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Understanding arithmetic progressions (AP) is fundamental in mathematics, especially in solving problems involving sequences and series. In this comprehensive guide, we will explore a specific problem: given that the sum of the first 10 terms of an AP is 50, and the fifth term is three times the second term, how can we determine the first term? We'll walk through the problem step-by-step, explain relevant concepts, and provide detailed solutions to enhance your understanding of arithmetic progressions.

Understanding the Basics of Arithmetic Progression (AP)

Before diving into the problem, let's review some essential concepts related to arithmetic progressions.

What is an Arithmetic Progression?

An arithmetic progression (AP) is a sequence of numbers where the difference between any two successive terms is constant. This constant difference is called the common difference and is denoted by 'd'.

Example of an AP: 2, 5, 8, 11, 14, ...

In this sequence:
- First term (a): 2
- Common difference (d): 3

General Term of an AP

The nth term of an AP is given by:
$$a_n = a + (n - 1)d$$
where:
- a = first term
- d = common difference
- n = position of the term

Sum of the First n Terms of an AP

The sum of the first n terms, denoted by S_n, can be calculated using:
$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

This formula is crucial for solving problems involving the sum of a sequence of terms.

Analyzing the Given Problem

Let's restate the problem:

> In an AP, the sum of the first 10 terms is 50, and the fifth term is three times the second term. Find the first term.

To solve this, we'll identify what information is given and what we need to find.

Given:
- $S_{10} = 50$
- $a_5 = 3 \times a_2$

Find:
- The first term a

Step-by-Step Solution Approach

We'll approach the problem systematically:

Step 1: Express Known Terms in Terms of a and d

Using the general term formula:
- $a_2 = a + d$
- $a_5 = a + 4d$

The relation $a_5 = 3 \times a_2$ becomes:
$$a + 4d = 3(a + d)$$

Step 2: Simplify the Relation Between a and d

From the above:
$$a + 4d = 3a + 3d$$

Rearranged:
$$ a + 4d - 3a - 3d = 0 $$
$$ -2a + d = 0 $$
Therefore:
$$ d = 2a $$

This relation between d and a is vital for further calculations.

Step 3: Use the Sum Formula to Find a and d

Recall:
$$ S_{10} = \frac{10}{2} [2a + (10 - 1)d] = 50 $$
Simplify:
$$ 5 [2a + 9d] = 50 $$

Substitute $d = 2a$:
$$ 5 [2a + 9(2a)] = 50 $$
$$ 5 [2a + 18a] = 50 $$
$$ 5 \times 20a = 50 $$
$$ 100a = 50 $$
$$ a = \frac{50}{100} = \frac{1}{2} $$

Now, find d:
$$ d = 2a = 2 \times \frac{1}{2} = 1 $$

Final Answer and Interpretation

The first term a of the arithmetic progression is $\frac{1}{2}$.

This conclusion aligns with the initial conditions:
- The sequence starts at 0.5.
- The common difference is 1.
- The sequence's terms: 0.5, 1.5, 2.5, 3.5, 4.5, 5.5, 6.5, 7.5, 8.5, 9.5.

Verify the key conditions:
- Sum of first 10 terms:

$$ S_{10} = \frac{10}{2} [2 \times 0.5 + 9 \times 1] = 5 [1 + 9] = 5 \times 10 = 50 $$

- Fifth term:

$$ a_5 = a + 4d = 0.5 + 4 \times 1 = 4.5 $$

- Second term:

$$ a_2 = a + d = 0.5 + 1 = 1.5 $$

- Check the relation:

$$a_5 = 3 \times a_2 \Rightarrow 4.5 = 3 \times 1.5 \Rightarrow 4.5 = 4.5$$

All conditions are satisfied, confirming the solution's correctness.

Key Takeaways and Tips for Solving AP Problems

When tackling similar problems, keep these points in mind:

1. **Identify knowns and unknowns:** Clearly list what is given and what needs to be found.
2. **Express terms in terms of a and d:** Use the general formula for the nth term.
3. **Write equations based on given conditions:** Relations between terms often lead to equations involving a and d.
4. **Substitute and simplify:** Use algebraic manipulation to find relations between variables.
5. **Verify the solution:** Plug the found values back into the original conditions for confirmation.

Additional Practice Problems

To strengthen your understanding, try solving these problems:

- In an AP, the sum of the first 15 terms is 120, and the third term is twice the first term. Find the first term and the common difference.
- The 7th term of an AP is 20, and the sum of the first 7 terms is 70. Find the first term and the common difference.
- If the first term of an AP is 5, and the sum of the first n terms is 55 when $n = 5$, find the common difference.

Conclusion

Mastering arithmetic progressions involves understanding the fundamental formulas, recognizing relationships between terms, and applying algebraic techniques to solve for unknowns. The problem we've dissected demonstrates how to use the sum formula and term relations to find the first term of an AP with given conditions. With practice, you'll develop confidence in solving a wide variety of sequence and series problems, which are essential for higher mathematics and various real-world applications.

By following systematic approaches and verifying your solutions, you'll enhance both your problem-solving skills and mathematical intuition. Keep practicing different types of AP problems to deepen your understanding and become proficient in this fundamental area of mathematics.

Frequently Asked Questions

In an AP, the sum of the first 10 terms is 50, and the fifth term is 3 times the second term. How do you find the first term?

Let the first term be a and the common difference be d. Given sum $S_{10} = 50$, and the fifth term $T_5 = 3 \times T_2$. Using formulas, set up equations: $T_2 = a + d$, $T_5 = a + 4d$. Given $T_5 = 3T_2$, so $a + 4d = 3(a + d)$. Solving gives $a = -d$. Using the sum formula, substitute $a = -d$ and solve for d to find the values of a and d.

What is the significance of the relation 'fifth term is 3 times the second term' in solving for the first term in an AP?

This relation allows us to establish an equation connecting the first term and common difference, specifically $a + 4d = 3(a + d)$. Simplifying this helps express a in terms of d, which is essential for solving the sum condition and finding the first term.

How do you use the sum of the first 10 terms to determine the first term in an AP?

The sum of the first n terms of an AP is given by $S_n = (n/2)[2a + (n-1)d]$. For n=10, plug in the sum (50) and solve the resulting equation along with other relations (like the term ratio) to find the first term a.

If the fifth term of an AP is three times the second term, and the sum of first 10 terms is 50, what steps are taken to find the first term?

First, write equations for T_2 and T_5: $T_2 = a + d$, $T_5 = a + 4d$. Use the relation $T_5 = 3T_2$ to find a in terms of d. Next, apply the sum formula for the first 10 terms and substitute $a = -d$ to solve for d. Finally, find a using the relation between a and d.

Can you provide a step-by-step method to determine the first term of an AP given the sum of first 10 terms and a relation between the second and fifth terms?

Yes. Step 1: Write $T_2 = a + d$ and $T_5 = a + 4d$. Step 2: Use $T_5 = 3 T_2$ to get a relation between a and d. Step 3: Express a as -d from this relation. Step 4: Use the sum formula $S_{10} = (10/2)[2a + 9d]$ and substitute $a = -d$. Step 5: Solve for d, then find a using $a = -d$. This yields the value of the first term.

Additional Resources

Arithmetic Progression (AP)

Understanding the behavior of sequences is fundamental in mathematics, especially when dealing with arithmetic progressions (AP). Today, we delve into a classic problem involving an AP: "In an AP, the sum of the first 10 terms is 50, and the fifth term is three times the second term. Find the first term." This problem not only tests our grasp of basic AP properties but also encourages us to develop problem-solving strategies that can be applied across various mathematical contexts.

Understanding the Basics of Arithmetic Progressions

Before analyzing the problem, it’s crucial to revisit the foundational concepts of AP.

What is an Arithmetic Progression?

An arithmetic progression is a sequence of numbers in which the difference between any two consecutive terms is constant. This constant difference is called the common difference and is usually denoted by d.

General form of an AP:

$$ a, a + d, a + 2d, a + 3d, \ldots $$

Where:

- a is the first term.
- d is the common difference.

Key characteristics:

- The n-th term of the AP:

$$ a_n = a + (n - 1)d $$

- The sum of the first n terms:

$$ S_n = \frac{n}{2} (2a + (n - 1)d) $$

Deconstructing the Problem Statement

The problem provides two critical pieces of information:

1. Sum of the first 10 terms is 50:

$$ S_{10} = 50 $$

2. The fifth term is three times the second term:

$$ a_5 = 3 \times a_2 $$

Our goal is to find the first term, a.

Step-by-Step Solution Approach

To solve for a, we will systematically use the given information and the properties of AP.

Step 1: Express the sum of the first 10 terms in terms of a and d

Using the sum formula:

$$ S_{10} = \frac{10}{2} [2a + (10 - 1)d] = 5 [2a + 9d] $$

Given:

$$ S_{10} = 50 $$

Thus,

$$ 5 [2a + 9d] = 50 $$

Dividing both sides by 5:

$$ 2a + 9d = 10 \quad \text{(Equation 1)} $$

Step 2: Express the fifth and second terms in terms of a and d

Using the general term formula:

$$ a_n = a + (n - 1)d $$

- The second term:

$$ a_2 = a + (2 - 1)d = a + d $$

- The fifth term:

$$ a_5 = a + (5 - 1)d = a + 4d $$

The problem states:

$$ a_5 = 3 \times a_2 $$

Substitute the expressions:

$$ a + 4d = 3(a + d) $$

Step 3: Solve for d in terms of a

Expand the right side:

$$ a + 4d = 3a + 3d $$

Bring all terms to one side:

$$ a + 4d - 3a - 3d = 0 $$

Simplify:

$$ -2a + d = 0 $$

Thus,

$$ d = 2a \quad \text{(Equation 2)} $$

Step 4: Substitute d into Equation 1 to find a

Recall Equation 1:

$$ 2a + 9d = 10 $$

Substitute $d = 2a$:

$$ 2a + 9(2a) = 10 $$

Simplify:

$$ 2a + 18a = 10 $$

$$ 20a = 10 $$

Divide both sides by 20:

$$ a = \frac{10}{20} = \frac{1}{2} $$

First term, a = 0.5

Verifying the Solution

While the calculations seem straightforward, verifying the solution ensures accuracy.

Calculate d:

From Equation 2:

$$ d = 2a = 2 \times 0.5 = 1 $$

Check the sum S_{10}:

Using the sum formula:

$$S_{10} = 5 [2a + 9d] = 5 [2 \times 0.5 + 9 \times 1] = 5 [1 + 9] = 5 \times 10 = 50$$

Matches the given sum.

Check the relationship between a_5 and a_2:

- $a_2 = a + d = 0.5 + 1 = 1.5$
- $a_5 = a + 4d = 0.5 + 4 \times 1 = 0.5 + 4 = 4.5$

Verify the ratio:

$$a_5 = 3 \times a_2 \Rightarrow 4.5 = 3 \times 1.5 \Rightarrow 4.5 = 4.5$$

This confirms the solution's correctness.

Summary and Key Takeaways

- The first term a of the AP is 0.5.
- The common difference d is 1.
- The problem demonstrates the importance of systematically translating verbal information into algebraic expressions and substituting appropriately.

Main lessons:

- Using the sum formula for the first n terms provides a powerful tool for solving AP-related problems.
- Expressing terms in terms of a and d simplifies the process.
- Verifying results with back-substitution ensures solution accuracy.

Additional Insights and Applications

This problem exemplifies core algebraic reasoning applied to sequences, which has broader applications such as:

- Financial calculations involving regular payments or interest.
- Engineering problems involving evenly spaced measurements.
- Computer science algorithms that process sequences or iterative data.

Moreover, mastering such problems enhances problem-solving skills, critical thinking, and the ability to interpret word problems effectively.

Conclusion

The exploration of this AP problem underscores the elegance and utility of fundamental mathematical principles. By carefully translating problem statements into equations, employing algebraic substitution, and verifying results, we arrive at a precise and reliable solution. Whether in academic settings or real-world applications, these analytical skills are invaluable tools for navigating the complexities of sequences and series.

In essence, the first term of the AP in this problem is 0.5, with a common difference of 1, beautifully illustrating the harmony of algebraic reasoning and sequence analysis.

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