HELP!After A Rotation, A(-3, 4) Maps To A'(4, 3), B(4, -5) Maps To B'(-5, -4), And C(1,6) Maps To C'(6,

HELP!After A Rotation, A(-3, 4) Maps To A'(4, 3), B(4, -5) Maps To B'(-5, -4), And C(1,6) Maps To C'(6,

Understanding how points transform under rotation is fundamental in geometry, especially when analyzing figures and their properties. The problem presented involves a rotation that maps three points, A, B, and C, to their respective images A', B', and C'. Specifically, points A(-3, 4) map to A'(4, 3), B(4, -5) maps to B'(-5, -4), and C(1, 6) maps to C'(6, ...). This scenario raises several questions: What is the center of rotation? What is the angle of rotation? How do we verify the rotation? And how can understanding this help in solving similar geometric problems? In this comprehensive guide, we'll explore each of these questions and provide step-by-step methods to analyze and solve rotation problems in coordinate geometry.

Understanding Rotation in Coordinate Geometry

Rotation is a fundamental transformation in Euclidean geometry that turns every point of a figure around a fixed point, called the center of rotation, by a specified angle, either clockwise or counterclockwise. When dealing with coordinate points, rotation can be precisely described using algebraic formulas.

What Is a Rotation?

A rotation is a transformation that:

- Moves points around a fixed point called the center of rotation (denoted as (h, k))
- Turns the figure through a certain angle (denoted as □)
- Maintains the shape and size of the figure, making it an isometry

Key Features of Rotation

- Center of rotation (h, k): The fixed point around which the rotation occurs.
- Rotation angle (□): The measure of the rotation, in degrees or radians. Positive angles typically denote counterclockwise rotation, while negative angles indicate clockwise rotation.
- Direction: Rotation can be clockwise or counterclockwise.

Analyzing the Given Points and Their Images

Let's revisit the problem's data:

- Original points:
- A(-3, 4)
- B(4, -5)
- C(1, 6)

- Images after rotation:
- A'(4, 3)
- B'(-5, -4)
- C'(6, ...)

Notice that the image of point C is incomplete, but based on the pattern, it likely maps to (6, ...). For completeness, we'll assume C' is (6, y), and later, we'll verify this.

The main goal is to determine:

- The center of rotation (h, k)
- The angle of rotation (□)
- The direction of rotation (clockwise or counterclockwise)

Steps to Find the Center and Angle of Rotation

To analyze the rotation, follow these systematic steps:

1. Use the Known Points and Their Images

Identify pairs of points and their images:

Original Point	Image Point	Coordinates
A	A'	(-3, 4) □ (4, 3)
B	B'	(4, -5) □ (-5, -4)
C	C'	(1, 6) □ (6, ?) (to be determined)

2. Find the Center of Rotation (h, k)

The center of rotation is equidistant from each original point and its image, and it lies along the perpendicular bisector of the segment connecting each point and its image.

Method:

- Find the midpoint of each pair of points (original and image).
- Determine the perpendicular bisector of each segment.
- The intersection point of these bisectors is the center of rotation.

Calculations:

- For A and A':
- Midpoint M_A:

$$
\left(\frac{-3 + 4}{2}, \frac{4 + 3}{2} \right) = \left(\frac{1}{2}, \frac{7}{2} \right)
$$

- For B and B':
- Midpoint M_B:

$$
\left(\frac{4 + (-5)}{2}, \frac{-5 + (-4)}{2} \right) = \left(\frac{-1}{2}, -\frac{9}{2} \right)
$$

Next, determine the slopes of the segments:

- Slope of segment AA':

$$
m_{AA'} = \frac{3 - 4}{4 - (-3)} = \frac{-1}{7}
$$

- Slope of segment BB':

$$
m_{BB'} = \frac{-4 - (-5)}{-5 - 4} = \frac{1}{-9} = -\frac{1}{9}
$$

Perpendicular bisectors will have slopes that are negative reciprocals:

- Perpendicular bisector of AA' has slope:

$$
m_{pA} = 7
$$

- Perpendicular bisector of BB' has slope:

$$
m_{pB} = 9
$$

Now, write equations for these bisectors:

- For AA' midpoint $M_A (0.5, 3.5)$:

$$
y - 3.5 = 7 (x - 0.5)
$$

- For BB' midpoint $M_B (-0.5, -4.5)$:

$$
y + 4.5 = 9 (x + 0.5)
$$

Solve these two equations simultaneously to find the intersection point (h, k), which is the rotation center.

Equation 1:

$$
y = 7(x - 0.5) + 3.5 = 7x - 3.5 + 3.5 = 7x
$$

Equation 2:

\[
y = 9(x + 0.5) - 4.5 = 9x + 4.5 - 4.5 = 9x
\]

Set equal to find x:

\[
7x = 9x \Rightarrow 7x - 9x = 0 \Rightarrow -2x = 0 \Rightarrow x = 0
\]

Plug x = 0 into either equation:

\[
y = 7 \times 0 = 0
\]

Result:

\[
\boxed{
\text{Center of rotation } (h, k) = (0, 0)
}
\]

3. Verify the Center with the Third Point

Using point C(1, 6) and its image C'(6, ?). Assuming C' is (6, y), verify if the rotation center (0,0) maintains equal distances:

- Distance from center to C:

$$
d_{C} = \sqrt{(1 - 0)^2 + (6 - 0)^2} = \sqrt{1 + 36} = \sqrt{37}
$$

- Distance from center to C':

$$
d_{C'} = \sqrt{(6 - 0)^2 + y^2} = \sqrt{36 + y^2}
$$

For a pure rotation, these distances are equal:

$$
\sqrt{37} = \sqrt{36 + y^2} \Rightarrow 37 = 36 + y^2 \Rightarrow y^2 = 1 \Rightarrow y = \pm 1
$$

Assuming the rotation preserves orientation, and based on the pattern of the images, it's reasonable to select y = 1. Therefore, C' is (6, 1).

Calculating the Rotation Angle (□)

Now that we know:

- Center of rotation: (0, 0)
- Original point A: (-3, 4)
- Image point A': (4, 3)

We can find the rotation angle □ by analyzing the rotation of vector $\vec{A}$ to $\vec{A'}$.

Method:

- Find the angle between $\vec{A}$ and $\vec{A'}$ relative to the center.

Calculations:

- Calculate the arguments (angles) of the vectors:

$$
\text{Angle of } \vec{A} = \arctan \left(\frac{4}{-3} \right) = \arctan(-\tfrac{4}{3}) \approx -53.13^\circ
$$

- For $\vec{A'}$:

$$
\arctan \left(\frac{3}{4} \right) \approx 36.87^\circ
$$

- Determine the rotation angle:

$$
\theta = \text{angle of } \vec{A'} - \text{angle of } \vec{A} \approx 36.87^\circ - (-53.13^\circ) = 90
$$

Frequently Asked Questions

What is the center of rotation if A(-3, 4) maps to A'(4, 3) and B(4, -5) maps to B'(-5, -4)?

The center of rotation is at the origin (0, 0).

What is the angle of rotation that maps point A(-3, 4) to A'(4, 3)?

The rotation angle is 90 degrees clockwise (or 270 degrees counterclockwise).

Does the rotation preserve the distances between points in the figure?

Yes, rotations are isometries, so they preserve distances between points.

How can I verify the rotation maps B(4, -5) to B'(-5, -4)?

Calculate the rotation around the center and check if the images match; or verify the rotation angle and direction are consistent for both points.

What is the image of point C(1, 6) after the rotation?

Point C(1, 6) maps to C'(6, 1) after the rotation.

How do I determine the rotation matrix from the given points?

Use the coordinates of the original and image points to find the rotation angle and then construct the rotation matrix accordingly.

Is the rotation clockwise or counterclockwise based on the point mappings?

The mappings suggest a 90-degree clockwise rotation around the origin.

Can this rotation be represented by a transformation matrix?

Yes, for a 90-degree clockwise rotation around the origin, the matrix is [[0, 1], [-1, 0]].

What steps are involved in finding the rotation transformation from the

given points?

Determine the rotation angle and center, verify the mapping of multiple points, and then derive the rotation matrix or formula.

Are the points A, B, and C consistent with a single rotation transformation?

Yes, since all points map according to the same rotation, they are consistent with a single rotation transformation.

Additional Resources

HELP! After A Rotation, A(-3, 4) Maps To A'(4, 3), B(4, -5) Maps To B'(-5, -4), And C(1,6) Maps To C'(6, ...)

Understanding how geometric transformations, particularly rotations, work is fundamental in coordinate geometry, computer graphics, and various engineering applications. The problem at hand involves mapping three points through a rotation and analyzing the resulting transformations. This review aims to dissect the given scenario thoroughly, exploring the details of the rotation, its characteristics, and the implications of the mappings, providing clarity for students, educators, and professionals alike.

Introduction to Rotation in Coordinate Geometry

Rotation is a transformation that turns every point of a figure around a fixed point called the center of rotation by a specific angle and in a specified direction (clockwise or counterclockwise). It preserves the shape and size of the figure, making it an isometry—meaning distances and angles are maintained.

Key features of rotations include:

- Preservation of distances and angles.
- The center of rotation remains fixed.
- The angle of rotation determines how much points are rotated.
- The direction (positive or negative angle) determines clockwise or counterclockwise rotation.

In the given problem, the transformations involve rotating points A, B, and C to their respective images A', B', and C'. Analyzing these mappings can help us determine the rotation's center, angle, and direction.

Details of the Given Points and Mappings

Let's first clearly state the points and their images:

- Original points:
- A(-3, 4)
- B(4, -5)
- C(1, 6)

- Mapped points:
- A'(4, 3)
- B'(-5, -4)
- C'(6, ...)

The incomplete data for C' introduces an element of uncertainty that needs addressing. To proceed, we will analyze the possible rotation that maps A to A', B to B', and C to C', assuming C' is (6, y) for some y-coordinate.

Determining the Rotation Parameters

The goal is to find the center of rotation and the angle of rotation that maps each original point to its image.

Step 1: Analyzing the Rotation Center

Since rotation preserves distances from the center, the following should hold:

- The distance from the center to A should equal the distance from the center to A'.
- Similarly for B and B', and C and C'.

Suppose the center of rotation is at point O(h, k). Then:

- Distance OA = Distance OA'
- Distance OB = Distance OB'
- Distance OC = Distance OC'

Expressed mathematically:

$(-3 - h)^2 + (4 - k)^2 = (4 - h)^2 + (3 - k)^2$ ...(1)

$(4 - h)^2 + (-5 - k)^2 = (-5 - h)^2 + (-4 - k)^2$ ...(2)

$(1 - h)^2 + (6 - k)^2 = (6 - h)^2 + (y - k)^2$ ...(3)

Equation (3) involves the unknown y-coordinate of C', which is incomplete. For now, focus on equations (1) and (2).

Step 2: Solving for the Center O(h, k)

Expanding equations (1) and (2):

Equation (1):

$(-3 - h)^2 + (4 - k)^2 = (4 - h)^2 + (3 - k)^2$

□ $(h + 3)^2 + (4 - k)^2 = (h - 4)^2 + (3 - k)^2$

Expanding:

$(h^2 + 6h + 9) + (16 - 8k + k^2) = (h^2 - 8h + 16) + (9 - 6k + k^2)$

Simplify both sides:

Left: $h^2 + 6h + 9 + 16 - 8k + k^2 = h^2 + 6h + 25 - 8k + k^2$

Right: $h^2 - 8h + 16 + 9 - 6k + k^2 = h^2 - 8h + 25 - 6k + k^2$

Subtract common terms h^2 and k^2 from both sides:

$(6h + 25 - 8k) = (-8h + 25 - 6k)$

Bring all to one side:

$6h + 25 - 8k + 8h - 25 + 6k = 0$

Combine like terms:

$(6h + 8h) + (-8k + 6k) + (25 - 25) = 0$

$14h - 2k = 0$

Simplify:

$7h - k = 0$ ...(A)

Similarly, equation (2):

$(4 - h)^2 + (-5 - k)^2 = (-5 - h)^2 + (-4 - k)^2$

Expand:

$(16 - 8h + h^2) + (25 + 10k + k^2) = (25 + 10h + h^2) + (16 + 8k + k^2)$

Simplify:

Left: $16 - 8h + h^2 + 25 + 10k + k^2 = h^2 - 8h + 41 + 10k + k^2$

Right: $25 + 10h + h^2 + 16 + 8k + k^2 = h^2 + 10h + 41 + 8k$

Subtract h^2 and k^2:

$(-8h + 41 + 10k) = (10h + 41 + 8k)$

Bring all to one side:

$-8h + 41 + 10k - 10h - 41 - 8k = 0$

Combine like terms:

$(-8h - 10h) + (10k - 8k) + (41 - 41) = 0$

$-18h + 2k = 0$

Divide through by 2:

$-9h + k = 0$ ...(B)

Step 3: Solving for h and k

From equations (A) and (B):

(A): $7h - k = 0 \Rightarrow k = 7h$

(B): $-9h + k = 0 \Rightarrow k = 9h$

Set equal:

$7h = 9h \Rightarrow 7h - 9h = 0 \Rightarrow -2h = 0 \Rightarrow h = 0$

Then, $k = 7h = 0$

Center of rotation: O(0, 0)

This suggests the rotation occurs about the origin.

Determining the Rotation Angle

Having identified the center, the next step is to find the rotation angle □ that maps each point to its image.

The rotation formula about the origin:

- For a point (x, y), its image (x', y') after rotation by angle □:

x' = x cos □ - y sin □

y' = x sin □ + y cos □

Let's verify this for points A and B to find □.

Step 1: Using Point A(-3, 4) and A'(4, 3)

Set up the equations:

4 = (-3) cos □ - 4 sin □ ...(i)

3 = (-3) sin □ + 4 cos □ ...(ii)

Expressed as a system:

Equation (i): $4 = -3 \cos \square - 4 \sin \square$

Equation (ii): $3 = -3 \sin \square + 4 \cos \square$

Solve for $\cos \square$ and $\sin \square$:

From (i):

$4 = -3 \cos \square - 4 \sin \square$

From (ii):

$3 = -3 \sin \square + 4 \cos \square$

Rewrite as:

(1): $4 = -3 \cos \square - 4 \sin \square$

(2): $3 = -3 \sin \square + 4 \cos \square$

Express (2) as:

$3 = 4 \cos \square - 3 \sin \square$

Now, treat these as linear equations in $\cos \square$ and $\sin \square$.

Multiply (2) by 4:

$12 = 16 \cos \square - 12 \sin \square$

Multiply (1) by 3:

$12 = -9 \cos \theta - 12 \sin \theta$

Subtract these two equations:

$(12) - (12) = (16 \cos \theta + 9 \cos \theta) + (-12 \sin \theta + 12 \sin \theta)$

$0 = 25 \cos \theta + 0$

$\Rightarrow 25 \cos \theta = 0 \Rightarrow \cos \theta = 0$

Given $\cos \theta = 0$, from (2):

$3 = 4 \cdot 0 - 3 \sin \theta \Rightarrow 3 = -3 \sin \theta \Rightarrow \sin \theta = -1$

Since $\sin \theta = -1$ and $\cos \theta = 0$, $\theta = 270°$, or $-90°$, which is a rotation of 270° clockwise (or 90° counterclockwise).

Step 2: Validating with Point B(4, -5) and B'(-5, -4)

Check if the rotation of 270

Help After A Rotation A 3 4 Maps To A 4 3 B 4 5 Maps To B 5 4 And C 1 6 Maps To C 6

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