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When working with polynomial functions, one of the fundamental skills is factoring the polynomial based on its roots or zeros. Knowing the roots of a polynomial allows us to determine its complete factorization, which is essential for graphing, solving equations, and understanding the behavior of the function. In this article, we will explore how to utilize the given roots of a polynomial function, specifically F(4)=0, F(-1)=0, and F(3/2)=0, to select the factors of F(x). We will walk through the concepts step-by-step, providing a detailed explanation suitable for students and enthusiasts eager to deepen their understanding of polynomial factorization.

Understanding Polynomial Roots and Factors

Before diving into specific calculations, it’s crucial to understand the relationship between the roots of a polynomial and its factors.

What Are Roots of a Polynomial?

Roots, zeros, or solutions of a polynomial are the values of x for which the polynomial evaluates to zero. For example, if F(c) = 0, then x = c is a root of the polynomial F(x).

Connecting Roots and Factors

The Fundamental Theorem of Algebra states that every polynomial can be factored into linear factors over the complex numbers (and into real factors if roots are real). Specifically, if a real polynomial has a root at x = c, then (x - c) is a factor of that polynomial.

For example:
- If 4 is a root of F(x), then (x - 4) is a factor.
- If -1 is a root, then (x + 1) is a factor.
- If 3/2 is a root, then (x - 3/2) is a factor.

In most cases, especially when dealing with real coefficients, it is preferable to express all factors with integer or rational coefficients. Therefore, factors involving fractions are often rewritten to clear the denominator.

Using the Given Roots to Find Factors of F(x)

Given the roots:
- F(4) = 0
- F(-1) = 0
- F(3/2) = 0

We want to find the factors of F(x). The initial step is to translate each root into a corresponding factor.

Step 1: Write the Basic Factors

Based on the roots:
- Root at x = 4 corresponds to factor (x - 4)
- Root at x = -1 corresponds to factor (x + 1)
- Root at x = 3/2 corresponds to factor (x - 3/2)

However, to work with polynomial expressions more conveniently, especially when multiplying factors, it’s better to clear fractions in the factors related to roots that are rational numbers.

Step 2: Rationalize Factors with Fractions

The factor corresponding to the root x = 3/2 is (x - 3/2). To clear the fraction, multiply numerator and denominator by 2:

$$
x - \frac{3}{2} = \frac{2x - 3}{2}
$$

Since the scalar factor (denominator 2) doesn't affect the roots, the factor is equivalent to (2x - 3), which is a polynomial with integer coefficients.

Therefore:

- The factor corresponding to root at x = 4 is (x - 4)
- The factor corresponding to root at x = -1 is (x + 1)
- The factor corresponding to root at x = 3/2 is (2x - 3)

Step 3: Form the Complete Factorization of F(x)

Assuming the polynomial is monic (leading coefficient of 1), the factors are multiplied together:

$$
F(x) = k \times (x - 4)(x + 1)(2x - 3)
$$

where k is a constant coefficient that can be determined if additional information about the polynomial (such as a specific value of F(x) at a point) is provided.

Constructing the Polynomial with the Given Factors

Suppose we want to expand the factors to obtain the polynomial expression for F(x). This process involves multiplying the factors step-by-step.

Step 1: Multiply the Binomials (x - 4) and (x + 1)

$$
(x - 4)(x + 1) = x \times x + x \times 1 - 4 \times x - 4 \times 1
$$
$$
= x^2 + x - 4x - 4
$$
$$
= x^2 - 3x - 4
$$

Step 2: Multiply the Result by (2x - 3)

$$
(x^2 - 3x - 4)(2x - 3)
$$

Using distributive property (FOIL method):

$$
x^2 \times 2x = 2x^3
$$
$$
x^2 \times (-3) = -3x^2
$$
$$
-3x \times 2x = -6x^2
$$
$$
-3x \times (-3) = 9x
$$
$$
-4 \times 2x = -8x
$$
$$
-4 \times (-3) = 12
$$

Now, sum all these:

$$
2x^3 + (-3x^2 - 6x^2) + (9x - 8x) + 12
$$

$$
= 2x^3 - 9x^2 + x + 12
$$

Therefore, the polynomial (up to constant factor k) is:

$$
F(x) = k \times (2x^3 - 9x^2 + x + 12)
$$

If the polynomial is monic (leading coefficient of 1), then $k = \frac{1}{2}$.

Determining the Constant Coefficient k

In many problems, additional information such as a specific value of F(x) at a point is provided to determine k. For example, if F(0) is known, you can substitute x = 0 into the expanded polynomial and solve for k.

Suppose F(0) = 0 (which would be consistent with having the root at x=0), then:

$$
F(0) = k \times (2 \times 0^3 - 9 \times 0^2 + 0 + 12) = k \times 12
$$

If F(0) = 0, then:

$$
k \times 12 = 0 \implies k = 0
$$

which makes F(x) the zero polynomial, so more information would be needed.

Alternatively, if no such information is provided, the general form of the polynomial with known roots is:

$$
F(x) = a \times (x - 4)(x + 1)(2x - 3)
$$

where a is any non-zero constant.

Summary of Factors for F(x)

Based on the roots provided, the factors of F(x) are:

- $\boxed{(x - 4)}$
- $\boxed{(x + 1)}$
- $\boxed{(2x - 3)}$

The complete polynomial, assuming a leading coefficient of 1, is:

$$
F(x) = (x - 4)(x + 1)(2x - 3) = 2x^3 - 9x^2 + x + 12
$$

or, with an arbitrary leading coefficient a:

$$
F(x) = a \times (x - 4)(x + 1)(2x - 3)
$$

Additional Considerations and Applications

Understanding how to find factors from roots is fundamental in polynomial algebra. It has several practical applications:

- Solving polynomial equations: Factoring simplifies solving for roots.
- Graphing polynomials: Roots determine intercepts on the x-axis.
- Polynomial interpolation: Roots are essential when constructing polynomials passing through specific points.
- Factor theorem application: Validates factors based on roots.

Moreover, knowing how to handle roots with fractional values ensures flexibility in solving more complex polynomial problems.

Conclusion

In summary, given the roots of a polynomial function, you can directly determine its factors. For the roots at x = 4, x = -1, and x = 3/2, the corresponding factors are (x - 4), (x + 1), and (2x - 3), respectively. Multiplying these factors provides the polynomial's explicit form, which can be further adjusted by a constant coefficient depending on additional information. Mastery of this process enables a deeper understanding of polynomial behavior and enhances problem-solving skills in algebra and calculus.

Remember, always verify your roots and factors, and use the fundamental theorem of algebra as your guide when constructing polynomial functions from their roots. With practice, selecting factors of a polynomial based on given zeros will become

Frequently Asked Questions

What are the roots of the polynomial function F(x) based on the given information?

The roots of F(x) are x = 4, x = -1, and x = 3/2, because these values make F(x) equal to zero.

How can I express F(x) using its roots?

F(x) can be written as F(x) = k(x - 4)(x + 1)(x - 3/2), where k is a constant multiplier.

What additional information do I need to fully determine F(x)?

You need to know the value of the leading coefficient k or a specific point on the graph to find the exact form of F(x).

Why are the factors of F(x) important in understanding its behavior?

Factors reveal the roots and multiplicities of the polynomial, helping to analyze where the function crosses or touches the x-axis and its overall shape.

How do the given roots influence the graph of F(x)?

The roots at x = 4, -1, and 3/2 indicate that the graph crosses the x-axis at these points, with the specific shape depending on the multiplicity of each root.

Additional Resources

I Need Help! Navigating Polynomial Factors with Given Roots

In the realm of algebra, understanding how to determine the factors of a polynomial function based on its roots is an essential skill for students and educators alike. When given specific zeros of a function, the task becomes to reconstruct the polynomial's factors that correspond to these roots. This process not only deepens comprehension of polynomial behavior but also hones problem-solving skills critical for higher-level mathematics. In this article, we explore how to select the factors of a polynomial function \(F(x) \) given the roots \(F(4) = 0 \), \(F(-1) = 0 \), and \(F(3/2) = 0 \), presenting a comprehensive, analytical approach designed to clarify each step involved.

Understanding the Roots and Zeroes of a Polynomial

Roots and Zeroes: Definitions and Significance

At the core of polynomial functions lies the concept of roots or zeroes, which are the values of x that satisfy the equation $F(x) = 0$. These points are critical because they indicate where the graph of the polynomial intersects or touches the x-axis.

For a polynomial $F(x)$, if $x = r$ is a root, then $(x - r)$ is a factor of $F(x)$. Conversely, every factor of the polynomial corresponds to a root, and the degree of the root (multiplicity) determines how the graph behaves at that point—for example, whether it crosses the x-axis or just touches it.

Roots from Given Conditions

The problem provides specific conditions:

- $F(4) = 0$
- $F(-1) = 0$
- $F(3/2) = 0$

Each of these indicates that $x = 4$, $x = -1$, and $x = 3/2$ are roots of the polynomial $F(x)$. Recognizing these roots allows us to identify the corresponding factors directly:

- $(x - 4)$
- $(x + 1)$
- $\left(x - \frac{3}{2}\right)$

The next step involves constructing the polynomial as a product of these factors, considering their multiplicities, and understanding how the polynomial's degree and leading coefficient come into play.

Constructing the Polynomial Factors

Fundamental Theorem of Algebra and Its Application

The Fundamental Theorem of Algebra states that every polynomial of degree n has exactly n roots in the complex number system, counting multiplicities. When the roots are known, the polynomial can be expressed as:

\[

$$
F(x) = a \times (x - r_1)^{m_1} \times (x - r_2)^{m_2} \times \dots \times (x - r_k)^{m_k}
$$

where:

- a is a non-zero constant (leading coefficient),
- r_i are the roots,
- m_i are the multiplicities of the roots.

In our case, assuming each root is simple (multiplicity 1), the polynomial can initially be written as:

$$
F(x) = a \times (x - 4) \times (x + 1) \times \left(x - \frac{3}{2}\right)
$$

The degree of $F(x)$ is therefore 3, assuming no repeated roots.

Converting Roots into Factors

Expressing the roots as factors involves recognizing their algebraic form:

- For $x = 4$, the factor is $(x - 4)$.
- For $x = -1$, the factor is $(x + 1)$.
- For $x = 3/2$, the factor is $\left(x - \frac{3}{2}\right)$. To clear the fraction, multiply numerator and denominator:

$$
x - \frac{3}{2} = \frac{2x - 3}{2}
$$

To keep the polynomial in standard form, multiply the entire polynomial by 2 to eliminate denominators.

Formulating the Polynomial Expression

Step-by-Step Construction

1. Identify the factors:

$$
F(x) = a \times (x - 4) \times (x + 1) \times \left(x - \frac{3}{2} \right)
$$

2. Express the fractional factor as an equivalent polynomial:

$$
x - \frac{3}{2} = \frac{2x - 3}{2}
$$

3. Include the denominator as a coefficient:

$$
F(x) = a \times (x - 4) \times (x + 1) \times \frac{2x - 3}{2}
$$

4. Combine all factors into a single expression:

$$
F(x) = a \times \frac{(x - 4)(x + 1)(2x - 3)}{2}
$$

5. Decide on the leading coefficient a:

- When the polynomial is fully expanded, the coefficient a scales the polynomial. Typically, unless specified, $a = 1$ for simplicity, but it can be adjusted based on additional conditions such as specific $F(x)$ values at other points.

6. Eliminate the denominator:

Multiply numerator and denominator by 2:

$$
F(x) = a \times (x - 4)(x + 1)(2x - 3)
$$

Now, the polynomial is in a standard form with integer coefficients:

$$
F(x) = a \times (x - 4)(x + 1)(2x - 3)
$$

Expanding the Polynomial Factors

Performing Stepwise Expansion

To understand the polynomial's explicit form, expand the factors:

1. Multiply the first two factors:

$$
(x - 4)(x + 1) = x \times x + x \times 1 - 4 \times x - 4 \times 1 = x^2 + x - 4x - 4 = x^2 - 3x - 4
$$

2. Multiply the result by $(2x - 3)$:

$$
(x^2 - 3x - 4)(2x - 3)
$$

Using distributive property:

$$
x^2 \times 2x = 2x^3 \\
x^2 \times (-3) = -3x^2 \\
-3x \times 2x = -6x^2 \\
-3x \times (-3) = 9x \\
-4 \times 2x = -8x \\
-4 \times (-3) = 12
$$

Now, sum all terms:

$$
2x^3 + (-3x^2) + (-6x^2) + 9x - 8x + 12
$$

Combine like terms:

$$
2x^3 + (-3x^2 - 6x^2) + (9x - 8x) + 12 = 2x^3 - 9x^2 + x + 12
$$

3. Include the leading coefficient a:

$$
F(x) = a \times (2x^3 - 9x^2 + x + 12)
$$

If no further constraints are given, the simplest choice is $a = 1$:

$$
F(x) = 2x^3 - 9x^2 + x + 12
$$

This polynomial satisfies the initial roots provided: plugging in $x = 4$, -1, or $3/2$ yields zero.

Verifying the Roots and Final Polynomial

Verification of Roots

To confirm, substitute each root into the polynomial:

- $x = 4$:

$$
F(4) = 2(4)^3 - 9(4)^2 + 4 + 12 = 2 \times 64 - 9 \times 16 + 4 + 12 = 128 - 144 + 4 + 12 = 0
$$

- $x = -1$:

$$
F(-1) = 2(-1)^3 - 9(-1)^2 + (-1) + 12 = -2 - 9 + (-1) + 12 = 0
$$

- $x = 3/2$:

$$
F\left(\frac{3}{2}\right) = 2 \times \left(\frac{3}{2}\right)^3 - 9 \times \left(\frac{3}{2}\right)^2 + \frac{3}{2} + 12
$$

Calculate each term:

$$
\left(\frac{3}{2}\right)^3 = \frac{27}{8}
$$

\[
\left(\frac{3}{

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