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Introduction

The concept of wave-particle duality is fundamental in quantum mechanics, introducing the idea that particles exhibit wave-like behavior under certain conditions. One of the most significant developments in this domain was Louis de Broglie's hypothesis, which proposed that particles such as electrons and protons possess a wavelength inversely proportional to their momentum. This phenomenon, known as de Broglie wavelength, bridges classical and quantum physics and has profound implications for our understanding of microscopic particles, especially when they move at relativistic speeds.

In this article, we aim to derive and demonstrate the expression for the de Broglie wavelength of a particle with charge e, rest mass M_0, moving at relativistic velocities. We will systematically explore the necessary concepts, including relativistic momentum, energy, and the wave-particle duality principle, to establish a comprehensive understanding of this fundamental quantum property.

Fundamentals of Wave-Particle Duality

Historical Context
The wave-particle duality originated from experimental observations such as the photoelectric effect and electron diffraction, which could not be explained solely by classical physics. De Broglie postulated that particles have wave-like properties, characterized by a wavelength λ.

De Broglie Hypothesis
De Broglie hypothesized that the wavelength λ associated with a particle is related to its momentum p by:

$$
\lambda = \frac{h}{p}
$$

where:
- h is Planck's constant ($6.626 \times 10^{-34} \, \text{Js}$)
- p is the relativistic momentum of the particle

This relation is valid regardless of whether the particle is moving at non-relativistic or relativistic speeds, provided p is appropriately calculated.

Relativistic Dynamics of a Particle

Rest Mass and Rest Energy
A particle's rest mass M_0 is an intrinsic property, and its rest energy E_0 is given by Einstein's mass-energy equivalence:

$$
E_0 = M_0 c^2
$$

where c is the speed of light in vacuum ($3 \times 10^8 \, \text{m/s}$).

Total Energy and Relativistic Momentum

When a particle moves at a velocity v close to c, its total energy E and relativistic momentum p are given by:

$$
E = \gamma m_0 c^2
$$

$$
p = \gamma m_0 v
$$

where γ is the Lorentz factor:

$$
\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}
$$

The Lorentz factor accounts for relativistic effects at high velocities.

Deriving the De Broglie Wavelength at Relativistic Speeds

Step 1: Expressing Momentum in Terms of Velocity

The relativistic momentum p of a particle is:

$$
p = \gamma m_0 v
$$

This relation indicates that as $v \to c$, $p \to \infty$, meaning particles require infinite energy to reach the speed of light.

Step 2: Rewriting p in Terms of Total Energy

Since total energy E relates to γ as:

$$
E = \gamma m_0 c^2
$$

we can rewrite p as:

$$
p = \frac{E}{c^2} v
$$

but it is more straightforward to directly relate p and E via the relativistic energy-momentum relation:

$$
E^2 = (pc)^2 + (m_0 c^2)^2
$$

which leads to:

$$
p = \frac{\sqrt{E^2 - m_0^2 c^4}}{c}
$$

Step 3: Expressing p in Terms of E

The relativistic relation for momentum becomes:

$$
p = \frac{\sqrt{E^2 - m_0^2 c^4}}{c}
$$

This expression is valid for any velocity, including relativistic speeds.

Step 4: De Broglie Wavelength Expression

Using de Broglie's hypothesis:

$$
\lambda = \frac{h}{p}
$$

we substitute p with the relativistic expression:

$$
\boxed{
\lambda = \frac{h c}{\sqrt{E^2 - M_0^2 c^4}}
}
$$
This is the general formula for the de Broglie wavelength of a relativistic particle with total energy E.

Special Case: Particle Accelerated to Relativistic Speeds

When the Particle Is Accelerated to High Energy
Suppose a particle is accelerated from rest to a high velocity v, gaining a total energy E. Its de Broglie wavelength becomes:
$$
\lambda = \frac{h c}{\sqrt{E^2 - M_0^2 c^4}}
$$

- At non-relativistic speeds: $E \approx M_0 c^2 + \frac{1}{2} M_0 v^2$, and the wavelength reduces to the classical form:
$$
\lambda \approx \frac{h}{p}
$$
- At relativistic speeds: The wavelength diminishes significantly as p (and E) increases, indicating more wave-like behavior at higher energies.

Incorporating Charge e into the Model

While the de Broglie wavelength derivation primarily depends on the particle's momentum and energy, the charge e plays a crucial role in interactions with electromagnetic fields. For instance:

- In electron diffraction experiments, the observed wavelength is directly related to the electron's momentum.
- In particle accelerators, electromagnetic fields accelerate charged particles (like electrons or protons), and their relativistic momentum determines their de Broglie wavelength, influencing diffraction and scattering phenomena.

The charge e does not directly alter the de Broglie wavelength but determines how the particle interacts with external fields, which can influence its energy E and velocity v.

Summary and Key Takeaways

- The de Broglie wavelength of a relativistic particle with rest mass M_0 and total energy E is:
$$
\boxed{
\lambda = \frac{h c}{\sqrt{E^2 - M_0^2 c^4}}
}
$$
- The relativistic energy-momentum relation is essential for deriving this formula.
- For particles accelerated to high energies, their wavelength decreases, highlighting the wave-like behavior becoming more prominent at microscopic scales.
- The particle's charge e affects its interactions but not directly its de Broglie wavelength; however, in experimental contexts, charge determines how the particle interacts with electromagnetic fields and detectors.

Practical Applications and Implications

Understanding the relativistic de Broglie wavelength is crucial for:

- Designing and interpreting experiments in electron microscopy, where the wavelength determines the resolution.
- Analyzing scattering experiments in particle physics, where wave properties influence cross-sections.
- Quantum field theory, where particle-wave duality underpins the behavior of particles at high energies.

Conclusion

In conclusion, the de Broglie wavelength of a particle with charge e, rest mass M_0, moving at relativistic speeds, is derived from the fundamental principles of special relativity and quantum mechanics. It provides a profound insight into the dual nature of matter and the behavior of particles at high energies. Recognizing how energy, momentum, and wavelength interrelate at relativistic regimes is essential for advancing research in modern physics, from fundamental particles to advanced technological applications.

References:

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Frequently Asked Questions

What is the expression for the de Broglie wavelength of a relativistic particle with charge e and rest mass Mo?

The de Broglie wavelength $\lambda = h / p$, where p is the relativistic momentum $p = \gamma M_0 v$. Therefore, $\lambda = h / (\gamma M_0 v)$, with $\gamma = 1 / \sqrt{1 - v^2/c^2}$.

How does the relativistic momentum differ from the classical momentum for a particle of rest mass Mo moving at velocity v?

Relativistic momentum $p = \gamma M_0 v$, which accounts for effects of special relativity, increasing significantly as v approaches c, unlike classical $p = M_0 v$.

Derive the de Broglie wavelength for a relativistic particle starting from the momentum expression.

Starting with $p = \gamma M_0 v$, the de Broglie wavelength is $\lambda = h / p = h / (\gamma M_0 v)$. This shows how λ depends on velocity through γ.

What role does the Lorentz factor γ play in the de Broglie wavelength of a relativistic particle?

$\gamma = 1 / \sqrt{1 - v^2/c^2}$ increases as v approaches c, leading to a decrease in the de Broglie wavelength $\lambda = h / (\gamma M_0 v)$, indicating shorter wavelengths at higher speeds.

How does the de Broglie wavelength change as the particle's velocity approaches the speed of light?

As v approaches c, γ □ □ , so p □ □ and λ = h / p □ 0. The wavelength becomes extremely short, reflecting high momentum in relativistic regimes.

Is the de Broglie wavelength dependent on the charge E of the particle?

No, the de Broglie wavelength depends on the particle's momentum, which is related to mass and velocity, not directly on its charge E.

Can the de Broglie wavelength be measured experimentally for relativistic particles? If so, how?

Yes, it can be measured using electron diffraction experiments, where the interference pattern reveals the wavelength, confirming the wave nature of relativistic particles.

How does the relativistic de Broglie wavelength compare to the non-relativistic case?

In the non-relativistic limit ($v \ll c$), $\gamma \approx 1$, so $\lambda \approx h / (Mo\ v)$. In the relativistic case, $\gamma > 1$, making λ shorter for the same velocity.

What is the significance of the de Broglie wavelength in understanding quantum behavior of high-speed particles?

It demonstrates that particles exhibit wave-like properties even at relativistic speeds, essential for understanding phenomena in quantum field theory and high-energy physics.

How does the relativistic energy relate to the de Broglie wavelength of a particle?

The total relativistic energy $E = \gamma\ Mo\ c^2$ relates to momentum $p = \gamma\ Mo\ v$, which determines the de Broglie wavelength $\lambda = h / p$, linking energy to wave properties.

Additional Resources

Show That The De Broglie Wavelength Of A Particle, Of Charge E, Rest Mass Mo, Moving At Relativistic Speeds

In the fascinating world of quantum mechanics, the wave-particle duality principle asserts that every particle exhibits both particle-like and wave-like properties. A cornerstone of this principle is the De Broglie wavelength—a measure that links a particle's momentum to its wave nature. For particles like electrons, protons, or even more exotic entities moving at relativistic speeds (speeds close to the speed of light), understanding how their De Broglie wavelength behaves is crucial for both theoretical insights and experimental applications. In this article, we will derive and analyze the expression for the De Broglie wavelength of a particle with charge E and rest mass Mo moving at relativistic speeds, elucidating the underlying physics and mathematical steps involved.

Understanding the Conceptual Foundations

Before diving into the derivation, it's essential to grasp the foundational concepts:

- Wave-Particle Duality: The idea that particles can display wave-like behavior, characterized by a wavelength linked to their momentum.
- Relativistic Dynamics: When particles move at speeds approaching the speed of light, classical Newtonian mechanics no longer suffice; instead, Einstein's special relativity must be used.
- De Broglie Hypothesis: Louis de Broglie proposed that the wavelength λ of a particle is inversely proportional to its momentum p, given by $\lambda = \frac{h}{p}$, where h is Planck's constant.

Step 1: Recall the Non-Relativistic De Broglie Wavelength

The fundamental relation for a particle with momentum p is:

$$
\boxed{
\lambda = \frac{h}{p}
}
$$

In non-relativistic regimes, the momentum p is simply $p = m v$, where m is the particle's mass and v its velocity.

However, at relativistic speeds, this relation needs to be modified to account for relativistic momentum.

Step 2: Relativistic Momentum

2.1. Basic Definitions

- Rest Mass (M_0): The invariant mass of the particle when at rest.
- Total Energy (E): The energy of the particle, including rest energy and kinetic energy.
- Relativistic Momentum (p): The momentum of a particle moving at velocity v, given by Einstein's theory.

2.2. The Relativistic Momentum Formula

The relativistic momentum p is expressed as:

$$
p = \gamma M_0 v
$$

where γ is the Lorentz factor:

$$
\boxed{
\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}
}
$$

and c is the speed of light in vacuum.

Step 3: Expressing Energy and Momentum Relativistically

The total relativistic energy E of a particle with rest mass M_0 moving at speed v:

$$
E = \gamma M_0 c^2
$$

The Lorentz factor relates energy and momentum through the energy-momentum relation:

$$
E^2 = (p c)^2 + (M_0 c^2)^2
$$

This fundamental relation links the three quantities: energy, momentum, and rest mass.

Step 4: Deriving the Relativistic De Broglie Wavelength

Given the relation:

$$
\lambda = \frac{h}{p}
$$

and knowing that:

$$
p = \gamma M_0 v
$$

we substitute to find:

$$
\boxed{
\lambda = \frac{h}{\gamma M_0 v}
}
$$

This expression shows how the wavelength depends on the Lorentz factor γ, the rest mass M_0, and the velocity v.

Step 5: Expressing λ in Terms of Total Energy

Alternatively, since the total energy E is often more accessible experimentally, it's advantageous to express λ in terms of E. From the energy-momentum relation:

$$
E^2 = (p c)^2 + (M_0 c^2)^2
$$

we solve for p:

$$
p = \frac{\sqrt{E^2 - M_0^2 c^4}}{c}
$$

Substituting into the wavelength formula:

$$
\lambda = \frac{h c}{\sqrt{E^2 - M_0^2 c^4}}
$$

This form is particularly useful when the particle's energy E is known, such as in high-energy physics experiments.

Complete Expression for the Relativistic De Broglie Wavelength

Bringing everything together, the De Broglie wavelength of a particle with rest mass M_0 and total energy E moving at relativistic speeds is:

$$
\boxed{
\lambda = \frac{h c}{\sqrt{E^2 - M_0^2 c^4}}
}
$$

Alternatively, in terms of particle velocity:

$$
\lambda = \frac{h}{\gamma M_0 v} = \frac{h}{\frac{E}{c^2} v}
$$

where the Lorentz factor γ links v and E via:

$$
E = \gamma M_0 c^2
$$

Additional Insights and Applications

1. Behavior at Non-Relativistic Speeds

When $v \ll c$, $\gamma \approx 1$, and the expression reduces to the classical de Broglie wavelength:

$$
\lambda \approx \frac{h}{M_0 v}
$$

which aligns with the familiar non-relativistic quantum mechanics.

2. Behavior at Ultra-Relativistic Speeds

As $v \to c$, $\gamma \to \infty$, and the wavelength λ approaches zero, indicating the particle's wave-like nature becomes increasingly localized, corresponding to high momentum.

3. Charge E and Particle Dynamics

While the charge E itself doesn't directly influence the wavelength (since it pertains to electromagnetic interactions), knowing the particle's charge is vital for understanding how it interacts with electromagnetic fields, accelerators, or detectors. The key point is that the De Broglie wavelength depends fundamentally on the particle's momentum or energy, regardless of its charge.

Practical Example: Electron at Relativistic Speeds

Suppose an electron ($M_0 \approx 9.11 \times 10^{-31}$ kg) is accelerated to a high energy E of 1 MeV ($1\, \text{MeV} = 1.602 \times 10^{-13}$ Joules). The rest energy of the electron:

$$
M_0 c^2 \approx 0.511\, \text{MeV}
$$

The total energy:

$$
E = 1\, \text{MeV}
$$

Calculate the relativistic De Broglie wavelength:

$$
\lambda = \frac{h c}{\sqrt{E^2 - M_0^2 c^4}}
$$

Given:

- $h c \approx 1.2398 \times 10^{-6}$ eV·m

- $E = 1 \times 10^{6}$ eV

- $M_0 c^2 = 0.511 \times 10^{6}$ eV

Compute numerator:

$$
h c \approx 1.2398 \times 10^{-6}\, \text{eV·m}
$$

Compute denominator:

$$
\sqrt{(1 \times 10^{6})^2 - (0.511 \times 10^{6})^2} \text{ eV} = \sqrt{10^{12} - 0.261 \times 10^{12}} = \sqrt{0.739 \times 10^{12}} \approx 8.6 \times 10^{5}\, \text{eV}
$$

Therefore:

$$
\lambda \approx \frac{1.2398 \times 10^{-6}}{8.6 \times 10^{5}} \text{ meters} \approx 1.44 \times 10^{-12}\, \text{meters}
$$

This extremely small wavelength illustrates the high localization of the electron's wavefunction at relativistic energies, which is fundamental in high-energy physics and electron microscopy.

Conclusion: The Significance of the Relativistic De Broglie Wavelength

Understanding the De Broglie wavelength of a particle with charge E and rest mass Mo moving at relativistic speeds is a vital aspect of modern physics. It bridges quantum mechanics with special relativity, providing insights into the behavior of particles in high-energy regimes. The derived formulas serve as essential tools in

PARTICLE PHYSICS, ACCELERATOR DESIGN, AND QUANTUM FIELD THEORY, ENABLING SCIENTISTS TO PREDICT AND INTERPRET PHENOMENA RANGING FROM ELECTRON DIFFRACTION PATTERNS TO THE FORMATION OF PARTICLE JETS IN COLLIDERS

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