Help!! ASAP PLEASE!!!What Is The Solution To The System Of Eqautions:x[tex]x^{2} -2x +y= 8\\x-y=2[/tex]

Help!! ASAP PLEASE!!! What Is The Solution To The System Of Equations: \(x^{2} - 2x + y = 8 \) and \(x - y = 2 \)?

Are you struggling to find the solution to this system of equations? You're not alone! Many students and math enthusiasts often encounter systems that seem complex at first glance but can be simplified with the right approach. In this article, we will guide you step-by-step through solving the system:

\[
\begin{cases}
x^{2} - 2x + y = 8 \\
x - y = 2
\end{cases}
\]

By understanding the methods involved, you'll be able to solve similar problems confidently and efficiently.

Understanding the System of Equations

Before diving into the solution, let's analyze the structure of the given system:

- The first equation, \(x^2 - 2x + y = 8 \), is a quadratic in \(x \) with a linear term in \(y \).
- The second equation, \(x - y = 2 \), is a linear equation relating \(x \) and \(y \).

Our goal is to find all pairs \((x, y)\) that satisfy both equations simultaneously.

Approach to Solving the System

There are several methods to solve systems like this one:

- Substitution Method: Express one variable in terms of the other and substitute into the other equation.
- Elimination Method: Add or subtract equations to eliminate a variable.
- Graphical Method: Plot both equations and identify the intersection points.

Given the equations, the substitution method is most straightforward here because the second equation directly relates \(x \) and \(y \).

Step-by-Step Solution

Let's proceed with substitution:

Step 1: Express y in terms of x

From the second equation:

$$
x - y = 2 \implies y = x - 2
$$

Step 2: Substitute $y = x - 2$ into the first equation

The first equation:

$$
x^{2} - 2x + y = 8
$$

becomes:

$$
x^{2} - 2x + (x - 2) = 8
$$

Step 3: Simplify the resulting equation

Combine like terms:

$$
x^{2} - 2x + x - 2 = 8
$$

$$
x^{2} - x - 2 = 8
$$

Bring all terms to one side:

$$
x^{2} - x - 2 - 8 = 0
$$

$$
x^{2} - x - 10 = 0
$$

Step 4: Solve the quadratic equation for x

Quadratic equation:

$$
x^{2} - x - 10 = 0
$$

Use the quadratic formula:

$$
x = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}
$$

where $a = 1$, $b = -1$, and $c = -10$:

$$
x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-10)}}{2(1)} = \frac{1 \pm \sqrt{1 + 40}}{2} = \frac{1 \pm \sqrt{41}}{2}
$$

Thus, the solutions for x:

$$
x = \frac{1 + \sqrt{41}}{2} \quad \text{or} \quad x = \frac{1 - \sqrt{41}}{2}
$$

Step 5: Find corresponding y values

Recall $y = x - 2$, so:

- For $x = \frac{1 + \sqrt{41}}{2}$:

$$
y = \frac{1 + \sqrt{41}}{2} - 2 = \frac{1 + \sqrt{41} - 4}{2} = \frac{\sqrt{41} - 3}{2}
$$

- For $x = \frac{1 - \sqrt{41}}{2}$:

$$
y = \frac{1 - \sqrt{41}}{2} - 2 = \frac{1 - \sqrt{41} - 4}{2} = \frac{-\sqrt{41} - 3}{2}
$$

Final solutions:

\[
\boxed{
\left(\frac{1 + \sqrt{41}}{2}, \frac{\sqrt{41} - 3}{2} \right) \quad \text{and} \quad \left(\frac{1 -
\sqrt{41}}{2}, \frac{-\sqrt{41} - 3}{2} \right)
}
\]

Interpreting the Solutions

The solutions involve irrational numbers due to the square root of 41. These solutions represent the points where the parabola defined by the quadratic equation intersects with the line derived from the second equation.

Summary:

- The system has two real solutions.
- Both solutions involve irrational numbers because \(\sqrt{41}\) is irrational.
- Each solution pair \((x, y)\) satisfies both equations simultaneously.

Additional Tips for Solving Similar Systems

- Always look for the simplest substitution or elimination approach.
- When dealing with quadratics, be prepared to use the quadratic formula.
- Rationalize and simplify expressions where possible.
- Check solutions by substituting back into the original equations.

Conclusion

Solving systems of equations involving quadratic and linear equations can seem intimidating initially, but with a systematic approach, they become manageable. The key steps involve expressing one variable in terms of the other, substituting into the quadratic, solving the quadratic, and then back-substituting to find the paired variable. In our case, we've successfully identified the solutions to the system:

\[
\boxed{
\left(\frac{1 + \sqrt{41}}{2}, \frac{\sqrt{41} - 3}{2} \right) \quad \text{and} \quad \left(\frac{1 -
\sqrt{41}}{2}, \frac{-\sqrt{41} - 3}{2} \right)
}
\]

Keep practicing similar problems to strengthen your algebra skills!

Frequently Asked Questions

What are the steps to solve the system of equations: $x^2 - 2x + y = 8$ and $x - y = 2$?

First, solve the second equation for y: $y = x - 2$. Then substitute this into the first equation: $x^2 - 2x + (x - 2) = 8$. Simplify and solve for x, then find y using $y = x - 2$.

How do I substitute y from the second equation into the first?

Replace y in the first equation with $(x - 2)$, resulting in $x^2 - 2x + (x - 2) = 8$. Simplify to get a quadratic in terms of x.

What is the simplified quadratic equation after substitution?

After substitution, the equation becomes $x^2 - 2x + x - 2 = 8$, which simplifies to $x^2 - x - 2 = 8$. Then, bring all to one side: $x^2 - x - 10 = 0$.

How do I solve the quadratic equation $x^2 - x - 10 = 0$?

Use the quadratic formula $x = [1 \pm \sqrt{(1^2 - 41(-10))}]/(21)$. Calculate discriminant: $1 + 40 = 41$. So, $x = [1 \pm \sqrt{41}]/2$.

What are the solutions for x in the quadratic equation?

$x = [1 + \sqrt{41}]/2$ and $x = [1 - \sqrt{41}]/2$, which are approximately $x \approx 3.70$ and $x \approx -2.70$.

How do I find the corresponding y-values for each x?

Use $y = x - 2$ for each x. For $x \approx 3.70$, $y \approx 1.70$; for $x \approx -2.70$, $y \approx -4.70$.

What are the final solutions to the system of equations?

The solutions are approximately $(x, y) \approx (3.70, 1.70)$ and $(-2.70, -4.70)$.

Can this system be solved graphically?

Yes, graphing the parabola $x^2 - 2x + y = 8$ and the line $x - y = 2$ will show their intersection points, which correspond to the solutions.

Additional Resources

Understanding and Solving the System of Equations: A Step-by-Step Approach

When faced with a system of equations like:

$$
\begin{cases}
x^{2} - 2x + y = 8 \\
x - y = 2
\end{cases}
$$

the key is to methodically analyze and solve for the unknown variables x and y. This type of problem is fundamental in algebra, often encountered in various mathematical contexts, including algebraic modeling, calculus, and applied sciences. Let's delve into the process of solving such a system, exploring various methods, their applications, and the significance of each step.

Understanding the System of Equations

Before solving, it’s essential to understand what the system represents and the nature of its equations.

Types of Equations in the System

- First Equation: $x^{2} - 2x + y = 8$

- This is a quadratic equation in terms of x, with a linear term in y.
- It resembles a parabola when viewed in the xy-plane, but since it involves both x and y, it can be viewed as a conic section (specifically, a parabola shifted or scaled).

- Second Equation: $x - y = 2$

- This is a linear equation, representing a straight line in the xy-plane.

Geometric Interpretation

- The second equation defines a straight line: every point (x, y) satisfying $y = x - 2$.
- The first equation can be rewritten as $y = 8 - x^{2} + 2x$, which is a parabola opening downward or upward depending on the sign conventions, but here, it's explicitly quadratic in x.

Understanding these helps visualize the problem: the solution set is the intersection point(s) of the parabola and the line.

Methods for Solving the System

Several techniques are at your disposal for solving such systems:

1. Substitution Method

This is the most straightforward approach when one equation is already solved for one variable.

- Step 1: Solve the linear equation for one variable.
- Step 2: Substitute into the quadratic equation.
- Step 3: Solve the resulting quadratic in one variable.
- Step 4: Back-substitute to find the other variable.

2. Graphical Method

Plotting both equations on the same coordinate plane to find their intersection points visually.

- Useful for approximate solutions.
- Good for understanding the nature of solutions (none, one, or multiple).

3. Elimination Method

Less practical here due to the quadratic nature but useful if the system consists of two linear equations or can be manipulated into linear form.

4. Using Algebraic Manipulation and Factoring

- Particularly effective when the quadratic can be factored easily.
- Critical for solving the quadratic equations that emerge.

Step-by-Step Solution Using Substitution

Let's illustrate the solution process in detail.

Step 1: Solve the linear equation for y

Given:

$$
x - y = 2
$$

Rearranged:

$$
y = x - 2
$$

This simplifies the problem, allowing substitution into the quadratic.

Step 2: Substitute $y = x - 2$ into the quadratic equation

Original quadratic:

$$
x^{2} - 2x + y = 8
$$

Substitute:

$$
x^{2} - 2x + (x - 2) = 8
$$

Simplify:

$$
x^{2} - 2x + x - 2 = 8
$$

$$
x^{2} - x - 2 = 8
$$

Bring all to one side:

$$
x^{2} - x - 2 - 8 = 0
$$

$$
x^{2} - x - 10 = 0
$$

Step 3: Solve the quadratic in x

Quadratic equation:

$$
x^{2} - x - 10 = 0
$$

Apply quadratic formula:

$$
x = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}
$$

where $a=1$, $b=-1$, $c=-10$.

Calculate discriminant:

$$
\Delta = (-1)^2 - 4 \times 1 \times (-10) = 1 + 40 = 41
$$

Solution:

$$
x = \frac{1 \pm \sqrt{41}}{2}
$$

Since $\sqrt{41}$ is irrational (~6.4), solutions are:

$$
x_1 = \frac{1 + \sqrt{41}}{2}
$$
$$
x_2 = \frac{1 - \sqrt{41}}{2}
$$

Numerical approximations:

$$
x_1 \approx \frac{1 + 6.4}{2} \approx \frac{7.4}{2} \approx 3.7
$$
$$
x_2 \approx \frac{1 - 6.4}{2} \approx \frac{-5.4}{2} \approx -2.7
$$

Finding Corresponding y Values

Recall:

$$
y = x - 2
$$

Calculate for each x:

- For $x_1 \approx 3.7$:

$$
y_1 \approx 3.7 - 2 = 1.7
$$

- For $x_2 \approx -2.7$:

$$
y_2 \approx -2.7 - 2 = -4.7
$$

Final solutions:

$$
\boxed{
\begin{cases}
x \approx 3.7,\quad y \approx 1.7 \\
x \approx -2.7,\quad y \approx -4.7
\end{cases}
}
$$

These are approximate solutions due to the irrational square root.

Verifying the Solutions

It's crucial to verify solutions by substituting back into the original equations.

- For $(x, y) \approx (3.7, 1.7)$:

- First equation:

$$
(3.7)^2 - 2(3.7) + 1.7 \approx 13.69 - 7.4 + 1.7 = 7.99 \approx 8
$$

- Second equation:

$$
3.7 - 1.7 = 2
$$

- For $(x, y) \approx (-2.7, -4.7)$:

- First equation:

$$
(-2.7)^2 - 2(-2.7) + (-4.7) \approx 7.29 + 5.4 - 4.7 = 7.99 \approx 8
$$

- Second equation:

$$
-2.7 - (-4.7) = 2
$$

Both solutions satisfy the system within reasonable approximation, confirming their validity.

Alternative Methods and Deeper Insights

While substitution is effective here, sometimes other methods are preferable or necessary:

Graphical Solution

Plotting $y = x - 2$ and $y = 8 - x^{2} + 2x$:

- The linear graph: a straight line with slope 1, intercept -2.
- The quadratic graph: a parabola opening downward (since the quadratic term is positive, but the equation is in $y = 8 - x^2 + 2x$), which can be rewritten as $y = -x^{2} + 2x + 8$), with vertex at:

$$
x_{vertex} = -\frac{b}{2a} = -\frac{2}{2 \times (-1)} = 1
$$

$$
y_{vertex} = - (1)^2 + 2(1) + 8 = -1 + 2 + 8 = 9
$$

Plotting these shows two intersection points, matching our algebraic solutions.

Numerical Approximation and Technology

Using graphing calculators, Desmos, or algebra software like WolframAlpha can provide quick visual and numerical solutions, especially for more complex systems.

Understanding the Nature of the Solutions

- Number of solutions: The quadratic discriminant ($\Delta = 41$) is positive, indicating two real solutions.
- Type of solutions: Both solutions are real and distinct.
- Graphical interpretation: The parabola and line intersect at two points, confirming the algebraic results.

Additional Considerations and Applications

Real-World Contexts

Systems like these appear across various fields:

- Physics: Modeling projectile motion with linear and quadratic terms.
- Economics: Optimization problems with quadratic cost functions.
- Engineering: Circuit analysis involving quadratic voltage or current

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