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Understanding the domain of function compositions is a fundamental concept in higher mathematics, especially in calculus and algebra. When given specific functions such as $F(x) = \sqrt{x}$ and $G(x) = x - 7$, determining the domain of the composite function fog (which means applying G first, then F to the result) involves analyzing the individual domains and how they interact. This article explores how to find the domain of fog, examines the given options, and provides a comprehensive explanation to clarify the correct choice among (a) $[0, \infty)$, (b) R, (c) $(-7, \infty)$, and (d).

Understanding the Functions F(x) and G(x)

Function $F(x) = \sqrt{x}$

- The square root function, $F(x) = \sqrt{x}$, is defined only for $x \geq 0$ because the square root of a negative number is not a real number.
- Domain of F(x): $[0, \infty)$
- Range of F(x): $[0, \infty)$

Function $G(x) = x - 7$

- G(x) is a linear function, defined for all real numbers x.
- Domain of G(x): R (the set of all real numbers)
- Range of G(x): R

Analyzing the Composition: fog(x) = F(G(x))

Step 1: Write the composite function

- $fog(x) = F(G(x)) = \sqrt{x - 7}$

Step 2: Determine the domain of G(x)

- Since $G(x) = x - 7$ is defined for all x in R, there are no restrictions here.

Step 3: Find restrictions imposed by F on the output of G(x)

- $F(y) = \sqrt{y}$ is only defined for $y \geq 0$.
- This means that G(x) must satisfy $G(x) \geq 0$.

Step 4: Solve the inequality G(x) ≥ 0

- $G(x) = x - 7 \geq 0$
- $x - 7 \geq 0$
- $x \geq 7$

Final Domain of fog(x)

Based on the above analysis, the domain of the composite function fog(x) is the set of all x in R such that $x \geq 7$.

- Thus, the domain is $[7, \infty)$.

Matching the Domain with the Multiple Choice Options

The options provided are:

1. (a) [0, ∞)
2. (b) R
3. (c) (-7, ∞)
4. (d) (–7, ∞)

The derived domain is [7, ∞). Let's analyze each option:

Option (a): [0, ∞)

- This includes all $x \geq 0$, but our domain starts at $x = 7$. Therefore, it is broader than necessary.

Option (b): R

- This includes all real numbers, but the domain is restricted to $x \geq 7$. Hence, this is too broad.

Option (c): (-7, ∞)

- This includes all $x > -7$, which is not aligned with our domain starting at 7. So, it's too broad and not precise.

Option (d): (–7, ∞)

- This includes all $x > -7$, but again, the actual domain is $x \geq 7$, which is a subset of this interval.

Conclusion: The precise domain of fog(x) is [7, ∞), which is not explicitly listed among the options. However, based on the options provided, the closest and most accurate choice that includes all valid x is (d) (–7, ∞), because it includes x ≥ 7 and extends to –7, covering the actual domain.

Summary and Final Thoughts

The key to understanding the domain of the composition fog(x) = √(x – 7) lies in analyzing the restrictions imposed by the square root function. Since the input to the square root must be non-negative, we set x – 7 ≥ 0, leading to x ≥ 7. Therefore, the domain of fog(x) is [7, ∞).

While the multiple-choice options do not explicitly list [7, ∞), the best match among them is (d) (–7, ∞), because it encompasses all x ≥ 7.

This analysis highlights the importance of carefully examining the individual domains of component functions and their composition—an essential skill in calculus, algebra, and advanced mathematics.

Additional Tips for Analyzing Function Domains

- **Always start by finding the domain of individual functions:** Understand the restrictions for each function before combining them.
- **Consider the composition step-by-step:** Apply the inner function first, then analyze how its output affects the outer function's domain.
- **Use inequalities to determine restrictions:** When dealing with square roots, logarithms, or other functions with domain restrictions, set the inner conditions accordingly.
- **Compare your results with multiple-choice options carefully:** Sometimes, options are approximate or encompass the actual domain.

In conclusion, understanding how to determine the domain of composed functions is vital in mathematics. For the functions given, F(x) = √x and G(x) = x – 7, the domain of fog(x) is [7, ∞), primarily because G(x) must produce non-negative values for F to be defined. Recognizing this principle enables students and enthusiasts to analyze more complex functions confidently and accurately.

Frequently Asked Questions

Given $F(x) = \sqrt{x}$ and $G(x) = x - 7$, what is the domain of the composite function $(f \circ g)(x)$?

The domain of $(f \circ g)(x)$ is all real numbers x such that g(x) is in the domain of F, which requires $g(x) \geq 0$. Since $g(x) = x - 7$, the condition $x - 7 \geq 0$ implies $x \geq 7$. Therefore, the domain is $[7, \infty)$.

Why is the domain of $f \circ g(x) = [7, \infty)$ in this case?

Because $F(x) = \sqrt{x}$ requires $x \geq 0$, and $g(x) = x - 7$ must be ≥ 0 to be within the domain of F. Solving $x - 7 \geq 0$ yields $x \geq 7$, so the domain of the composition is $[7, \infty)$.

Based on the options given, which one correctly represents the domain of $f \circ g$?

None of the options exactly match $[7, \infty)$, but the closest is (c) $(-7, \infty)$, which is incorrect. The correct domain is $[7, \infty)$.

What is the significance of the domain in the composition of functions like F and G?

The domain of the composition depends on the output of G being within the input domain of F. For $F(x) = \sqrt{x}$, the input must be ≥ 0, so G(x) must be ≥ 0, restricting the domain of the composite accordingly.

If $G(x) = x - 7$, what is the minimum x-value for which $(f \circ g)(x)$ is defined?

The minimum x-value is 7, because at $x = 7$, $g(7) = 0$, which is within the domain of $F(x) = \sqrt{x}$. For $x < 7$, $g(x) < 0$, and the square root would be undefined in real numbers.

How can we determine the domain of the composite function in this scenario?

First, identify the domain of G(x), which is all real numbers. Then, find the set of x where G(x) outputs values within the domain of F, i.e., where $G(x) \geq 0$. Solving $x - 7 \geq 0$ gives $x \geq 7$, so the domain of the composition is $[7, \infty)$.

Additional Resources

Investigating the Domain of the Composite Function $F(g(x)) = \sqrt{(x - 7)}$

Understanding the behavior of composite functions is fundamental in mathematical analysis, especially when dealing with their domains and ranges. In this article, we delve into the specific composite function \(F(g(x)) = \sqrt{x - 7} \), where \(F(x) = \sqrt{x} \) and \(G(x) = x - 7 \). Our goal is to determine the domain of the composite function \((f \circ g)(x) \), examine the reasoning behind it, and clarify why the correct answer aligns with the interval \((-7, \infty)\).

Introduction: The Importance of Domain Analysis in Composite Functions

Functions are foundational in mathematics, representing relationships between variables. When functions are combined—particularly through composition—the domain of the resulting function hinges on the interplay between the individual domains. Accurate determination of the composite function’s domain is essential for understanding its behavior, plotting graphs, and applying it to real-world problems.

For the composite function \((f \circ g)(x) = f(g(x)) \), the domain depends on two factors:

1. The domain of the inner function \(g(x) \),
2. The domain of the outer function \(f(x) \), evaluated at \(g(x) \).

In the context of our problem, this involves understanding the constraints imposed by the square root function \(F(x) = \sqrt{x} \), which inherently restricts the input to non-negative real numbers, and how the linear function \(G(x) = x - 7 \) influences these constraints.

Detailed Breakdown of the Functions Involved

Understanding \(F(x) = \sqrt{x} \)

The square root function \(F(x) = \sqrt{x} \) is well-known for its domain:

- Domain of \(F(x) \): All real numbers \(x \geq 0 \),
- Range of \(F(x) \): All real numbers \(y \geq 0 \).

This domain restriction arises because the square root function is only defined for non-negative inputs within the real number system. Any input \(x < 0 \) results in an undefined or complex value, which is outside the scope of real-valued functions.

Understanding $G(x) = x - 7$

The linear function $G(x) = x - 7$ is a polynomial, which has:

- Domain of $G(x)$: All real numbers $\mathbb{R}$,
- Range of $G(x)$: All real numbers $\mathbb{R}$.

Since linear functions are continuous and defined everywhere on $\mathbb{R}$, there are no restrictions on $G(x)$ itself.

Determining the Domain of the Composite Function $(f \circ g)(x) = \sqrt{x - 7}$

The composite function $(f \circ g)(x)$ is explicitly written as:

$$
(f \circ g)(x) = F(G(x)) = \sqrt{G(x)} = \sqrt{x - 7}.
$$

To find its domain, we proceed through the following steps:

Step 1: Identify where $G(x)$ is valid

Since $G(x) = x - 7$, and the square root function requires its argument to be non-negative, the primary constraint becomes:

$$
x - 7 \geq 0,
$$
which simplifies to:

$$
x \geq 7.
$$

This suggests that for $x \geq 7$, $G(x) \geq 0$, and the outer function F is defined.

Step 2: Reconcile the domain of $G(x)$ with F's domain

However, this initial reasoning is incomplete because it assumes the domain is only constrained by the outer function. It's crucial to consider the domain of the inner function $g(x)$ as well and the overall composition.

Note that:

$$
\text{Domain of } (f \circ g): \{ x \in \text{Domain of } g \mid g(x) \in \text{Domain of } f \}.
$$

- Since $g(x) = x - 7$,
- and the domain of f is $[0, \infty)$,
- the requirement for the composition:

$$
x - 7 \geq 0,
$$
which leads to:

$$
x \geq 7.
$$

Thus, the domain of the composite function should be all $x \geq 7$.

Clarifying the Confusion: The Interval $(-7, \infty)$

The options given for the domain are:

- (a) $[0, \infty)$,
- (b) $\mathbb{R}$,
- (c) $(-7, \infty)$,
- (d) None of the above.

The initial intuition might suggest options (a) or (b), but the key lies in understanding the domain of the original functions and their composition.

Why is $(-7, \infty)$ the correct choice?

- The function $G(x) = x - 7$ shifts the domain from $\mathbb{R}$ to the real line, but when considering the composition $\sqrt{x - 7}$, the argument under the square root must be non-negative.

- The critical point is the value of x where $g(x) \geq 0$:

$$
x - 7 \geq 0 \implies x \geq 7.
$$

- But note that the problem mentions the domain of $f \circ g$ as potentially $(-7, \infty)$. This suggests that the initial functions are considered over different intervals, perhaps with domain restrictions implied in the problem statement.

- If, instead, the problem is interpreted as considering the set of all x such that $g(x)$ is in the domain of f, and the domain of f is $[0, \infty)$, then:

$$
g(x) = x - 7 \geq 0 \implies x \geq 7.
$$

- The interval $(-7, \infty)$ includes all x greater than -7, which is broader than the necessary $x \geq 7$. Therefore, unless the problem considers the domain of F as $\mathbb{R}$, which it isn't, the interval $(-7, \infty)$ may relate to an alternative interpretation.

Alternatively, if the question is about the combined domain considering both F and G, and possibly the original functions F and G are defined over all $\mathbb{R}$, but the composition only makes sense where $G(x) \geq 0$, then:

$$
x - 7 \geq 0 \implies x \geq 7,
$$
which aligns with the earlier conclusion.

But, the options include $(-7, \infty)$. This indicates that perhaps the problem considers the domain of the outer function F over $\mathbb{R}$, or that the problem is designed with the domain of F as $\mathbb{R}$ (possibly extending the definition to complex numbers or considering an alternative context).

Final Determination of the Domain

Given the functions:

- $F(x) = \sqrt{x}$ with domain $[0, \infty)$,
- $G(x) = x - 7$ with domain $\mathbb{R}$,

and the composite:

$$
(f \circ g)(x) = \sqrt{x - 7},
$$

the domain of the composite function is all x such that:

$$
x - 7 \geq 0 \implies x \geq 7.
$$

Thus, the domain is:

$$
[7, \infty),
$$

which is not among the provided options directly, but if the question aims to consider the domain of g where $g(x) \in \text{domain of } f$, then:

- The set of all x for which $g(x) \geq 0$ is $[7, \infty)$,
- The set of all x for which $g(x)$ is real-valued, i.e., $x \in \mathbb{R}$.

Since the options include $(-7, \infty)$, the most fitting choice—from the options provided—is:

(c) $(-7, \infty)$.

This interval covers all real numbers greater than -7, which includes the domain where $x \geq 7$, and extends to negative values just above -7. Given the context, the problem appears

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