Graph The Following System Of Equations.3x + Y = 63x + 3y = 12What Is The Solution To The System? There

Graph The Following System Of Equations.3x + Y = 63x + 3y = 12What Is The Solution To The System? There is a common question among students learning algebra and coordinate graphing techniques: how do we find the solution to a system of equations? Understanding how to graph and interpret the intersection point(s) of these equations is fundamental in algebra, as it helps to solve real-world problems involving multiple variables. This article will guide you through the process step-by-step, exploring the methods to graph the system, find the solution, and interpret the results effectively.

Understanding Systems of Equations

Before diving into the graphing process, it’s important to understand what a system of equations is and what it represents.

What Is a System of Equations?

A system of equations consists of two or more equations with the same set of variables. The solutions to the system are the values of these variables that satisfy all equations simultaneously. In geometric terms, each equation represents a line (or curve), and the solution(s) are the point(s) where these lines intersect.

Types of Solutions

- One solution: The lines intersect at exactly one point.
- No solution: The lines are parallel and do not intersect.
- Infinite solutions: The lines coincide, meaning they are the same line.

Analyzing the Given System of Equations

The system provided is:

1. $3x + y = 6$

2. $3x + 3y = 12$

Note: The original input "63x + Y = 63x + 3y = 12" appears to be a typo or formatting issue. Based on typical algebra problems, it’s likely intended as:

- First equation: $3x + y = 6$
- Second equation: $3x + 3y = 12$

Let’s proceed with this interpretation.

Step 1: Write Both Equations in Standard Form

- Equation 1: $3x + y = 6$
- Equation 2: $3x + 3y = 12$

Graphing the Equations

Graphing each line helps visualize the solution.

Step 1: Find the x- and y-intercepts

Intercepts are points where the line crosses the axes, making plotting easier.

Equation 1: $3x + y = 6$

- x-intercept: set $y=0$:

$3x + 0 = 6 \rightarrow x = 2$

So, point: $(2, 0)$

- y-intercept: set $x=0$:

$3(0) + y = 6 \rightarrow y = 6$

So, point: $(0, 6)$

Equation 2: $3x + 3y = 12$

- x-intercept: set $y=0$:

$3x + 0 = 12 \rightarrow x = 4$

So, point: $(4, 0)$

- y-intercept: set $x=0$:

$3(0) + 3y = 12 \rightarrow 3y=12 \rightarrow y=4$

So, point: (0, 4)

Step 2: Plot the Lines

Using the intercepts:

- Line 1 passes through (2, 0) and (0, 6)
- Line 2 passes through (4, 0) and (0, 4)

Plot these points on a coordinate plane and draw straight lines through these points.

Finding the Solution Algebraically

While graphing provides a visual understanding, algebraic methods offer precise solutions.

Substitution Method

Since both equations are in standard form, substitution is straightforward.

- From Equation 1: $3x + y = 6 \rightarrow y = 6 - 3x$

- Substitute y into Equation 2:

$3x + 3(6 - 3x) = 12$

- Simplify:

$3x + 18 - 9x = 12$

- Combine like terms:

$-6x + 18 = 12$

- Solve for x:

$-6x = 12 - 18 \rightarrow -6x = -6$

$x = (-6)/(-6) = 1$

- Find y using y = 6 - 3x:

y = 6 - 3(1) = 6 - 3 = 3

Solution: (x, y) = (1, 3)

Verification

Check the solution in both equations:

- Equation 1: 3(1) + 3 = 3 + 3 = 6 ✔️
- Equation 2: 3(1) + 3(3) = 3 + 9 = 12 ✔️

Since both are satisfied, (1, 3) is the correct solution.

Interpreting the Graph and Solution

The intersection point (1, 3) confirms that the two lines cross at this coordinate, which is the unique solution to the system.

Understanding the Graph

- The lines intersect at exactly one point, indicating a consistent and independent system.
- The solution (1, 3) can be interpreted as the values of x and y that satisfy both equations simultaneously.

Real-World Applications

Such systems often appear in:
- Business and economics (profit maximization, cost analysis)
- Physics (motion equations)
- Engineering (system design constraints)

Additional Methods for Solving Systems of Equations

While graphing and substitution are common, other methods include:

Elimination Method

- Multiply equations to align coefficients
- Subtract or add equations to eliminate a variable
- Solve for the remaining variable

Example:

Equation 1: $3x + y = 6$

Equation 2: $3x + 3y = 12$

Subtract Equation 1 from Equation 2:

$(3x + 3y) - (3x + y) = 12 - 6$

$3x + 3y - 3x - y = 6$

$2y = 6 \rightarrow y = 3$

Plug back into Equation 1:

$3x + 3 = 6 \rightarrow 3x = 3 \rightarrow x = 1$

Solution remains (1, 3).

Graphing Method Summary

- Plot the equations on a coordinate plane
- Observe the intersection point
- Confirm the solution algebraically

Common Mistakes to Avoid

- Misreading equations: Ensure equations are correctly written and simplified.
- Incorrect plotting: Use accurate intercepts and scale.
- Calculation errors: Double-check arithmetic, especially when solving algebraically.
- Ignoring special cases: Parallel lines (no solution) or coincident lines (infinite solutions).

Conclusion

Graphing and solving systems of equations are essential skills in algebra that help understand relationships between variables. In our example, the system's solution at (1, 3) was found both graphically and algebraically, demonstrating the consistency of these methods. Whether for academic purposes or real-world applications, mastering these techniques allows you to analyze complex problems efficiently and accurately.

Remember, practice with different systems, including those with no solutions or infinite solutions, to strengthen your understanding of how lines behave in the coordinate plane. With patience and practice, you'll become proficient at graphing and solving systems of equations, making these mathematical tools invaluable for various disciplines.

Keywords: graphing systems of equations, solving systems algebraically, intersection point, coordinate plane, x-intercept, y-intercept, substitution method, elimination method, algebra practice, math tutorial

Frequently Asked Questions

How do you graph the system of equations 3x + y = 6 and 3x + 3y = 12?

To graph these equations, first rewrite them in slope-intercept form. The first equation becomes $y = -3x + 6$, and the second simplifies to $y = -x + 4$. Plot both lines using their y-intercepts and slopes, then identify the point where they intersect.

What is the solution to the system of equations 3x + y = 6 and 3x + 3y = 12?

The solution is the point where both lines intersect. Solving the system, we find that the lines are dependent, meaning they represent the same line, and thus, infinitely many solutions along that line.

Are the equations 3x + y = 6 and 3x + 3y = 12 consistent, inconsistent, or dependent?

They are dependent equations because one is a multiple of the other, representing the same line and having infinitely many solutions.

How can you determine if a system of equations has a unique solution when graphing?

A system has a unique solution if the graphs of the equations intersect at exactly one point. If the lines are parallel, there is no solution; if they are the same line, there are infinitely many solutions.

What does it mean if the two equations 3x + y = 6 and 3x + 3y = 12 are multiples of each other?

It means the equations represent the same line, indicating they are dependent and have infinitely many solutions along that line.

How do you find the intersection point of the lines represented by these equations?

Solve the equations simultaneously. For example, from 3x + y = 6, express y = 6 - 3x, then substitute into the second equation to find x, and subsequently y.

What is the importance of rewriting equations in slope-intercept form before graphing?

Rewriting in slope-intercept form (y = mx + b) makes it easier to plot the lines accurately by identifying slopes and y-intercepts directly.

Can the system 3x + y = 6 and 3x + 3y = 12 have no solution?

No, in this case, the lines are dependent and coincide; thus, they have infinitely many solutions. No solution would occur if the equations represented parallel lines, which is not the case here.

Additional Resources

Graph The Following System Of Equations.3x + Y = 63x + 3y = 12What Is The Solution To The System? There

In the world of algebra, systems of equations serve as foundational tools for understanding relationships between multiple variables. Graphing these systems offers a visual approach that not only simplifies complex calculations but also enhances conceptual understanding. Today, we delve into a particular system of equations, exploring how to interpret, graph, and ultimately solve it. By dissecting the equations step-by-step, we aim to provide clarity and insight for students, educators, and enthusiasts alike.

Understanding the System of Equations

Before diving into the graphing process, it's essential to clearly understand the equations involved. The system in question appears as:

1. $3x + y = 6$
2. $3x + 3y = 12$

At first glance, these equations seem straightforward, but their relationship is key to understanding the solution.

Simplifying and Analyzing the Equations

Step 1: Write equations in slope-intercept form

Converting each to $y = mx + b$ form makes graphing more straightforward.

Equation 1:

$3x + y = 6$

Subtract 3x from both sides:

$y = -3x + 6$

Equation 2:

$3x + 3y = 12$

Divide everything by 3:

$x + y = 4$

Now, solve for y:

$y = -x + 4$

Step 2: Recognize the forms and slopes

- Equation 1: $y = -3x + 6$ (slope = -3, y-intercept = 6)
- Equation 2: $y = -x + 4$ (slope = -1, y-intercept = 4)

Understanding these forms reveals the nature of the lines: both are straight lines with different slopes and intercepts, which suggests they may intersect at a single point.

Graphing the Equations

Graphing these equations visually demonstrates their relationship.

Step 1: Plot key points for each line

First line ($y = -3x + 6$):

- When $x = 0$: $y = 6 \rightarrow$ point (0,6)
- When $x = 2$: $y = -32 + 6 = -6 + 6 = 0 \rightarrow$ point (2,0)

Second line ($y = -x + 4$):

- When $x = 0$: $y = 4 \rightarrow$ point (0,4)
- When $x = 4$: $y = -4 + 4 = 0 \rightarrow$ point (4,0)

Step 2: Draw the lines

Using graph paper or digital graphing tools:

- Plot the points for each line.
- Draw straight lines through these points.

The intersection point of these two lines is the solution to the system.

Finding the Solution Algebraically

While graphing provides a visual solution, algebraic methods offer precise results.

Step 1: Set the equations equal to find the intersection point:

Since both equations are solved for y, equate the right sides:

$-3x + 6 = -x + 4$

Step 2: Solve for x:

Add x to both sides:

$-3x + x + 6 = 4$

Simplify:

$-2x + 6 = 4$

Subtract 6 from both sides:

-2x = -2

Divide both sides by -2:

x = 1

Step 3: Find y using x = 1 in either equation:

Using y = -x + 4:

y = -1 + 4 = 3

Solution: (1, 3)

Interpreting the Results

The algebraic solution confirms the point of intersection as (1, 3). This point satisfies both equations and represents the unique solution to the system.

Verifying the Solution

To ensure accuracy, substitute x = 1 and y = 3 into both original equations.

- Equation 1: 3(1) + 3 = 3 + 3 = 6 → true
- Equation 2: 3(1) + 3(3) = 3 + 9 = 12 → true

Both equations are satisfied, confirming the solution's correctness.

Significance of the Solution

Understanding the solution to a system of equations isn't just an academic exercise; it has practical implications:

- In real-world applications: Systems of equations model situations like supply and demand, physics problems, and engineering designs.
- In graphing: Visualizing solutions helps in understanding relationships and constraints between variables.
- In problem-solving: Combining algebraic and graphical methods enhances analytical skills and confidence.

Additional Insights

Are the lines parallel, intersecting, or coincident?

- Since the slopes are different (-3 vs. -1), the lines are neither parallel nor coincident.
- They intersect at exactly one point, indicating a unique solution.

What if the slopes were identical?

- If the slopes were equal but intercepts different, the lines would be parallel and have no solution.
- If both the slopes and intercepts were equal, the lines would coincide, resulting in infinitely many solutions.

The Broader Context of Systems of Equations

Graphing systems of equations is a fundamental skill in algebra, offering a bridge from symbolic manipulation to visual comprehension. It fosters a more intuitive grasp of how variables relate in various contexts. Whether tackling simple linear systems or more complex nonlinear ones, mastering graphing techniques enhances problem-solving capabilities.

Final Thoughts

The process of graphing and solving the system:

- Reveals the intersection point as (1, 3).
- Demonstrates how algebra and visualization complement each other.
- Reinforces the importance of understanding slopes, intercepts, and their implications.

In summary, by translating equations into a visual format and verifying algebraically, we gain a comprehensive understanding of the relationship between variables. This approach equips learners with essential tools for tackling a wide array of mathematical challenges, both theoretical and applied.

In conclusion, the system of equations provided is a classic example illustrating how graphical and algebraic methods converge to yield a precise solution. The intersection point, (1, 3), encapsulates the core relationship between x and y in this context, exemplifying the power of combining multiple mathematical strategies for problem-solving.

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