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If U(t) = (sin(4t), Cos(6t), T) And V(t) = (t, Cos(6t), Sin(4t)), Use Formula 4 Of This Theorem To Find

Understanding the behavior of vector functions is fundamental in calculus, especially when analyzing curves and their properties in three-dimensional space. In this article, we will explore how to apply Formula 4 of a specific theorem to the given vector functions U(t) and V(t). We will break down the problem step-by-step, providing a comprehensive explanation suitable for students and enthusiasts seeking to deepen their understanding of vector calculus concepts.

Overview of the Given Vector Functions

Before diving into the application of the theorem, it's essential to understand the structure of the vector functions involved.

Function U(t)

The vector function U(t) is defined as:

$$
U(t) = (\sin(4t), \cos(6t), t)
$$

- First component: $\sin(4t)$ — oscillates sinusoidally with amplitude 1 and frequency 4.
- Second component: $\cos(6t)$ — oscillates sinusoidally with amplitude 1 and frequency 6.
- Third component: t — linear, increasing without bound as t increases.

This combination creates a space curve with complex oscillatory behavior in the x-y plane and a linear progression along the z-axis.

Function V(t)

The vector function V(t) is given as:

$$
V(t) = (t, \cos(6t), \sin(4t))
$$

- First component: t — linear with t.
- Second component: $\cos(6t)$ — oscillatory with frequency 6.
- Third component: $\sin(4t)$ — oscillatory with frequency 4.

V(t) traces a different but related path in space, with an interplay between linear and oscillatory behaviors.

Understanding the Theorem and Its Formula

The theorem referenced involves properties of vector functions, especially in the context of their derivatives, dot products, cross products, and perhaps their geometric interpretations such as curvature, torsion, or other differential geometric quantities.

Formula 4 of this theorem, while not explicitly provided here, generally pertains to one of the fundamental identities involving derivatives of vector functions, such as:

- The derivative of the dot product: $\frac{d}{dt} [U(t) \cdot V(t)] = U'(t) \cdot V(t) + U(t) \cdot V'(t)$
- The derivative of the cross product: $\frac{d}{dt} [U(t) \times V(t)] = U'(t) \times V(t) + U(t) \times V'(t)$

Given the context, it is likely that Formula 4 involves calculating derivatives of the dot or cross products to analyze the relationship between U(t) and V(t), such as their orthogonality, parallelism, or other properties.

Step-by-Step Application of Formula 4

Assuming that Formula 4 relates to derivatives involving dot or cross products, here’s how to proceed.

Step 1: Compute Derivatives of U(t) and V(t)

To apply most derivative formulas, we need the derivatives of U(t) and V(t):

$$
U(t) = (\sin(4t), \cos(6t), t)
$$

$$
U'(t) = \left(\frac{d}{dt} \sin(4t), \frac{d}{dt} \cos(6t), \frac{d}{dt} t \right)
$$

Calculating each component:

- $\frac{d}{dt} \sin(4t) = 4 \cos(4t)$

- $\frac{d}{dt} \cos(6t) = -6 \sin(6t)$
- $\frac{d}{dt} t = 1$

Thus,

$$
U'(t) = (4 \cos(4t), -6 \sin(6t), 1)
$$

Similarly, for V(t):

$$
V(t) = (t, \cos(6t), \sin(4t))
$$

$$
V'(t) = \left(1, -6 \sin(6t), 4 \cos(4t) \right)
$$

Step 2: Calculate the Dot Product $U(t) \cdot V(t)$ and Its Derivative

The dot product:

$$
U(t) \cdot V(t) = \sin(4t) \times t + \cos(6t) \times \cos(6t) + t \times \sin(4t)
$$

Simplify:

$$
U(t) \cdot V(t) = t \sin(4t) + \cos^2(6t) + t \sin(4t) = 2 t \sin(4t) + \cos^2(6t)
$$

Derivative of the dot product:

$$
\frac{d}{dt} [U(t) \cdot V(t)] = \frac{d}{dt} \left(2 t \sin(4t) + \cos^2(6t) \right)
$$

Calculate each term separately:

- $\frac{d}{dt} \left(2 t \sin(4t) \right)$

Apply product rule:

$$
2 \sin(4t) + 2 t \times 4 \cos(4t) = 2 \sin(4t) + 8 t \cos(4t)
$$

- $\frac{d}{dt} \left(\cos^2(6t) \right)$

Use chain rule:

$$
2 \cos(6t) \times (-6 \sin(6t)) = -12 \cos(6t) \sin(6t)
$$

Therefore, the derivative of the dot product:

$$
\frac{d}{dt} [U(t) \cdot V(t)] = 2 \sin(4t) + 8 t \cos(4t) - 12 \cos(6t) \sin(6t)
$$

Step 3: Apply Formula 4 to Find the Desired Quantity

Depending on what Formula 4 specifies, possible objectives include:

- Orthogonality check: Verify if $U(t) \cdot V(t) = 0$ at specific points.
- Derivative relationships: Find the rate of change of the dot product or cross product.
- Cross product analysis: Compute $U(t) \times V(t)$ and its derivative.

For illustration, let's compute $U(t) \times V(t)$.

Step 4: Compute the Cross Product $U(t) \times V(t)$

Recall:

$$
U(t) = (x_1, y_1, z_1) = (\sin(4t), \cos(6t), t)
$$
$$
V(t) = (x_2, y_2, z_2) = (t, \cos(6t), \sin(4t))
$$

The cross product:

$$
U(t) \times V(t) = \left(y_1 z_2 - z_1 y_2, z_1 x_2 - x_1 z_2, x_1 y_2 - y_1 x_2 \right)
$$

Calculate each component:

- First component:

$$
\cos(6t) \times \sin(4t) - t \times \cos(6t) = \cos(6t) \sin(4t) - t \cos(6t) = \cos(6t) (\sin(4t) - t)

- Second component:

$$
t \times t - \sin(4t) \times \sin(4t) = t^2 - \sin^2(4t)
$$

- Third component:

$$
\sin(4t) \times \cos(6t) - \cos(6t) \times t = \sin(4t) \cos(6t) - t \cos(6t) = \cos(6t) (\sin(4t) - t)
$$

Hence,

$$
U(t) \times V(t) = \left(\cos(6t) (\sin(4t) - t), \quad t^2 - \sin^2(4t), \quad \cos(6t) (\sin(4t) - t) \right)
$$

Step 5: Derive the Cross Product (if required)

Applying derivative formulas (like Formula 4), we could proceed to differentiate $U(t) \times V(t)$ to analyze how the cross product evolves, which relates to the twisting or torsion of the space curves.

Conclusion

Applying Formula 4 of the theorem to the given vector functions involves several steps: computing derivatives, dot products, cross products, and their derivatives. These

Frequently Asked Questions

What is the explicit form of the vectors U(t) and V(t) in the given problem?

U(t) = (sin(4t), cos(6t), t) and V(t) = (t, cos(6t), sin(4t)).

What does Formula 4 of the theorem refer to in the context of vector functions?

While the specific theorem isn't provided, Formula 4 typically relates to the derivative of the cross product or a similar operation involving vector functions, often used to find quantities like the derivative of a cross product, magnitude, or related vector calculus expressions.

How can we apply Formula 4 to find the derivative of the cross product U(t) × V(t)?

Applying Formula 4 to find d/dt [U(t) × V(t)] involves taking the derivatives of U(t) and V(t) separately and then combining them according to the product rule for cross products: d/dt [U × V] = U' × V + U × V'.

What are the derivatives U'(t) and V'(t) for the given vectors?

U'(t) = (4 cos(4t), -6 sin(6t), 1) and V'(t) = (1, -6 sin(6t), 4 cos(4t)).

Using the derivatives, how do we compute U'(t) × V(t) and U(t) × V'(t)?

To compute these, we perform the cross product of the derivatives with the original vectors: U'(t) × V(t) and U(t) × V'(t), using the standard determinant method for cross products.

What is the significance of applying Formula 4 in this problem?

Applying Formula 4 helps to find the derivative of the cross product U(t) × V(t), which can be useful in understanding properties like the rate of change of the area of the parallelogram spanned by U(t) and V(t), or in computing related vector calculus quantities.

Additional Resources

Vector Calculus Analysis: Exploring U(t) and V(t) with Formula 4 of the Theorem

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Introduction to the Problem and Its Significance

In the realm of vector calculus, understanding the intricate relationships between vector functions is fundamental for applications spanning physics, engineering, and mathematics. The functions:

$$
U(t) = (\sin(4t), \cos(6t), t)
$$

and

$$
V(t) = (t, \cos(6t), \sin(4t))
$$

offer a compelling case study into the behavior of vector-valued functions, especially when evaluated through the lens of a specific theorem—referred to here as Formula 4. While the exact statement of the theorem isn't provided, such formulas typically involve derivatives, dot products, cross products, or other vector operations essential for analyzing curves, tangents, and curvature.

Our goal is to apply Formula 4 to these functions, unpack its meaning, and explore the implications for the geometric and algebraic properties of these vectors. This deep dive is designed for enthusiasts and experts alike, offering a comprehensive, step-by-step exploration akin to a detailed product review or expert analysis.

Understanding the Functions U(t) and V(t)

Before delving into the theorem, it’s vital to understand the structure and behavior of U(t) and V(t).

2.1 Overview of U(t)

$$
U(t) = (\sin(4t), \cos(6t), t)
$$

- Components:
- x-component: $\sin(4t)$ — oscillates sinusoidally with frequency 4.
- y-component: $\cos(6t)$ — oscillates sinusoidally with frequency 6.
- z-component: t — linear growth, creating a helical or spiral-like pattern in 3D space.

- Behavior:
- The sinusoidal components induce periodic oscillations, while the linear t component causes the overall path to ascend or descend along the z-axis.
- The combination results in a complex, potentially twisted trajectory that can be visualized as a combination of sine and cosine waves with different frequencies.

2.2 Overview of V(t)

$$
V(t) = (t, \cos(6t), \sin(4t))
$$

- Components:
- x-component: t, linear in t.
- y-component: $\cos(6t)$, oscillates with frequency 6.

- z-component: $\sin(4t)$, oscillates with frequency 4.

- Behavior:
- The linear t in the x-direction creates a steady progression along the x-axis.
- The oscillatory y and z components introduce a wave-like pattern perpendicular to the x-axis.
- The combined effect results in a 3D path that resembles a wave propagating along the x-direction.

Applying Formula 4 of the Theorem: Theoretical Foundations

While the original statement of Formula 4 isn't explicitly provided, similar formulas in vector calculus typically relate to the derivative of scalar or vector functions, dot products, cross products, or the calculation of quantities like curvature, torsion, or the derivative of a scalar product.

Common forms of such formulas include:

- Derivative of the dot product: $\frac{d}{dt}[U(t) \cdot V(t)]$
- Derivative of the cross product: $\frac{d}{dt}[U(t) \times V(t)]$
- Derivative of the magnitude of a vector: $\frac{d}{dt} \|U(t)\|$

Assuming Formula 4 pertains to the derivative of the dot product between two vector functions, a typical expression would be:

$$
\frac{d}{dt}[U(t) \cdot V(t)] = U'(t) \cdot V(t) + U(t) \cdot V'(t)
$$

This is a direct application of the product rule for derivatives extended to vector functions. It’s fundamental in analyzing how two curves relate—whether they are orthogonal, parallel, or exhibit specific geometric properties.

Step-by-Step Application to U(t) and V(t)

Let’s proceed by explicitly computing the derivatives, dot products, and applying the presumed formula.

3.1 Computing Derivatives $U'(t)$ and $V'(t)$

Derivative of U(t):

\[
U(t) = (\sin(4t), \cos(6t), t)
\]

- \(x(t) = \sin(4t)\)

\[
\frac{d}{dt} \sin(4t) = 4 \cos(4t)
\]

- \(y(t) = \cos(6t)\)

\[
\frac{d}{dt} \cos(6t) = -6 \sin(6t)
\]

- \(z(t) = t\)

\[
\frac{d}{dt} t = 1
\]

Thus:

\[
U'(t) = (4 \cos(4t), -6 \sin(6t), 1)
\]

Derivative of V(t):

\[
V(t) = (t, \cos(6t), \sin(4t))
\]

- \(x(t) = t\)

\[
\frac{d}{dt} t = 1
\]

- \(y(t) = \cos(6t)\)

\[
\frac{d}{dt} \cos(6t) = -6 \sin(6t)
\]

- \(z(t) = \sin(4t)\)

\[

\frac{d}{dt} \sin(4t) = 4 \cos(4t)
\]

Thus:

\[
V'(t) = (1, -6 \sin(6t), 4 \cos(4t))
\]

3.2 Computing the Dot Product \(U(t) \cdot V(t)\)

Original dot product:

\[
U(t) \cdot V(t) = \sin(4t) \cdot t + \cos(6t) \cdot \cos(6t) + t \cdot \sin(4t)
\]

Simplify:

- \(\sin(4t) \cdot t + t \cdot \sin(4t) = 2 t \sin(4t)\)

- \(\cos(6t) \cdot \cos(6t) = \cos^2(6t)\)

Therefore:

\[
U(t) \cdot V(t) = 2 t \sin(4t) + \cos^2(6t)
\]

3.3 Differentiating \(U(t) \cdot V(t)\)

Applying the product rule:

\[
\frac{d}{dt}[U(t) \cdot V(t)] = U'(t) \cdot V(t) + U(t) \cdot V'(t)
\]

Calculating each term separately:

First term: \(U'(t) \cdot V(t)\)

\[
U'(t) = (4 \cos(4t), -6 \sin(6t), 1)
\]
\[
V(t) = (t, \cos(6t), \sin(4t))
\]

Dot product:

$$
(4 \cos(4t)) \cdot t + (-6 \sin(6t)) \cdot \cos(6t) + (1) \cdot \sin(4t)
$$

Simplify:

$$
4 t \cos(4t) - 6 \sin(6t) \cos(6t) + \sin(4t)
$$

Note:

$$
\sin(6t) \cos(6t) = \frac{1}{2} \sin(12t)
$$

Thus:

$$
-6 \times \frac{1}{2} \sin(12t) = -3 \sin(12t)
$$

So the first term simplifies to:

$$
4 t \cos(4t) - 3 \sin(12t) + \sin(4t)
$$

Second term: $U(t) \cdot V'(t)$

$$
U(t) = (\sin(4t), \cos(6t), t)
$$
$$
V'(t) = (1, -6 \sin(6t), 4 \cos(4t))
$$

Dot product:

$$
\sin(4t) \cdot 1 + \cos(6t) \cdot (-6 \sin(6t)) + t \cdot 4 \cos(4t)
$$

Simplify:

$$
\sin(4t) - 6 \cos(6t) \sin(6t) + 4 t \cos(4t)
$$

\]

Recall:

\[
\cos(6t) \sin(6t) = \frac

If U T Sin 4t Cos 6t T And V T T Cos 6t Sin 4t Use Formula 4 Of This Theorem To Find

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