If We Want To Evaluate The Integral I=x2+49x+29dx We Use The Trigonometric Substitution X= And Dx=d And

If We Want To Evaluate The Integral I = ∫ (x^2 + 49x + 29) dx We Use The Trigonometric Substitution X = And Dx = d And

When tackling integrals involving quadratic expressions, especially those that resemble the form of a sum of squares, trigonometric substitution becomes a powerful technique. It simplifies the integral by transforming the algebraic expression into a trigonometric form, which is often easier to integrate. In this article, we will explore the process of evaluating the integral $I = \int (x^2 + 49x + 29)\,dx$ using trigonometric substitution, covering the theoretical background, step-by-step procedures, and practical examples to enhance your understanding.

Understanding the Integral and Its Structure

Before diving into substitution methods, it's crucial to analyze the integral:

$I = \int (x^2 + 49x + 29)\,dx$

This is a polynomial integral, and typically, such integrals are straightforward to evaluate using algebraic methods like completing the square or direct integration after simplification. However, when the quadratic expression resembles a form suitable for trigonometric substitution—particularly when it can be expressed as a sum or difference involving a perfect square—this method can offer an elegant solution.

Why Use Trigonometric Substitution?

Trigonometric substitution is particularly effective for integrals involving expressions like:

- $\sqrt{(a^2 - x^2)}$
- $\sqrt{(a^2 + x^2)}$
- $\sqrt{(x^2 - a^2)}$

These forms correspond to identities involving sine, cosine, and tangent functions, which simplify the radicals and algebraic expressions.

In our case, the quadratic expression $x^2 + 49x + 29$ can be rewritten to fit one of these forms by completing the square. Once in a suitable form, a trigonometric substitution can be applied to evaluate the integral more straightforwardly.

Step-by-Step Process for Evaluating the Integral

Step 1: Complete the Square

The first step involves rewriting the quadratic expression $x^2 + 49x + 29$ as a perfect square plus a constant. This makes it easier to identify a substitution.

- The quadratic is:

$x^2 + 49x + 29$

- Complete the square:

1. Take half of the coefficient of x (which is 49):

$49 / 2 = 24.5$

2. Square this:

$(24.5)^2 = 600.25$

3. Rewrite the quadratic as:

$x^2 + 49x + 600.25 - 600.25 + 29$

$= (x + 24.5)^2 - 600.25 + 29$

$= (x + 24.5)^2 - 571.25$

- Thus, the integral becomes:

$I = \int [(x + 24.5)^2 - 571.25]\, dx$

Step 2: Substitution to Simplify the Expression

Let:

$u = x + 24.5$

Then:

$du = dx$

Rewriting the integral:

$I = \int [u^2 - 571.25] du$

This is a standard polynomial integral, and we can proceed with direct integration.

Step 3: Integrate the Simplified Expression

- Integrate term by term:

$I = \int u^2 du - 571.25 \int du$

- Recall:

$\int u^2 du = (u^3) / 3$

$\int du = u$

- Therefore:

$I = (u^3) / 3 - 571.25 u + C$

- Substitute back $u = x + 24.5$:

$I = [(x + 24.5)^3] / 3 - 571.25 (x + 24.5) + C$

This completes the integral evaluation using algebraic methods.

Incorporating Trigonometric Substitution

While the algebraic method is straightforward here, suppose the quadratic expression was more complex or involved radicals, making trigonometric substitution more advantageous.

Let's consider a general approach using trigonometric substitution, especially if the quadratic takes the form suitable for such techniques.

Scenario: When to Use Trigonometric Substitution

- For integrals involving $\sqrt{(a^2 - x^2)}$, use $x = a \sin \theta$
- For $\sqrt{(a^2 + x^2)}$, use $x = a \tan \theta$
- For $\sqrt{(x^2 - a^2)}$, use $x = a \sec \theta$

In our case, after completing the square, the quadratic becomes:

$(x + 24.5)^2 - 571.25$

Suppose we consider the integral involving $\sqrt{u^2 - a^2}$, which resembles the form:

$\int dx / \sqrt{x^2 - a^2}$

or similar variations, trigonometric substitution becomes a natural choice.

Applying Trigonometric Substitution to a Similar Integral

Suppose we had an integral like:

$I = \int dx / \sqrt{x^2 - a^2}$

- The substitution:

$x = a \sec \theta$

- Then:

$dx = a \sec \theta \tan \theta \, d\theta$

- And:

$\sqrt{x^2 - a^2} = \sqrt{a^2 \sec^2 \theta - a^2} = a \tan \theta$

- Substituting into the integral:

$I = \int a \sec \theta \tan \theta \, d\theta / a \tan \theta = \int \sec \theta \, d\theta$

- Which integrates to:

$\int \sec \theta \, d\theta = \ln |\sec \theta + \tan \theta| + C$

- Reverting back to x:

$\sec \theta = x / a$

$\tan \theta = \sqrt{\sec^2 \theta - 1} = \sqrt{x^2 / a^2 - 1} = \sqrt{x^2 - a^2} / a$

- So the integral evaluates to:

$I = \ln |x / a + \sqrt{x^2 - a^2} / a| + C$

- Simplify:

$I = \ln |x + \sqrt{x^2 - a^2}| + C$

This illustrates how trigonometric substitution simplifies the integral.

Practical Example: Evaluating a Modified Integral

Let's consider a similar integral where the quadratic expression involves radicals:

$J = \int dx / \sqrt{x^2 + 49}$

- Recognize the form:

$\sqrt{x^2 + a^2}$

- The suitable substitution:

$x = a \tan \theta$

- Here, $a = 7$

- Then:

$dx = 7 \sec^2 \theta \, d\theta$

- And:

$\sqrt{x^2 + 49} = \sqrt{7^2 \tan^2 \theta + 49} = 7 \sec \theta$

- Substituting:

$J = \int 7 \sec^2 \theta \, d\theta / 7 \sec \theta = \int \sec \theta \, d\theta$

- The integral:

$\int \sec \theta \, d\theta = \ln |\sec \theta + \tan \theta| + C$

- Reverting back:

$\sec \theta = x / 7$

$\tan \theta = \sqrt{\sec^2 \theta - 1} = \sqrt{x^2 / 49 - 1} = \sqrt{x^2 - 49} / 7$

- Final expression:

$J = \ln |x / 7 + \sqrt{x^2 - 49} / 7| + C$

Simplify:

$J = \ln |x + \sqrt{x^2 - 49}| + C$

This demonstrates the power of trigonometric substitution in evaluating integrals involving quadratic radicals.

Summary of Key Steps in Trigonometric Substitution

To effectively evaluate integrals using trigonometric substitution, follow these systematic steps:

1. Identify the form of the radical or quadratic expression.
2. Complete the square if necessary.
3. Select the appropriate substitution:
- $x = a \sin \theta$ for $\sqrt{a^2 - x^2}$
- $x = a \tan \theta$ for $\sqrt{a^2 + x^2}$
- $x = a \sec \theta$ for $\sqrt{x^2 - a^2}$
4. Compute dx in terms of dθ.
5. Rewrite the integral in terms of θ, simplifying radicals using identities.
6. Integrate in θ.
7. Back-substitute to x using inverse trigonometric functions.

Benefits of Trigonometric Substitution

- Simplifies radicals involving quadratic expressions.
- Converts complicated algebraic integrals into standard trigonometric integrals.
- Provides a systematic approach to integrals that are otherwise complex to evaluate algebraically.
- Enhances understanding of the relationship between algebraic and trigonometric functions.

Conclusion

Evaluating the integral $I = \int (x^2 + 49x + 29)\, dx$ demonstrates the importance of choosing the right technique based on the quadratic expression's structure. While completing the square and direct integration are straightforward methods, trigonometric substitution offers a powerful alternative, especially for more complex radicals or quadratic forms. Understanding when and how to apply this method broadens your problem-solving toolkit in calculus, enabling you to tackle a wide array of integrals efficiently.

By mastering the process of completing the square and selecting suitable trigonometric substitutions, you can simplify and evaluate integrals with confidence. Whether you are dealing with polynomial expressions, radicals, or more intricate functions, these techniques form the foundation of integral calculus and are essential for advanced mathematical problem-solving.

Keywords: integral evaluation, trigonometric substitution, quadratic integrals, calculus techniques, completing the square, radical simplification, calculus tutorial, integral

Frequently Asked Questions

What is the integral $I = \int(x^2 + 49x + 29)\,dx$ that we need to evaluate?

The integral $I = \int(x^2 + 49x + 29)\,dx$ is a polynomial integral that can be approached using algebraic methods or substitution techniques, but in this context, we're considering trigonometric substitution for a specific form.

Which substitution is appropriate for evaluating the integral involving $x^2 + 49x + 29$?

A suitable substitution depends on completing the square for the quadratic expression or recognizing a form suitable for trigonometric substitution, such as $x = 7 \tan \theta$, to simplify the integral.

What is the specific form of the substitution used in trigonometric substitution for this integral?

The typical substitution is $x = 7 \tan \theta$, which transforms the quadratic into a form involving $\sec^2 \theta$, facilitating easier integration.

How is dx expressed in terms of dθ after choosing the substitution $x = 7 \tan \theta$?

Differentiating $x = 7 \tan \theta$ gives $dx = 7 \sec^2 \theta \, d\theta$.

Why is trigonometric substitution useful for evaluating integrals of quadratic expressions?

Trigonometric substitution simplifies the quadratic expressions into trigonometric identities, making the integral easier to evaluate, especially when the quadratic resembles forms like $a^2 + x^2$.

How do you complete the square for the quadratic expression

$x^2 + 49x + 29$?

Complete the square as follows: $x^2 + 49x + (49/2)^2 - (49/2)^2 + 29 = (x + 24.5)^2 - 24.5^2 + 29$, simplifying the integral after substitution.

What is the general process for evaluating the integral after applying the trigonometric substitution?

Substitute x in terms of θ, express dx in terms of dθ, rewrite the integral in terms of θ, simplify using trigonometric identities, integrate, and then back-substitute to x if needed.

Are there alternative methods to evaluate the integral besides trigonometric substitution?

Yes, completing the square and then integrating directly, or using standard polynomial integration techniques, are alternative methods. Trigonometric substitution is particularly useful for integrals involving quadratic expressions that match certain identities.

Additional Resources

If We Want To Evaluate The Integral $I = x^2 + 49x + 29$ dx, We Use The Trigonometric Substitution x = ... and dx = ...

Introduction

In the realm of calculus, integrating quadratic expressions often presents a challenge that invites creative techniques beyond basic algebra. One such technique is trigonometric substitution, an elegant method that leverages the properties of trigonometric functions to simplify integrals involving square roots or quadratic polynomials.

This article embarks on an in-depth exploration of evaluating the integral:

$$
I = \int (x^2 + 49x + 29) \, dx,
$$

focusing on the application of trigonometric substitution. We will examine the reasoning behind choosing appropriate substitutions, the step-by-step process, and the nuances that make this method effective. Our goal is to present a comprehensive, rigorous analysis suitable for educators, students, and professionals seeking a thorough understanding of the technique.

Understanding the Integral

The integral under consideration is:

$$
I = \int (x^2 + 49x + 29) \, dx,
$$

which involves a quadratic polynomial. At first glance, this looks straightforward: a polynomial integral can be directly integrated term by term:

$$
I = \frac{x^3}{3} + \frac{49x^2}{2} + 29x + C,
$$

where C is the constant of integration.

However, the context of applying trigonometric substitution suggests that there's an underlying motivation—perhaps the integral was part of a larger problem involving a radical expression, or the quadratic is to be transformed into a perfect square to facilitate substitution.

In many cases, trigonometric substitution is most beneficial when integrating functions involving expressions like $\sqrt{a^2 - x^2}$, $\sqrt{a^2 + x^2}$, or $\sqrt{x^2 - a^2}$.

Given that, our focus is to explore how to adapt the quadratic polynomial integral into a form suitable for trigonometric substitution, which often involves completing the square.

Completing the Square: The First Step

Before applying any substitution, it's essential to understand the quadratic polynomial's structure.

Step 1: Complete the square for the quadratic $x^2 + 49x + 29$.

We proceed as follows:

$$
x^2 + 49x + 29 = \left(x^2 + 49x + \left(\frac{49}{2}\right)^2\right) - \left(\frac{49}{2}\right)^2 + 29,
$$

which simplifies to:

$$
\left(x + \frac{49}{2}\right)^2 - \frac{2401}{4} + 29.
$$

Express the constants with a common denominator:

$$
- \frac{2401}{4} + 29 = - \frac{2401}{4} + \frac{116}{4} = - \frac{2285}{4}.
$$

Thus,

$$
x^2 + 49x + 29 = \left(x + \frac{49}{2}\right)^2 - \frac{2285}{4}.
$$

This form reveals the quadratic as a difference of a square and a constant.

When Is Trigonometric Substitution Appropriate?

Having expressed the quadratic as:

$$
x^2 + 49x + 29 = \left(x + \frac{49}{2}\right)^2 - \frac{2285}{4},
$$

we notice it resembles the form $x^2 - a^2$, which suggests a substitution involving tangent or secant functions.

Key observation:

- If the integrand involves $\sqrt{(x + b)^2 - a^2}$, then the substitution $x + b = a \sec \theta$ or $x + b = a \tan \theta$ is suitable.

In our case, the quadratic is not under a radical yet, but if the integral were of the form:

$$
\int \frac{dx}{\sqrt{x^2 - a^2}},
$$

or similar expressions, trigonometric substitution would be ideal.

But, since the integral involves a polynomial, direct integration is straightforward unless we desire to evaluate an indefinite integral involving radicals or other functions derived from this quadratic.

Hypothetical Scenario: The Integral Under a Radical

Suppose the integral was:

$$
I = \int \sqrt{x^2 + 49x + 29} \, dx,
$$

which is a common context where trigonometric substitution applies.

Expressed in completed square form:

$$
I = \int \sqrt{\left(x + \frac{49}{2}\right)^2 - \frac{2285}{4}} \, dx.

\]

Let:

$$
x + \frac{49}{2} = a \sec \theta,
$$

where:

$$
a = \sqrt{\frac{2285}{4}} = \frac{\sqrt{2285}}{2}.
$$

Then,

$$
dx = a \sec \theta \tan \theta \, d\theta,
$$

and the integrand becomes:

$$
\sqrt{a^2 \sec^2 \theta - a^2} = a \sqrt{\sec^2 \theta - 1} = a \tan \theta.
$$

This leads to the integral:

$$
I = \int a \tan \theta \times a \sec \theta \tan \theta \, d\theta = a^2 \int \tan^2 \theta \sec \theta \, d\theta.
$$

Such an integral involves standard identities and can be tackled with substitution and known formulas.

General Procedure for Trigonometric Substitution in Quadratic Integrals

Based on the above, the general steps when employing trigonometric substitution are:

1. Complete the square to express the quadratic in the form $(x + b)^2 \pm a^2$.
2. Identify the appropriate substitution:
- $x + b = a \sec \theta$ when the quadratic is of the form $(x + b)^2 - a^2$,
- $x + b = a \tan \theta$ when the quadratic is of the form $(x + b)^2 + a^2$,
- $x + b = a \sinh \theta$ for hyperbolic substitution.
3. Calculate dx in terms of θ.
4. Rewrite the integral in terms of θ, simplifying using identities.
5. Integrate with respect to θ.
6. Back-substitute to express the result in terms of x.

Applying Trigonometric Substitution: Step-by-Step Example

Let's formalize the process with a concrete example, assuming the integral:

$$
I = \int \sqrt{x^2 + 49x + 29} \, dx,
$$

which involves radicals.

Step 1: Complete the square:

$$
x^2 + 49x + 29 = \left(x + \frac{49}{2}\right)^2 - \frac{2285}{4}.
$$

Set:

$$
u = x + \frac{49}{2},
$$

then:

$$
I = \int \sqrt{u^2 - \left(\frac{\sqrt{2285}}{2}\right)^2} \, du.
$$

Step 2: Choose substitution:

$$
u = a \sec \theta,
$$
where:

$$
a = \frac{\sqrt{2285}}{2}.
$$

Step 3: Compute du:

$$
du = a \sec \theta \tan \theta \, d\theta.
$$

Step 4: Rewrite the integral:

$$
I = \int \sqrt{a^2 \sec^2 \theta - a^2} \times a \sec \theta \tan \theta \, d\theta,
$$

\]

which simplifies to:

\[
I = \int a \tan \theta \times a \sec \theta \tan \theta \, d\theta = a^2 \int \tan^2 \theta \sec \theta \, d\theta.
\]

Step 5: Simplify and integrate:

Using the identity \(\tan^2 \theta = \sec^2 \theta - 1\):

\[
I = a^2 \int (\sec^2 \theta - 1) \sec \theta \, d\theta = a^2 \int (\sec^3 \theta - \sec \theta) \, d\theta.
\]

This separates into:

\[
I = a^2 \left(\int \sec^3 \theta \, d\theta - \int \sec \theta \, d\theta \right).
\]

The integrals:

- \(\int \sec \theta \, d\theta = \ln |\sec \theta + \tan \theta| + C\),
- \(\int \sec^3 \theta \, d\theta\) has a standard solution:

\[
\int \sec^3 \theta \, d\theta = \frac{1}{2} \sec \theta \tan \

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