations — all in the left half-plane. This suggests that the system's response to sinusoidal inputs will eventually settle into a steady-state oscillation without unbounded growth.

The frequency response of the system gives insight into how it reacts to sinusoidal inputs at various frequencies. Since the input frequency here is 4 rad/sec, evaluating the transfer function at $s = j \omega$ with $\omega = 4$ rad/sec allows us to determine the magnitude and phase shift of the output relative to the input.

Analyzing the Input Signal $U(t) = 3 \sin(4t)$

Sinusoidal Input Characteristics

The input signal is a sinusoid with amplitude 3 and angular frequency 4 rad/sec:

- Amplitude: 3
- Frequency: $f = \frac{\omega}{2\pi} \approx \frac{4}{6.283} \approx 0.637$ Hz
- Angular frequency: $\omega = 4$ rad/sec

Such signals are common in AC circuits, vibrations, and wave-based systems, making their analysis fundamental in understanding steady-state behaviors.

Relevance of Sinusoidal Inputs in System Analysis

Sinusoidal signals are essential because:

- They allow the use of frequency response techniques.
- They help in understanding how systems amplify or attenuate signals at different frequencies.
- They facilitate the analysis of phase shifts, which are crucial for control system stability and synchronization.

Calculating the System’s Response to the Sinusoidal

Input

Laplace Transform Approach

The system's output in response to the sinusoidal input can be analyzed using the Laplace transform:

- The Laplace transform of the input \(U(t) = 3 \sin(4t) \) is:

\[
U(s) = \frac{3 \times 4}{s^2 + 16} = \frac{12}{s^2 + 16}
\]

- The output \(Y(s) \) in Laplace domain is given by:

\[
Y(s) = G(s) \times U(s) = \frac{s + 3}{s^2 + 9s + 15} \times \frac{12}{s^2 + 16}
\]

This combined transfer function describes how the input propagates through the system.

Frequency Response Analysis

To analyze the steady-state sinusoidal response, evaluate \(G(s) \) at \(s = j \omega = j 4 \):

\[
G(j 4) = \frac{j 4 + 3}{(j 4)^2 + 9 (j 4) + 15}
\]

Calculating numerator and denominator:

- Numerator:

\[
j 4 + 3 = 3 + j 4
\]

- Denominator:

\[
(j 4)^2 + 9 (j 4) + 15 = -16 + j 36 + 15 = (-16 + 15) + j 36 = -1 + j 36
\]

The magnitude and phase of \(G(j 4) \) are:

- Magnitude:

\[
|G(j 4)| = \frac{\sqrt{3^2 + 4^2}}{\sqrt{(-1)^2 + 36^2}} = \frac{\sqrt{9 + 16}}{\sqrt{1 +

$$
1296}} = \frac{\sqrt{25}}{\sqrt{1297}} = \frac{5}{\sqrt{1297}} \approx \frac{5}{36.0} \approx 0.139
$$

- Phase:

$$
\angle G(j 4) = \arctan \left(\frac{4}{3} \right) - \arctan \left(\frac{36}{-1} \right)
$$

Since the denominator is negative real part, the phase calculation involves considering quadrants:

$$
\angle G(j 4) \approx \arctan(1.333) - \arctan(-36) \approx 53.13^\circ - (-88.37^\circ) = 141.5^\circ
$$

The phase shift indicates the output sinusoid will be attenuated by approximately 0.139 and shifted by roughly 141.5 degrees relative to the input.

Steady-State Output Signal

Using the magnitude and phase, the steady-state output $y_{ss}(t)$ is:

$$
y_{ss}(t) = |G(j 4)| \times 3 \times \sin(4t + \phi)
$$

where ϕ is the phase shift:

$$
\phi \approx 141.5^\circ \approx 2.47 \text{ radians}
$$

Thus,

$$
y_{ss}(t) \approx 0.139 \times 3 \times \sin(4t + 2.47) \approx 0.417 \sin(4t + 2.47)
$$

This result indicates the output sinusoid has a reduced amplitude and a phase delay relative to the input.

Implications and Applications

Practical Significance of the Analysis

Understanding the system's response to sinusoidal inputs is crucial for several reasons:

- System Design: Engineers can design filters or controllers to enhance or suppress specific frequency components.
- Signal Processing: Recognize how signals are attenuated or phase-shifted, affecting communication quality.
- Control Systems: Ensure stability and desired transient/steady-state behavior.

Applications in Real-World Systems

The principles discussed are applicable in:

- Electrical Circuits: Analyzing RLC circuits with sinusoidal voltage sources.
- Mechanical Systems: Vibration analysis in mechanical structures subjected to sinusoidal forces.
- Communication Systems: Signal modulation and filtering.
- Control Engineering: Designing controllers that compensate for phase shifts and amplitude attenuation at specific frequencies.

Conclusion

The analysis of a transfer function \(G(s) = \frac{s + 3}{s^2 + 9s + 15} \) with a sinusoidal input \(U(t) = 3 \sin(4t

Frequently Asked Questions

What is the transfer function G(s) provided in the problem?

The transfer function is G(s) = (s + 3) / (s^2 + 9s + 15).

What is the input signal applied to the system?

The input signal is a sinusoidal function: u(t) = 3 sin(4t).

How do you find the steady-state output of the system for the given input?

By calculating the system's frequency response at ω = 4 rad/sec and multiplying the input amplitude by the magnitude of G(j4), then determining the output as a sinusoid with that amplitude and phase shift.

What is the method to determine the output signal Y(t) for

sinusoidal input?

Use the frequency response method: evaluate $G(j\omega)$ at $\omega = 4$, find the magnitude and phase, then express the output as a sinusoid with the same frequency, scaled and phase-shifted accordingly.

Calculate the magnitude of the transfer function G(j4) at ω=4 rad/sec.

Calculate $G(j4) = (j4 + 3) / ((j4)^2 + 9j4 + 15)$. The magnitude is $|G(j4)| = \sqrt{(Re)^2 + (Im)^2}$, where Re and Im are the real and imaginary parts of $G(j4)$.

What is the approximate steady-state output amplitude for the given input?

The amplitude is approximately $3 \times |G(j4)|$, where $|G(j4)|$ is calculated as above. The exact value depends on the computed magnitude of $G(j4)$.

How does the transfer function influence the system's response to the sinusoidal input?

The transfer function determines the gain and phase shift at the input frequency, affecting the amplitude and phase of the steady-state output signal.

Additional Resources

In-Depth Analysis of the Transfer Function $G(s) = \frac{s+3}{s^2 + 9s + 15}$ with Input Signal $u(t) = 3 \sin(4t)$

Introduction

Understanding the behavior of a dynamic system in response to a particular input is fundamental in control systems and signal processing. The specific case of analyzing a transfer function with a sinusoidal input provides valuable insights into the system's steady-state and transient responses, frequency characteristics, and potential resonances. In this review, we will explore the transfer function:

$$
G(s) = \frac{s + 3}{s^2 + 9s + 15}
$$

and analyze its response to the sinusoidal input:

$$
u(t) = 3 \sin(4t)
$$

We will delve into the mathematical derivation of the system's output, interpret the physical meaning of the transfer function, examine its frequency response, and explore the implications of the sinusoidal input in the context of system behavior.

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1. Understanding the Transfer Function $G(s)$

1.1. Structure and Characteristics

The transfer function $G(s)$ is a ratio of two polynomials in the complex frequency domain. Its numerator and denominator reveal the zeros and poles of the system:

- Numerator: $s + 3$ (a first-order polynomial)
- Denominator: $s^2 + 9s + 15$ (a second-order polynomial)

This structure suggests that the system is a second-order system with a zero at $s = -3$, and poles determined by the quadratic denominator.

1.2. Poles and Zeros Analysis

- Zeros: The zero at $s = -3$ influences the system's frequency response, especially at frequencies near zero.
- Poles: Find the roots of the denominator:

$$
s^2 + 9s + 15 = 0
$$

Using the quadratic formula:

$$
s = \frac{-9 \pm \sqrt{81 - 60}}{2} = \frac{-9 \pm \sqrt{21}}{2}
$$

- Roots:

$$
s_{1,2} = \frac{-9 \pm \sqrt{21}}{2}
$$

Since $\sqrt{21} \approx 4.58$, the poles are approximately at:

$$
s_1 \approx \frac{-9 + 4.58}{2} \approx -2.21
$$
$$
s_2 \approx \frac{-9 - 4.58}{2} \approx -6.79
$$

Both poles are real and negative, indicating a stable system with no oscillatory poles. The pole at approximately -2.21 is closer to the imaginary axis than the other, influencing transient response speed.

1.3. System Stability and Response

- Stability: Since all poles have negative real parts, the system is BIBO stable (Bounded Input, Bounded Output).
- Type: It is a second-order system, but without complex conjugate poles, it exhibits overdamped or critically damped behavior depending on damping ratios.

---

2. Mathematical Response to Sinusoidal Input

2.1. Time-Domain Approach

Given the sinusoidal input:

$$
u(t) = 3 \sin(4t)
$$

The output $y(t)$ can be obtained via the frequency response approach or Laplace transform methods.

2.2. Frequency Response Method

Since the input is sinusoidal, the steady-state output will also be sinusoidal at the same frequency but with a different amplitude and phase:

$$
y(t) = |G(j\omega)| \times 3 \sin(4t + \phi)
$$

where:

- $\omega = 4$ rad/sec (frequency of input)
- $|G(j\omega)|$ is the magnitude of the transfer function evaluated at $s = j\omega$
- ϕ is the phase shift introduced by the system at ω

2.3. Calculating the Frequency Response at $\omega = 4$

Evaluate $G(s)$ at $s = j4$:

$$
G(j4) = \frac{j4 + 3}{(j4)^2 + 9(j4) + 15}
$$

Compute numerator:

\[

$$
N = j4 + 3
$$

Compute denominator:

$$
D = (j4)^2 + 9(j4) + 15
$$

- $(j4)^2 = -16$
- $9(j4) = j36$

So,

$$
D = -16 + j36 + 15 = (-16 + 15) + j36 = -1 + j36
$$

Therefore:

$$
G(j4) = \frac{3 + j4}{-1 + j36}
$$

---

3. Magnitude and Phase of $G(j4)$

3.1. Magnitude Calculation

$$
|G(j4)| = \frac{\sqrt{(3)^2 + (4)^2}}{\sqrt{(-1)^2 + (36)^2}} = \frac{\sqrt{9 + 16}}{\sqrt{1 + 1296}} = \frac{\sqrt{25}}{\sqrt{1297}} = \frac{5}{\sqrt{1297}}
$$

Numerically,

$$
\sqrt{1297} \approx 36.0
$$

Thus:

$$
|G(j4)| \approx \frac{5}{36} \approx 0.1389
$$

3.2. Phase Calculation

The phase of numerator:

$$
\angle N = \arctan \left(\frac{4}{3} \right) \approx 53.13^\circ
$$

The phase of denominator:

$$
\angle D = \arctan \left(\frac{36}{-1} \right) \approx \arctan(-36) \approx -88.4^\circ
$$

But since the real part of D is negative and imaginary part positive, the phase is in the second quadrant:

$$
\angle D = 180^\circ - \arctan \left(\frac{36}{1} \right) \approx 180^\circ - 88.4^\circ = 91.6^\circ
$$

Alternatively, to be precise:

$$
\angle D = \arctan \left(\frac{36}{-1} \right) \approx -88.4^\circ
$$

Since the real part is negative and imaginary positive, the actual phase is:

$$
\angle D \approx 180^\circ - 88.4^\circ = 91.6^\circ
$$

Now, the overall phase shift:

$$
\phi = \angle N - \angle D \approx 53.13^\circ - 91.6^\circ \approx -38.47^\circ
$$

This indicates the output sinusoid will lag the input by approximately 38.5 degrees.

---

4. Final Expression for Steady-State Output

Using the magnitude and phase:

$$
y(t) \approx 3 \times 0.1389 \times \sin(4t + 38.5^\circ) \approx 0.4167 \sin(4t + 38.5^\circ)
$$

This shows the system's response is a sinusoid with amplitude approximately 0.417 and a phase lag of about 38.5 degrees relative to the input.

---

5. Transient vs. Steady-State Response

5.1. Transient Response Components

- The transient response is primarily dictated by the system poles.
- Given the pole locations at approximately $s \approx -2.21$ and -6.79, the transient components decay exponentially with time constants:

$$
\tau_1 = \frac{1}{|s_1|} \approx \frac{1}{2.21} \approx 0.45 \text{ sec}
$$
$$
\tau_2 = \frac{1}{|s_2|} \approx \frac{1}{6.79} \approx 0.147 \text{ sec}
$$

- The transient response will die out quickly within a few time constants, leaving only the steady-state sinusoid.

5.2. Long-Term Behavior

- After initial transients, the output will asymptotically approach:

$$
y_{ss}(t) \approx 0.417 \sin(4t + 38.5^\circ)
$$

---

6. Physical Interpretation and System Behavior

6.1. System Response Characteristics

- The system acts as a bandpass filter at the input frequency $\omega = 4$ rad/sec, attenuating the input amplitude due to the low magnitude response.
- The phase lag indicates the system's delay in responding to the sinusoidal input.

6.2. Effect of Zero at $s = -3$

- The zero at \(s =

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