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Introduction

Understanding age-related problems can often seem daunting, especially when they involve multiple people and complex conditions. One such intriguing problem involves Amy, Ben, and Chris, whose ages are interconnected through averages and historical data. The problem states: "Help! 10 points! The average of Amy's, Ben's, and Chris's ages is 6. Four years ago, Chris was the same age as one of his friends," which invites us to delve into basic algebra, reasoning, and problem-solving strategies. These types of problems are not only excellent exercises for sharpening mathematical skills but also useful in real-life scenarios where understanding relationships and changes over time are essential.

In this article, we will explore how to analyze such age problems systematically, provide step-by-step solutions, and offer insights into solving similar problems efficiently. Whether you're a student preparing for exams, a teacher designing practice questions, or just a curious mind, this comprehensive guide will help you grasp the underlying concepts and improve your problem-solving skills.

Understanding the Problem: Breaking Down the Statement

Before jumping into calculations, it’s crucial to interpret the problem carefully. Let’s break down the given information:

- The average age of Amy, Ben, and Chris today is 6.
- Four years ago, Chris was the same age as one of his friends (this could be Amy or Ben, but the problem will clarify this).

The goal is to find the current ages of Amy, Ben, and Chris based on these clues.

Key points to consider:

- The average age formula:

$$
\frac{\text{Amy's age} + \text{Ben's age} + \text{Chris's age}}{3} = 6
$$

- The ages four years ago:

$$
\text{Amy's age} - 4, \quad \text{Ben's age} - 4, \quad \text{Chris's age} - 4
$$

- The specific relationship involving Chris's age four years ago:

$$
\text{Chris's age four years ago} = \text{Amy's age four years ago} \quad \text{or} \quad \text{Ben's age four years ago}
$$

Now, let’s formalize these points into mathematical expressions.

Setting Up Variables and Equations

To solve the problem systematically, assign variables to each person’s current age:

- Let A = Amy's current age
- Let B = Ben's current age
- Let C = Chris's current age

Given the problem statement, the average age is 6:

Equation 1: The average of Amy, Ben, and Chris today

$$
\frac{A + B + C}{3} = 6
$$

Multiplying both sides by 3:

$$
A + B + C = 18
$$

This is our first key equation.

Equation 2: Chris's age four years ago

Four years ago, Chris's age was:

$$
C - 4
$$

Similarly, Amy's and Ben's ages four years ago:

$$
A - 4, \quad B - 4
$$

The problem states: "Four years ago, Chris was the same age as one of his friends." This gives us two possible conditions:

- Condition 1: $C - 4 = A - 4$ (Chris was the same age as Amy four years ago)
- Condition 2: $C - 4 = B - 4$ (Chris was the same age as Ben four years ago)

Simplify both conditions:

- Condition 1:

$$
C - 4 = A - 4 \Rightarrow C = A
$$

- Condition 2:

$$
C - 4 = B - 4 \Rightarrow C = B
$$

Thus, Chris was the same age as either Amy or Ben four years ago, which implies:

- Case 1: $C = A$
- Case 2: $C = B$

In each case, we can substitute back into the first equation to find the values of A, B, and C.

Case 1: Chris is the same age as Amy now ($C = A$)

Substitute $C = A$ into the sum:

$$
A + B + C = 18 \Rightarrow A + B + A = 18 \Rightarrow 2A + B = 18
$$

Express B in terms of A:

$$
B = 18 - 2A
$$

Since ages are positive integers, A must satisfy:

$$
A > 0, \quad B > 0
$$

Calculate possible values of A:

- For $A = 1$:

$$
B = 18 - 2(1) = 16
$$

- For $A = 2$:

$$
B = 18 - 4 = 14
$$

- For $A = 3$:

$$
B = 18 - 6 = 12
$$

- For $A = 4$:

$$
B = 18 - 8 = 10
$$

- For $A = 5$:

$$
B = 18 - 10 = 8
$$

- For $A = 6$:

$$
B = 18 - 12 = 6
$$

- For $A = 7$:

$$
B = 18 - 14 = 4
$$

- For $A = 8$:

\[

$$
B = 18 - 16 = 2
$$

At $A = 9$:

$$
B = 18 - 18 = 0
$$

But ages cannot be zero or negative in this context, so the maximum A is 8.

Now, the corresponding ages are:

A	B	C (since $C = A$)
1	16	1
2	14	2
3	12	3
4	10	4
5	8	5
6	6	6
7	4	7
8	2	8

Similarly, check if these are reasonable ages in real life (e.g., ages being plausible for children).

Case 2: Chris is the same age as Ben now ($C = B$)

Substitute $C = B$:

$$
A + B + C = 18 \Rightarrow A + B + B = 18 \Rightarrow A + 2B = 18
$$

Express A as:

$$
A = 18 - 2B
$$

Again, A, B must be positive integers:

- For $B = 1$:

$$
A = 18 - 2 = 16
$$

- For $B = 2$:

$$
A = 18 - 4 = 14
$$

- For $B = 3$:

$$
A = 18 - 6 = 12
$$

- For $B = 4$:

$$
A = 18 - 8 = 10
$$

- For $B = 5$:

$$
A = 18 - 10 = 8
$$

- For $B = 6$:

$$
A = 18 - 12 = 6
$$

- For $B = 7$:

$$
A = 18 - 14 = 4
$$

- For $B = 8$:

$$
A = 18 - 16 = 2
$$

Again, ages should be plausible.

The possible solutions are:

B	C (since $C = B$)	A
1	1	16
2	2	14
3	3	12
4	4	10
5	5	8

6	6	6
7	7	4
8	8	2

Interpreting the Results

From these possible solutions, we observe:

- Ages vary widely, but the sum remains consistent.
- The plausible ages depend on contextual factors, like whether children are of appropriate ages.
- The key point is that in each scenario, the sum $A + B + C = 18$.

Verifying the Conditions and Finalizing the Solution