In Exercises 43 Through 46, Solve The Given Separable Initial Value Problem. 43. Dx/dy =2y;y=3 When X=0

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Separable differential equations are a fundamental class of differential equations where the variables can be separated on different sides of the equation. They are particularly accessible because they allow us to integrate both sides independently after rearranging. In this article, we will explore the process of solving a specific initial value problem (IVP) involving a separable differential equation. The initial value problem given is:

- Dx/dy = 2y
- with the initial condition x=0 when y=3

Our goal is to find the explicit function x=y that satisfies this differential equation and the initial condition. We will systematically go through the steps, including separation of variables, integration, applying the initial condition, and interpreting the solution.

Understanding the Problem

What Is a Separable Differential Equation?

A differential equation is called separable if it can be written in the form:

```
dx/dy = f(y)  g(x)
```

or, more generally, in a form that can be rearranged to:

```
h(x) dx = k(y) dy
```

In our case, the differential equation is:

```
dx/dy = 2y
```

This is already in a form where the derivative of x with respect to y depends solely on y, making it straightforward to separate variables.

Initial Condition and Its Significance

The initial condition y=3 when x=0 provides the starting point for the solution. It allows us to determine the constant of integration after integrating the differential equation.

Step-by-Step Solution Process

Step 1: Rewrite the Differential Equation

The given differential equation is:

$$dx/dy = 2y$$

Since the right side depends only on y, and the left is dx/dy, we recognize that we can treat x as a function of y, and rewrite as:

$$dx = 2y\ dy$$

Step 2: Separate Variables and Integrate

Because the variables are already separated, we can integrate both sides independently:

$$\int dx = \int 2y\ dy$$

Integrating each side gives:

$$x = y^2 + C$$

where C is the constant of integration.

Step 3: Apply the Initial Condition to Find C

Using the initial condition x=0 when y=3:

$$0 = (3)^2 + C$$

$$0 = 9 + C$$

$$C = -9$$

Thus, the particular solution to the IVP is:

$$x = y^2 - 9$$

Final Solution and Interpretation

Explicit Solution

The explicit solution to the initial value problem is:

```
x = y^2 - 9
```

This function satisfies both the differential equation and the initial condition, as verified by substitution.

Verification of the Solution

Differentiate x with respect to y:

```
dx/dy = 2y
```

which matches the original differential equation perfectly. Additionally, when y=3:

```
x = 3^2 - 9 = 9 - 9 = 0
```

```
which confirms the initial condition is satisfied.
```

Additional Insights and Applications of Separable Equations

General Approach to Separable Equations

1. Identify if the differential equation is separable.
2. Rewrite the equation to separate variables.
3. Integrate both sides with respect to their variables.
4. Apply initial conditions to find constants.
5. Express the solution explicitly or implicitly.

Common Applications

- Modeling population growth where the rate depends on the current population.
- Radioactive decay processes.
- Heat transfer problems.
- Chemical reactions where concentration changes over time.

Summary and Conclusion

In this problem, we demonstrated how to solve a straightforward separable initial value problem. By recognizing the separability of the differential equation, integrating both sides, and applying the initial condition, we obtained an explicit expression for x in terms of y. This process exemplifies the powerful method of solving separable equations and highlights their importance across various scientific disciplines.

Understanding the methodical approach to these equations equips students and practitioners with essential tools for tackling more complex differential equations encountered in mathematics, physics, engineering, and beyond.

Frequently Asked Questions

What is the initial value problem given in Exercise 43?

The initial value problem is dy/dx = 2y with the initial condition y = 3 when x = 0.

How do you separate variables in the differential equation dy/dx = 2y?

Rearranged as dy/y = 2 dx, allowing integration of both sides separately.

What is the general solution to the differential equation dy/dx = 2y?

The general solution is y = Ce^{2x}, where C is an arbitrary constant.

How do you find the specific solution using the initial condition y(0) = 3?

Substitute x = 0 and y = 3 into the general solution: 3 = C e^{0} => C = 3, so the specific solution is y = 3e^{2x}.

What is the value of y when x = 0 in the specific solution?

When x = 0, y = 3e^{0} = 3.

How can you verify that the solution y = 3e^{2x} satisfies the differential equation?

Differentiate y = 3e^{2x} to get dy/dx = 6e^{2x}, and since y = 3e^{2x}, dy/dx = 2 y, confirming it satisfies the original equation.

What is the importance of the initial condition in solving the IVP?

The initial condition allows us to determine the constant C and find the unique solution that passes through the given point.

Are separable equations always solvable using this method?

Most separable equations are solvable using variable separation, provided the functions involved are integrable.

What are common pitfalls when solving separable initial value problems?

Common pitfalls include incorrect separation of variables, forgetting to include the constant of integration, and neglecting to apply initial conditions to find the particular solution.

Additional Resources

Separable Initial Value Problem (IVP) Solutions: A Comprehensive Review

When exploring differential equations, especially first-order differential equations, the method of separable equations stands out as one of the most accessible and elegant techniques. In particular, solving initial value problems (IVPs) that are separable involves straightforward algebraic manipulation, making it a fundamental skill in the toolkit of students and professionals working in mathematics, physics, engineering, and related fields. The specific problem highlighted—Dx/dy = 2y with the initial condition x=0 when y=3—serves as a classic example demonstrating the power and clarity of this method. This review delves into the intricacies of solving such problems, breaking down the steps, discussing key features, and analyzing strengths and limitations.

Understanding Separable Differential Equations

Separable differential equations are those that can be expressed in the form:

$$
\frac{dx}{dy} = g(y) \cdot h(x)
$$

or equivalently,

$$
\frac{dx}{dy} = f(y) \cdot g(x)
$$

which allows for the separation of variables—placing all x-related terms on one side and y-related terms on the other. The core idea is to rewrite the equation so that each side involves only one variable, then integrate both sides to find the solution.

Key features of separable equations:

- Simplicity: They often involve straightforward algebraic manipulation.
- Universality: Many first-order differential equations can be transformed into separable form.
- Applicability: Useful in modeling phenomena where the rate of change depends on the current state, such as population growth, radioactive decay, and chemical reactions.

Step-by-Step Solution of the Given IVP

The initial value problem presented is:

$$
\frac{dx}{dy} = 2y, \quad \text{with} \quad x=0 \quad \text{when} \quad y=3
$$

This example is a textbook case showcasing the procedure for solving a separable differential equation with initial conditions.

Step 1: Recognize the form

The equation $\frac{dx}{dy} = 2y$ is already separable because the right side depends solely on y, and the left side involves x's derivative with respect to y.

Step 2: Separate variables

Since the right side involves only y, and the derivative is with respect to y, rewrite as:

$$
dx = 2y\,dy

\]

This step simplifies the differential equation into an integrable form where variables are separated.

Step 3: Integrate both sides

Integrate to find x as a function of y:

\[
\int dx = \int 2y\,dy
\]

which yields:

\[
x = y^2 + C
\]

where C is the constant of integration.

Step 4: Use initial conditions to find C

Apply the initial condition \(x=0\) when \(y=3\):

\[
0 = (3)^2 + C \Rightarrow 0 = 9 + C \Rightarrow C = -9
\]

Thus, the particular solution is:

\[
x = y^2 - 9
\]

Analysis of the Solution and Its Features

The completed solution, \(x = y^2 - 9\), is a simple quadratic relation describing the evolution of x in terms of y. Several features and considerations are worth discussing:

Features:

- Explicit form: The solution is expressed explicitly, making it easy to evaluate at any y.
- Initial condition incorporation: The constant C ensures the solution satisfies the initial condition, giving it real-world relevance.
- Continuous and smooth: The solution is continuous and differentiable over all real y, reflecting the

nature of the differential equation.

Pros:

- Ease of solution: The problem demonstrates how straightforward it is to solve simple separable equations.
- Educational clarity: It showcases core concepts such as separation of variables, integration, and applying initial conditions.
- Applicability: Similar techniques extend well to more complex problems, provided they are separable.

Cons:

- Limited scope: Not all differential equations are separable; many require other methods.
- Potential integration challenges: For more complex equations, integration may not be straightforward.
- Initial condition limitations: In some cases, initial conditions may lead to solutions with discontinuities or non-single-valued solutions.

Broader Context and Relevance

Separable equations like the one discussed are fundamental in modeling real-world phenomena. For example, in physics, the equation might represent a process where the rate of change of an entity depends solely on its current state. Understanding how to solve such equations efficiently and accurately is critical for scientists and engineers.

Applications include:

- Population dynamics: Modeling how populations grow or decline based on current size.
- Radioactive decay: Describing how the amount of a substance decreases over time.
- Chemical kinetics: Expressing reaction rates that depend on concentration.

Educational significance:

- Serves as an introductory step into differential equations.
- Builds intuition about the relationship between derivatives, integrals, and initial conditions.
- Provides a foundation for solving more complex non-separable equations via substitution or numerical methods.

Additional Considerations and Advanced Topics

While the example provided is straightforward, real-world problems often involve more complexities:

- Non-separable equations: Many real-world models are not readily separable; techniques such as integrating factors, substitutions, or numerical methods are necessary.
- Initial condition sensitivity: Some solutions may have multiple branches or singularities; understanding the domain of solutions is crucial.
- Existence and uniqueness: The Picard-Lindelöf theorem provides criteria for the existence and uniqueness of solutions, which is fundamental in advanced studies.

Conclusion

The process of solving the initial value problem $\frac{dx}{dy} = 2y$, with $x=0$ when $y=3$, exemplifies the elegance and effectiveness of the method of separation of variables. This technique's simplicity, combined with its broad applicability, makes it an essential first step in understanding differential equations. By integrating and applying initial conditions carefully, one obtains explicit solutions that can be analyzed, interpreted, and applied across various scientific domains. While the method has its limitations—particularly when equations are not separable—it remains a cornerstone technique with enduring relevance in mathematics and beyond. Understanding its features, strengths, and potential pitfalls prepares students and practitioners to approach more complex problems with confidence and clarity.

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