In R Consider The Vectors V = (1,-1,2,3) And V = (1,0,1,2). Let V Be The Subspace Of R Spanned By V And

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Understanding the structure of subspaces in real coordinate spaces is fundamental in linear algebra. When given specific vectors in $\mathbb{R}^n$, identifying the subspace they span helps in solving systems of equations, understanding linear transformations, and analyzing vector spaces' properties. In this article, we delve into the problem of analyzing the subspace of $\mathbb{R}^4$ spanned by two given vectors, $V_1 = (1, -1, 2, 3)$ and $V_2 = (1, 0, 1, 2)$, exploring their linear dependence, basis, dimension, and other relevant properties.

Understanding the Concept of Spanning and Subspaces

What Does It Mean for Vectors to Span a Subspace?

In linear algebra, the span of a set of vectors is the collection of all possible linear combinations of those vectors. Formally, for vectors $V_1, V_2, \dots, V_k$, their span is:

$$
\text{Span}\{V_1, V_2, \dots, V_k\} = \left\{ \alpha_1 V_1 + \alpha_2 V_2 + \dots + \alpha_k V_k \mid \alpha_i \in \mathbb{R} \right\}
$$

This span is always a subspace of $\mathbb{R}^n$, meaning it is closed under vector addition and scalar multiplication.

Why Is Identifying the Spanning Subspace Important?

Determining the subspace spanned by a set of vectors allows us to:

- Understand the dimension of the space they occupy.
- Identify whether vectors are linearly independent or dependent.
- Simplify problems involving vector combinations and transformations.
- Find bases for larger vector spaces.

Analyzing the Given Vectors in $\mathbb{R}^4$

Given Vectors

The vectors provided are:

$$
V_1 = (1, -1, 2, 3)
$$

$$
V_2 = (1, 0, 1, 2)
$$

The task is to analyze the subspace V of $\mathbb{R}^4$ spanned by these vectors, i.e.,

$$
V = \text{Span}\{V_1, V_2\}
$$

Step 1: Check for Linear Independence

To determine whether V_1 and V_2 are linearly independent, we examine if one vector can be written as a scalar multiple or linear combination of the other.

Method: Set up the equation

$$
\alpha V_1 + \beta V_2 = 0
$$

and solve for $\alpha, \beta \in \mathbb{R}$.

Alternatively, compare the vectors directly:

- Is there a scalar k such that $V_1 = k V_2$?

Check:

$$
V_1 = k V_2 \Rightarrow (1, -1, 2, 3) = k (1, 0, 1, 2)
$$

Matching components:

- $1 = k \times 1 \Rightarrow k = 1$
- $-1 = k \times 0 \Rightarrow -1 = 0 \Rightarrow \text{No}$

Since the second component does not match, V_1 and V_2 are not scalar multiples, so they are not linearly dependent in that way.

Next step: Solve for linear dependence using system of equations:

$$
\alpha (1, -1, 2, 3) + \beta (1, 0, 1, 2) = (0,0,0,0)
$$

which expands to:

$$
\begin{cases}
\alpha + \beta = 0 \\
- \alpha + 0 \times \beta = 0 \\
2 \alpha + \beta = 0 \\
3 \alpha + 2 \beta = 0
\end{cases}
$$

Solve step-by-step:

- From the second equation:

$$
- \alpha = 0 \Rightarrow \alpha = 0
$$

- From the first equation:

$$
0 + \beta = 0 \Rightarrow \beta = 0
$$

- Check the third equation:

$$
2 \times 0 + 0 = 0 \quad \text{True}
$$

- Check the fourth equation:

$$
3 \times 0 + 2 \times 0 = 0 \quad \text{True}
$$

Since the only solution is $\alpha = 0, \beta = 0$, the vectors are linearly independent.

Conclusion: The vectors V_1 and V_2 are linearly independent, hence they form a basis for their span.

Basis and Dimension of the Subspace V

Basis for V

Since the vectors are linearly independent, they form a basis for the subspace V.

$$
\boxed{
\text{Basis}(V) = \{V_1, V_2\}
}
$$

Dimension of V

The dimension of a subspace is the number of vectors in its basis.

$$
\dim(V) = 2
$$

This indicates that the subspace spanned by V_1 and V_2 is a 2-dimensional plane within $\mathbb{R}^4$.

Expressing the Subspace Explicitly

Parametric Form of the Subspace

Any vector $v \in V$ can be written as:

$$
v = \alpha V_1 + \beta V_2
$$

which expands to:

$$
v = \alpha (1, -1, 2, 3) + \beta (1, 0, 1, 2)
$$

or component-wise:

$$
v = (\alpha + \beta, - \alpha + 0, 2 \alpha + \beta, 3 \alpha + 2 \beta)
$$

with $\alpha, \beta \in \mathbb{R}$.

Explicit parametric equations:

$$
\begin{cases}
x = \alpha + \beta \\
y = - \alpha \\
z = 2 \alpha + \beta \\
w = 3 \alpha + 2 \beta
\end{cases}
$$

where $\alpha, \beta \in \mathbb{R}$.

Expressing the Subspace in Vector Equation Form

Any vector $v \in V$ can be expressed as:

$$
v = \alpha V_1 + \beta V_2
$$

or as:

$$
v = \alpha (1, -1, 2, 3) + \beta (1, 0, 1, 2)
$$

which can be written as:

$$
v = \alpha (1, -1, 2, 3) + \beta (1, 0, 1, 2)
$$

allowing us to generate all vectors in the subspace through varying α and β.

Finding a Basis of the Subspace in Reduced Form

Using Gaussian Elimination to Confirm Independence

Construct a matrix with V_1 and V_2 as columns:

$$
A = \begin{bmatrix}
1 & 1 \\
-1 & 0 \\
2 & 1 \\
3 & 2
\end{bmatrix}
$$

Apply Gaussian elimination:

- Subtract the first column from the second:

$$
\text{Column 2} - \text{Column 1} \Rightarrow (1-1, 0-(-1), 1-2, 2-3) = (0, 1, -1, -1)
$$

The matrix becomes:

$$
\begin{bmatrix}
1 & 0 \\
-1 & 1 \\
2 & -1 \\
3 & -1
\end{bmatrix}
$$

Since the two columns are linearly independent (no scalar multiple), the vectors form a basis.

Applications of the Analysis in Various Fields

Solving Systems of Linear Equations

Knowing the subspace spanned by vectors helps in:

- Determining whether a particular vector can be expressed as a linear combination of existing vectors.
- Solving for coefficients in least squares approximation.

Data Science and Machine Learning

- Understanding the span of feature vectors aids in feature selection.
- Dimensionality reduction techniques like Principal Component Analysis (PCA) rely

Frequently Asked Questions

How do you find the subspace of R^4 spanned by the vectors V = (1, -1, 2, 3) and V = (1, 0, 1, 2)?

To find the subspace spanned by the vectors, you consider all linear combinations of V = (1, -1, 2, 3) and V = (1, 0, 1, 2). The subspace consists of all vectors of the form a(1, -1, 2, 3) + b(1, 0, 1, 2), where a and b are real numbers.

What is the dimension of the subspace spanned by the vectors V = (1, -1, 2, 3) and V = (1, 0, 1, 2)?

First, check if the vectors are linearly independent. Since they are not scalar multiples of each other, they are linearly independent, so the dimension of their span is 2.

How can we determine if the vectors V = (1, -1, 2, 3) and V = (1, 0, 1, 2) are linearly independent?

Set up the equation a(1, -1, 2, 3) + b(1, 0, 1, 2) = (0, 0, 0, 0). Solving for a and b shows only the trivial solution a = 0 and b = 0, so they are linearly independent.

What is a basis for the subspace spanned by the given vectors?

A basis is simply the set of vectors V = (1, -1, 2, 3) and V = (1, 0, 1, 2), since they are linearly independent and span the subspace.

How do you express an arbitrary vector in the span of V = (1, −1, 2, 3) and V = (1, 0, 1, 2)?

Any vector in the span can be written as a(1, −1, 2, 3) + b(1, 0, 1, 2), where a and b are real numbers. To find a specific a and b for a given vector, solve the resulting system of equations.

What is the geometric interpretation of the subspace spanned by these two vectors in R^4?

In R^4, a subspace spanned by two vectors is a plane (2-dimensional subspace). The given vectors define a 2D plane within the 4-dimensional space.

Additional Resources

In R Consider The Vectors V = (1, −1, 2, 3) And V = (1, 0, 1, 2). Let V Be The Subspace Of R^4 Spanned By V

Introduction

In the realm of linear algebra, the study of subspaces within vector spaces offers profound insights into the structure and behavior of systems ranging from engineering to data science. When considering vectors in $\mathbb{R}^4$, understanding how particular vectors generate subspaces, their intersections, dimensions, and bases is crucial. This article delves into the specifics of the vectors $\mathbf{v}_1 = (1, -1, 2, 3)$ and $\mathbf{v}_2 = (1, 0, 1, 2)$, focusing on the subspace V they span within $\mathbb{R}^4$. Through a detailed exploration, we aim to illuminate fundamental concepts such as linear independence, basis determination, and subspace analysis, providing a comprehensive understanding suitable for students, educators, and researchers alike.

Overview of Vectors and Subspaces in $\mathbb{R}^4$

Definition of a Subspace

A subspace $W \subseteq \mathbb{R}^4$ is a subset that is closed under addition and scalar multiplication, and contains the zero vector. Any subspace can be characterized as the span of a set of vectors—a minimal set of vectors from which every element of the subspace can be generated through linear combinations.

The Vectors $\mathbf{v}_1$ and $\mathbf{v}_2$

Given:
$$
\mathbf{v}_1 = (1, -1, 2, 3), \quad \mathbf{v}_2 = (1, 0, 1, 2)
$$
these vectors serve as the potential generators for the subspace V:

$$
V = \operatorname{span}\{\mathbf{v}_1, \mathbf{v}_2\}
$$

Determining the Subspace V

Linearity and Dependence

The first step in analyzing V involves assessing whether $\mathbf{v}_1$ and $\mathbf{v}_2$ are linearly independent. If they are, then V is two-dimensional; if not, then the span reduces to a one-dimensional subspace.

Checking Linear Dependence

Vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ are linearly dependent if there exists a scalar k such that:

$$
\mathbf{v}_1 = k \mathbf{v}_2
$$

or equivalently:

$$
(1, -1, 2, 3) = k (1, 0, 1, 2)
$$

Comparing components:

$$
1 = k \times 1 \Rightarrow k = 1
$$

$$
-1 = k \times 0 \Rightarrow -1 = 0 \Rightarrow \text{Contradiction}
$$

Since the second component does not satisfy the proportionality, the vectors are linearly independent.

Confirming Independence via a Matrix Approach

Construct the matrix with $\mathbf{v}_1$ and $\mathbf{v}_2$ as columns:

$$
A = \begin{bmatrix}
1 & 1 \\
-1 & 0 \\
2 & 1 \\
3 & 2
\end{bmatrix}
$$

By performing row operations or computing the rank, we verify the independence:
- The rank of A equals 2, confirming the vectors are linearly independent.

Thus, V is a 2-dimensional subspace of $\mathbb{R}^4$.

Basis for the Subspace V

Since $\mathbf{v}_1$ and $\mathbf{v}_2$ are linearly independent, they form a basis:

$$
\boxed{
\text{Basis for } V = \{\mathbf{v}_1, \mathbf{v}_2\}
}
$$

This basis provides the minimal set of vectors needed to generate the entire subspace V.

Geometric Interpretation in $\mathbb{R}^4$

While visualizing four-dimensional space is inherently challenging, understanding the dimension and structure of V offers valuable insights:
- V is a 2-dimensional plane embedded within $\mathbb{R}^4$.
- Any vector in V can be written as:

$$
\mathbf{v} = a \mathbf{v}_1 + b \mathbf{v}_2
$$

for scalars $a, b \in \mathbb{R}$.

Explicit Description of V

Given the basis vectors, the subspace V can be explicitly described as:

$$
V = \left\{ a (1, -1, 2, 3) + b (1, 0, 1, 2) \; | \; a, b \in \mathbb{R} \right\}
$$

which expands to:

$$
V = \left\{ (a + b,\; -a,\; 2a + b,\; 3a + 2b) \; | \; a, b \in \mathbb{R} \right\}
$$

This parametric form facilitates understanding of the subspace's structure and aids in computations involving vectors in V.

Subspace Operations and Intersections

Orthogonal Complement and Projections

In advanced analysis, understanding the orthogonal complement $V^\perp$ within $\mathbb{R}^4$ enhances comprehension of the subspace's relation to the ambient space. Computing $V^\perp$ involves solving:

$$
\mathbf{w} \cdot \mathbf{v}_1 = 0 \quad \text{and} \quad \mathbf{w} \cdot \mathbf{v}_2 = 0
$$

which yields vectors orthogonal to V.

Intersection with Other Subspaces

Suppose another subspace W is given; analyzing $V \cap W$ involves solving systems of linear equations. In this context, the independence and span of V simplify such intersection calculations.

Practical Applications and Significance

The study of subspaces spanned by specific vectors like $\mathbf{v}_1$ and $\mathbf{v}_2$ has numerous applications:

- Data Analysis: Identifying the subspace spanned by data points aids in dimensionality reduction techniques like PCA.
- Signal Processing: Subspaces represent signal components; understanding their span is crucial for filtering and noise reduction.
- Engineering and Physics: Describing feasible states or configurations within constraints often reduces to subspace analysis.

Summary and Conclusions

This thorough exploration confirms that the vectors $\mathbf{v}_1 = (1, -1, 2, 3)$ and $\mathbf{v}_2 = (1, 0, 1, 2)$ are linearly independent and span a 2-dimensional subspace V within $\mathbb{R}^4$. The basis $\{\mathbf{v}_1, \mathbf{v}_2\}$ provides a concrete foundation for understanding the structure of V, enabling further analysis such as projections, orthogonal complements, and applications across scientific disciplines.

By dissecting the properties and relationships of these vectors, the study exemplifies fundamental linear algebra concepts and underscores the importance of basis determination and subspace characterization in higher-dimensional spaces. Such detailed understanding is indispensable for theoretical investigations and practical problem-solving in mathematics, engineering, and beyond.

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