If -5 Is A Root Of The Quadratic Equation 2x2+px-15=0 And The Quadratic Equation P(x^2+x)+k=0 Has Equal

If -5 Is A Root Of The Quadratic Equation 2x2+px-15=0 And The Quadratic Equation P(x^2+x)+k=0 Has Equal roots, then exploring the relationships between the coefficients, roots, and parameters involved becomes essential for understanding the underlying algebraic principles. This article delves into the detailed analysis of these two quadratic equations, their roots, and the conditions that tie them together. By understanding the implications of a given root and the nature of quadratic equations, students and mathematics enthusiasts can enhance their problem-solving skills and grasp key concepts in algebra.

Understanding the First Quadratic Equation: 2x^2 + px - 15 = 0

The significance of a known root in quadratic equations

When analyzing quadratic equations, knowing one root can significantly simplify the process of finding the other roots and the coefficients involved. If we are told that -5 is a root of the quadratic equation:

$$ 2x^2 + px - 15 = 0 $$

then it provides us with a direct way to determine the parameter p. This is because substituting the root into the equation must satisfy it, leading to a specific value of p.

Calculating the coefficient p using the known root

Substituting x = -5 into the quadratic equation:

$$ 2(-5)^2 + p(-5) - 15 = 0 $$

Calculating step-by-step:

1. $2 \times 25 = 50$
2. $p \times (-5) = -5p$
3. The equation becomes:

$$ 50 - 5p - 15 = 0 $$

Simplify:

$$ (50 - 15) - 5p = 0 $$
$$ 35 - 5p = 0 $$

Solve for p:

$$ -5p = -35 $$
$$ p = \frac{-35}{-5} = 7 $$

Thus, the value of p is 7.

Verifying the other root of the quadratic

Knowing p = 7, the quadratic becomes:

$$ 2x^2 + 7x - 15 = 0 $$

The roots can be found via the quadratic formula:

$$ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} $$

where:

- $a = 2$
- $b = 7$
- $c = -15$

Calculating the discriminant:

$$ \Delta = 7^2 - 4 \times 2 \times (-15) = 49 + 120 = 169 $$

Since $\sqrt{169} = 13$, the roots are:

$$ x = \frac{-7 \pm 13}{4} $$

This yields:

1. $x = \frac{-7 + 13}{4} = \frac{6}{4} = \frac{3}{2}$
2. $x = \frac{-7 - 13}{4} = \frac{-20}{4} = -5$

As expected, -5 is one root, and the other root is $\frac{3}{2}$.

Analyzing the Second Quadratic Equation: $P(x^2 + x) + k = 0$

Understanding the structure of the second quadratic

The second quadratic involves parameters p and k, and is expressed as:

$$ P(x^2 + x) + k = 0 $$

Given the context, it's important to understand what it means for this quadratic to have equal roots—that is, a repeated root or a double root. When a quadratic has equal roots, its discriminant is zero.

Expressing the quadratic in standard form

Rearranged, the quadratic becomes:

$$ P x^2 + P x + k = 0 $$

This is a quadratic in terms of x with coefficients:

- $a = P$
- $b = P$
- $c = k$

Conditions for Equal Roots in the Second Quadratic Equation

Discriminant condition for double roots

For the quadratic $P x^2 + P x + k = 0$, the discriminant Δ must be zero:

$$ \Delta = b^2 - 4ac = 0 $$

Substituting the coefficients:

$$ (P)^2 - 4 \times P \times k = 0 $$

This simplifies to:

$$P^2 - 4 P k = 0$$

or

$$P (P - 4k) = 0$$

This leads to two cases:

1. Case 1: $P = 0$
2. Case 2: $P - 4k = 0 \Rightarrow P = 4k$

Implications of the Conditions for the Parameters P and K

Case 1: P = 0

If P = 0, then the quadratic simplifies to:

$$0 \times x^2 + 0 \times x + k = 0$$

which reduces to:

$$k = 0$$

Thus, the quadratic becomes:

$$0 = 0$$

which is true for all x, implying infinitely many solutions, not a quadratic in the traditional sense. Therefore, for a quadratic with equal roots, P ≠ 0 in this context.

Case 2: $P = 4k$

When $P = 4k$, the quadratic becomes:

$$4k x^2 + 4k x + k = 0$$

Dividing through by k (assuming $k \neq 0$):

$$4 x^2 + 4 x + 1 = 0$$

The discriminant of this quadratic:

$$\Delta = 4^2 - 4 \times 4 \times 1 = 16 - 16 = 0$$

Thus, the roots are equal, confirming the condition for double roots.

The root can be found via the quadratic formula:

$$x = \frac{-4 \pm \sqrt{0}}{2 \times 4} = \frac{-4}{8} = -\frac{1}{2}$$

This indicates the double root of the quadratic in this case is $-\frac{1}{2}$.

Summary of key relationships:

- When $P = 4k$, the quadratic has a double root at $-\frac{1}{2}$.
- The parameters P and k are directly related through $P = 4k$.

Connecting Both Quadratics: Final Insights and Solutions

Linking the roots and parameters

Recall from earlier:

- The first quadratic's root is -5, with $p = 7$.
- The second quadratic has a double root at $-\frac{1}{2}$ when $P = 4k$.

The question focuses on the scenario where -5 is a root of the first quadratic and the second quadratic has equal roots.

Summary of key points:

- Given -5 is a root of $2x^2 + px -15 = 0$, we find $p = 7$.
- In the second quadratic $P(x^2 + x) + k = 0$, equal roots occur when $P = 4k$, with the double root at $-\frac{1}{2}$.
- The parameters P and k are related, with $P = 4k$, and the double root at $-\frac{1}{2}$.

Practical Applications and Problem-Solving Tips

How to approach similar algebraic problems

When faced with quadratic equations involving parameters and known roots, follow these steps:

1. Substitute known roots to find unknown parameters.
2. Use the discriminant condition ($\Delta = 0$) to determine when roots are equal.
3. Express parameters in terms of each other to explore relationships.
4. Verify solutions by substituting back into the original equations.

Conclusion: Key Takeaways from the Algebraic Analysis

Understanding the relationships between roots and coefficients in quadratic equations is fundamental in algebra. When a specific root is known, it allows for the straightforward determination of parameters like p. Additionally, the condition for equal roots, characterized by a zero discriminant, provides insights into the nature of the quadratic's roots and the relationships among parameters like P and k. By applying these principles, students can solve complex algebraic problems efficiently and deepen their understanding of quadratic equations.

Remember:

- The root-parameter relationship is crucial in solving quadratic equations.

Frequently Asked Questions

If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$, what is the value of p?

Substitute $x = -5$ into the equation: $2(-5)^2 + p(-5) - 15 = 0 \rightarrow 2(25) - 5p - 15 = 0 \rightarrow 50 - 5p - 15 = 0 \rightarrow 35 - 5p = 0 \rightarrow p = 7$.

Given p = 7, what is the value of k if the quadratic equation P(x^2 + x) + k = 0 has equal roots?

Rewrite as $P(x^2 + x) + k = 0$. For equal roots, the discriminant must be zero: $D = P^2 - 4 P k = 0 \rightarrow P^2 = 4 P k \rightarrow k = P/4$. Substituting $P = 7$, $k = 7/4$.

How do you verify that -5 is a root of the quadratic 2x^2 + px - 15 = 0?

Plug $x = -5$ into the quadratic and check if the expression equals zero: $2(-5)^2 + p(-5) - 15 = 0$. If it holds true for the value of p found, then -5 is a root.

What condition must p and k satisfy for the quadratic P(x^2 + x) + k = 0 to have equal roots?

The discriminant $D = P^2 - 4 P k$ must be zero, which implies $k = P/4$.

If the quadratic 2x^2 + px - 15 = 0 has -5 as a root, what are the possible values of p?

The only value of p that satisfies this condition is $p = 7$, as demonstrated by substitution.

Are the roots of the quadratic P(x^2 + x) + k = 0 real and equal when k = P/4? Why?

Yes, because setting the discriminant $D = P^2 - 4 P k = 0$ results in a perfect square trinomial, ensuring real and equal roots.

Additional Resources

If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $P(x^2 + x) + k = 0$ has equal roots, understanding the relationship between the roots and coefficients is crucial. This problem combines fundamental concepts of quadratic equations, roots, and parameter relationships, offering an insightful exploration into algebraic reasoning. In this comprehensive guide, we'll break down the problem step-by-step, analyze the given conditions, and derive the necessary expressions to understand what these conditions imply about the parameters p and k.

Understanding the Problem

The problem presents two quadratic equations:

1. First quadratic: $2x^2 + px - 15 = 0$
- We are told -5 is a root of this equation.

2. Second quadratic: $P(x^2 + x) + k = 0$
- It is given that this quadratic "has equal roots," meaning it has a repeated root or a double root.

Step 1: Analyzing the First Quadratic Equation

Using the Root -5

Since -5 is a root of the quadratic $2x^2 + px - 15 = 0$, substituting $x = -5$ into the equation should satisfy it:

$$ 2(-5)^2 + p(-5) - 15 = 0 $$

Calculating:

$$ 2 \times 25 - 5p - 15 = 0 $$
$$ 50 - 5p - 15 = 0 $$
$$ 35 - 5p = 0 $$

Solve for p:

$$ 5p = 35 $$
$$ p = 7 $$

Result:
$$
\boxed{
p = 7
}
$$

This value of p is now fixed. The first quadratic becomes:

$$ 2x^2 + 7x - 15 = 0 $$

Step 2: Exploring the Second Quadratic Equation

The form and roots

The second quadratic is:

$$ P(x^2 + x) + k = 0 $$

This can be rewritten as:

$$ P x^2 + P x + k = 0 $$

which is a quadratic in x, with coefficients:

- Quadratic coefficient: P
- Linear coefficient: P
- Constant term: k

Condition: The quadratic has equal roots

For this quadratic to have a double root, its discriminant must be zero:

$$\Delta = (P)^2 - 4 \times P \times k = 0$$

Simplify:

$$P^2 - 4 P k = 0$$
$$P (P - 4k) = 0$$

This gives two possibilities:

1. P = 0
2. P - 4k = 0

Step 3: Analyzing the Possible Values of P and K

Case 1: P = 0

Substitute P = 0 into the quadratic:

$$0 \times x^2 + 0 \times x + k = 0 \Rightarrow k = 0$$

But then the quadratic reduces to:

$$0 = 0$$

which is an identity, not a quadratic. Such an "equation" either has all real numbers as roots or is degenerate, not fitting the typical quadratic form with equal roots.

Conclusion:
This case is trivial or degenerate; thus, more meaningful solutions arise when:

Case 2: $P - 4k = 0$

Express k in terms of P:

$$k = \frac{P}{4}$$

Now, recall that P is a parameter in the quadratic:

$$P x^2 + P x + k = 0$$

which has a double root when the discriminant is zero, as established.

Step 4: Connecting P to the First Equation

Previously, p was determined as 7 from the root -5. The second quadratic involves P and k, and the question is about the nature of its roots. To analyze this, we need to understand the relationship between P and the parameters of the first quadratic.

Typically, in problems like this, P could be related to p or derived from conditions involving the roots of the first quadratic. Since the first quadratic has roots:

$$x_{1,2} = \frac{-p \pm \sqrt{p^2 - 4 \times 2 \times (-15)}}{2 \times 2}$$

Calculate the discriminant:

$$\Delta_1 = p^2 - 4 \times 2 \times (-15) = p^2 + 120$$

Plugging $p = 7$:

$$\Delta_1 = 49 + 120 = 169$$

Calculate the roots:

$$x_{1,2} = \frac{-7 \pm \sqrt{169}}{4} = \frac{-7 \pm 13}{4}$$

which gives:

- $x_1 = \frac{-7 + 13}{4} = \frac{6}{4} = 1.5$
- $x_2 = \frac{-7 - 13}{4} = \frac{-20}{4} = -5$

Note: The root -5 is one of the roots, as expected.

Step 5: Interpreting the Relationship Between the Roots and the Parameters P and K

Given that the second quadratic:

$$P x^2 + P x + k = 0$$

has equal roots, and P is related to the parameters of the first quadratic, perhaps through the roots or coefficients, the most straightforward assumption is that P is connected to p or to the roots themselves.

Possible assumptions:

- The parameter P might be equal to p, or related via the roots.
- Since roots of the first quadratic include -5, perhaps P relates to these roots.

Step 6: Final Conditions and Conclusion

Given the above:

- $p = 7$ (from root -5)
- $k = P / 4$ (from the double root condition)

The second quadratic has double roots when:

$$ P (P - 4k) = 0 $$

and either $P = 0$ or $P = 4k$.

Summary and Final Insights

Key Takeaways:

- The root -5 of the quadratic $2x^2 + px - 15 = 0$ leads directly to $p = 7$.
- The second quadratic $P(x^2 + x) + k = 0$ simplifies to $P x^2 + P x + k = 0$.
- For this quadratic to have equal roots, its discriminant must be zero: $P^2 - 4 P k = 0$, which yields $P = 0$ or $P = 4k$.
- When $P = 0$, the quadratic degenerates; thus, the meaningful case is $P = 4k$.
- The parameter k can then be expressed as $k = P / 4$.

Practical implications:

- If P is chosen as $P = p = 7$, then:

$$ k = \frac{P}{4} = \frac{7}{4} $$

- The quadratic:

$$ 7x^2 + 7x + \frac{7}{4} = 0 $$

has double roots, specifically at:

$$ x = -\frac{b}{2a} = -\frac{7}{2 \times 7} = -\frac{1}{2} $$

- This indicates a consistent set of parameters satisfying the initial conditions.

Conclusion

In problems combining roots, coefficients, and parameter relationships, methodical substitution and discriminant analysis are essential. Here, the key steps involved:

- Using the known root -5 to find p.
- Recognizing the double root condition for the second quadratic.
- Connecting the parameters P and k via the discriminant condition.
- Confirming the consistency of the parameters through substitution.

This analysis not only solves the specific problem but also provides a blueprint for tackling similar quadratic parameter problems in algebra. Understanding how roots influence coefficients and parameters enables deeper insights into the structure of quadratic equations and their solutions.

Remember: When dealing with quadratic equations involving parameters, always check the discriminant to determine the nature of roots, and use substitution to relate roots to coefficients for a complete understanding.

If 5 Is A Root Of The Quadratic Equation 2x2 Px 15 0 And The Quadratic Equation P X 2 X K 0 Has Equal

If 5 Is A Root Of The Quadratic Equation 2x2 Px 15 0 And The Quadratic Equation P X 2 X K 0 Has Equal

Related Articles

- Foundations Of Comprehension:Which Pair Of Contradictory Statements Is Implicit In The Passage?A.Poetry
- Dan Surveys 10 Of The 24 Students In His Calss About The Number Of People Intheir Families. The Results
- Iam Working On An Assignment Arguing That People Can Be Forced To Bevirtuous. I Need Two Statements Why

Back to Home