In The Fourier Series Expansion For The Function F(x) = {7 ITT ,-1 < X < 0 (-1,7), The Find Value

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Understanding Fourier series expansions is fundamental in analyzing periodic functions, especially in fields like signal processing, engineering, and applied mathematics. The process involves expressing a periodic function as an infinite sum of sines and cosines, which simplifies the analysis of complex periodic phenomena. In this comprehensive guide, we will explore how to find the Fourier series expansion for the piecewise function:

$$
F(x) =
\begin{cases}
7, & -1 < x < 0 \\
-1, & 0 < x < 7
\end{cases}
$$

and then determine the specific value of the Fourier series at a given point or over a specific interval.

Understanding the Problem and Function Characteristics

Function Definition and Periodicity

The function $F(x)$ is defined piecewise over the interval $-1 < x < 0$ and $0 < x < 7$, with constant values 7 and -1, respectively. To develop a Fourier series, the function must be extended periodically outside this primary interval. Typically, the period T is selected based on the interval over which the function repeats, which in this case can be taken as:

$$
T = 8
$$

since the combined interval from -1 to 7 spans 8 units.

Key points:

- The function is piecewise constant.
- It exhibits a jump discontinuity at $x=0$.
- It is extended periodically with period $T=8$.

Formulating the Fourier Series

The general Fourier series for a periodic function $F(x)$ with period T is:

$$
F(x) = a_0 + \sum_{n=1}^\infty \left[a_n \cos \left(\frac{2\pi n x}{T} \right) + b_n \sin \left(\frac{2\pi n x}{T} \right) \right]
$$

where the Fourier coefficients are calculated as:

$$
a_0 = \frac{2}{T} \int_{x_0}^{x_0 + T} F(x) \, dx
$$

$$
a_n = \frac{2}{T} \int_{x_0}^{x_0 + T} F(x) \cos \left(\frac{2\pi n x}{T} \right) dx
$$

$$
b_n = \frac{2}{T} \int_{x_0}^{x_0 + T} F(x) \sin \left(\frac{2\pi n x}{T} \right) dx
$$

Choosing $x_0 = -1$ makes sense because the function is defined from -1 to 7.

Calculating Fourier Coefficients for the Given Function

Step 1: Compute a_0 - The Average or DC Component

$$
a_0 = \frac{2}{T} \int_{-1}^{7} F(x) dx
$$

Given the piecewise nature:

$$
a_0 = \frac{2}{8} \left(\int_{-1}^0 7 \, dx + \int_0^7 (-1) \, dx \right)
$$

Calculations:

$$
a_0 = \frac{1}{4} \left(7 \times (0 - (-1)) + (-1) \times (7 - 0) \right) = \frac{1}{4} (7 \times 1 - 1 \times 7) = \frac{1}{4} (7 - 7) = 0
$$

Result: $a_0 = 0$

Step 2: Calculate a_n - Cosine Coefficients

$$
a_n = \frac{2}{8} \left(\int_{-1}^0 7 \cos \left(\frac{\pi n x}{4} \right) dx + \int_0^7 (-1) \cos \left(\frac{\pi n x}{4} \right) dx \right)
$$

Simplify:

$$
a_n = \frac{1}{4} \left(7 \int_{-1}^0 \cos \left(\frac{\pi n x}{4} \right) dx - \int_0^7 \cos \left(\frac{\pi n x}{4} \right) dx \right)
$$

Calculating the integrals:

$$
\int \cos(kx) dx = \frac{\sin(kx)}{k}
$$

where $k = \frac{\pi n}{4}$.

Hence:

$$
a_n = \frac{1}{4} \left[7 \left(\frac{\sin \left(\frac{\pi n x}{4} \right)}{\frac{\pi n}{4}} \right)_{x=-1}^{x=0} - \left(\frac{\sin \left(\frac{\pi n x}{4} \right)}{\frac{\pi n}{4}} \right)_{x=0}^{x=7} \right]
$$

Simplify denominators:

\[
a_n = \frac{1}{4} \left[7 \times \frac{4}{\pi n} \left(\sin 0 - \sin \left(-\frac{\pi n}{4}

\right) \right) - \frac{4}{\pi n} \left(\sin \left(\frac{7 \pi n}{4} \right) - 0 \right) \right]
\]

Recall that \(\sin(-x) = - \sin x\):

\[
a_n = \frac{1}{4} \times \frac{4}{\pi n} \left[7 \left(- \sin \left(\frac{\pi n}{4} \right) \right) - \sin \left(\frac{7 \pi n}{4} \right) \right]
\]

Simplify:

\[
a_n = \frac{1}{\pi n} \left[-7 \sin \left(\frac{\pi n}{4} \right) - \sin \left(\frac{7 \pi n}{4} \right) \right]
\]

Step 3: Calculate \(b_n\) - Sine Coefficients

Similarly,

\[
b_n = \frac{2}{T} \left(\int_{-1}^0 7 \sin \left(\frac{\pi n x}{4} \right) dx + \int_0^7 (-1) \sin \left(\frac{\pi n x}{4} \right) dx \right)
\]

which simplifies to:

\[
b_n = \frac{1}{4} \left[7 \int_{-1}^0 \sin \left(\frac{\pi n x}{4} \right) dx - \int_0^7 \sin \left(\frac{\pi n x}{4} \right) dx \right]
\]

Integrate:

\[
\int \sin(kx) dx = - \frac{\cos(kx)}{k}
\]

with \(k = \frac{\pi n}{4}\):

\[
b_n = \frac{1}{4} \left[7 \left(- \frac{\cos \left(\frac{\pi n x}{4} \right)}{\frac{\pi n}{4}} \right)_{x=-1}^{x=0} - \left(- \frac{\cos \left(\frac{\pi n x}{4} \right)}{\frac{\pi n}{4}} \right)_{x=0}^{x=7} \right]
\]

Simplify:

\[
b_n = \frac{1}{4} \times \left[- \frac{4}{\pi n} \left(\cos 0 - \cos \left(- \frac{\pi n}{4} \right) \right) + \frac{4}{\pi n} \left(\cos \left(\frac{7 \pi n}{4} \right) - \cos 0 \right) \right]
\]

Recall \(\cos(-x) = \cos x\):

\[
b_n = \frac{1}{4} \times \frac{4}{\pi n} \left[- \left(1 - \cos \left(\frac{\pi n}{4} \right) \right) + \left(\cos \left(\frac{7 \pi n}{4} \right) - 1 \right) \right]
\]

Simplify numerator:

\[
b_n = \frac{1}{\pi n} \left[- (1 - \cos \left(\frac{\

Frequently Asked Questions

What is the general Fourier series expansion for the function F(x) = 7 on the interval -1 < x < 0?

The function F(x) = 7, a constant function on the interval, has a Fourier series consisting of only the constant term (a_0/2), with all sine and cosine coefficients equal to zero. The Fourier series simplifies to F(x) = 7.

How do you compute the Fourier coefficients for F(x) = 7 on the interval -1 < x < 0?

Since F(x) = 7 is constant, the Fourier coefficients are straightforward: a_0 = (2/L) ∫_{-1}^{0} 7 dx, and a_n = 0, b_n = 0 for all n, because the sine and cosine integrals over a constant function cancel out or are zero for sine terms.

What is the value of the Fourier series at a point within the interval for F(x) = 7?

Within the interval -1 < x < 0, the Fourier series converges to the function value, which is 7, since the function is constant and the Fourier series accurately represents it.

How does the periodic extension affect the Fourier series of F(x) = 7?

The periodic extension of a constant function results in a square wave with jumps at the interval boundaries, but the Fourier series remains dominated by the constant term, converging to 7 within each period.

Are there any discontinuities or Gibbs phenomena associated with the Fourier series of F(x) = 7?

No, since F(x) = 7 is a constant and continuous function, its Fourier series converges uniformly to the function without Gibbs phenomena or discontinuities.

How can the Fourier series be used to find the value of F(x) at specific points?

For the constant function F(x) = 7, the Fourier series directly gives the value at any point within the interval, which is 7. For more complex functions, the series can be used to approximate the value via partial sums.

Additional Resources

Fourier Series Expansion for the Function F(x): An In-Depth Analytical Review

In the realm of mathematical analysis, Fourier series serve as a powerful tool for decomposing periodic functions into sums of sines and cosines. This technique not only simplifies complex functions but also provides insights into their frequency components, making it indispensable in fields ranging from signal processing to quantum physics. In this comprehensive review, we focus on the Fourier series expansion of a specific piecewise function, denoted as F(x), defined over a particular interval. We aim to elucidate the process of deriving its Fourier series, interpret the significance of each component, and explore the implications of its behavior over the specified domain.

Understanding the Function F(x)

Definition of the Function

At the heart of our analysis lies the function:

$$F(x) = \begin{cases}
7, & -1 < x < 0 \\
-1, & 0 < x < 7
\end{cases}$$

This is a piecewise constant function defined over the interval $-1 < x < 7$. It exhibits a jump discontinuity at $x=0$, with a constant value of 7 on $-1 < x < 0$ and -1 on $0 < x < 7$. The problem involves finding the Fourier series expansion of this function, which requires understanding its periodic extension.

Key Considerations:

- The function appears to be defined on a finite interval, but Fourier series inherently require periodicity. Therefore, we first need to determine the period T over which F(x) repeats.
- Typically, for Fourier series, the function is extended periodically beyond the initial interval, often assuming the function repeats every T.

Periodicity and Domain Setup

To proceed with the Fourier series, we define the period T. Although the problem statement mentions the interval $-1 < x < 0$ and $0 < x < 7$, the total length of the domain is 8 units. A natural choice is to consider the fundamental period:

$$T = 8$$

This makes the function periodic with period 8, repeating the pattern of the given piecewise function every 8 units.

Visualizing the Function:

- On $[-1, 0)$, $F(x) = 7$.
- On $[0, 7)$, $F(x) = -1$.
- The function repeats every 8 units, so:

$$F(x + 8) = F(x)$$

This periodic extension allows us to analyze the Fourier series over one period, say $[0, T)$, and then express the entire function as a sum of harmonics.

Mathematical Foundations of Fourier Series

Fourier Series Representation

Any reasonably well-behaved periodic function $F(x)$ with period T can be expressed as a Fourier series:

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2\pi n x}{T} + b_n \sin \frac{2\pi n x}{T} \right)$$

where:

- a_0 is the average (DC component) of the function over one period.
- a_n and b_n are Fourier coefficients representing the amplitudes of cosine and sine components, respectively.

The coefficients are computed as:

$$
a_n = \frac{2}{T} \int_{x_0}^{x_0 + T} F(x) \cos \frac{2\pi n x}{T} \, dx
$$

$$
b_n = \frac{2}{T} \int_{x_0}^{x_0 + T} F(x) \sin \frac{2\pi n x}{T} \, dx
$$

where x_0 is an arbitrary starting point of the interval, often chosen as 0 for simplicity.

Function Symmetry and Simplification

The nature of $F(x)$ influences the Fourier series:

- Even functions (symmetric about the y-axis) contain only cosine terms (a_n), with $b_n=0$.
- Odd functions (antisymmetric about the origin) contain only sine terms (b_n), with $a_n=0$.
- Neither: Both sine and cosine terms are present.

Given the function's jump at $x=0$, it is neither purely even nor odd. Therefore, both sine and cosine terms will contribute.

Calculating Fourier Coefficients for F(x)

Setting Up the Integrals

Given the period $T=8$, and the piecewise nature of $F(x)$, the Fourier coefficients are:

$$
a_n = \frac{2}{8} \left[\int_{-1}^{0} 7 \cos \frac{\pi n x}{4} \, dx + \int_{0}^{7} (-1) \cos \frac{\pi n x}{4} \, dx \right]
$$

$$
b_n = \frac{2}{8} \left[\int_{-1}^{0} 7 \sin \frac{\pi n x}{4} \, dx + \int_{0}^{7} (-1) \sin \frac{\pi n x}{4} \, dx \right]
$$

\]

Simplifying the constants:

\[
a_n = \frac{1}{4} \left[7 \int_{-1}^{0} \cos \frac{\pi n x}{4} \, dx - \int_{0}^{7} \cos \frac{\pi n x}{4} \, dx \right]
\]

\[
b_n = \frac{1}{4} \left[7 \int_{-1}^{0} \sin \frac{\pi n x}{4} \, dx - \int_{0}^{7} \sin \frac{\pi n x}{4} \, dx \right]
\]

Evaluating the Integrals

The integrals involve standard trigonometric functions:

1. Cosine integrals:

\[
\int \cos kx \, dx = \frac{\sin kx}{k}
\]

2. Sine integrals:

\[
\int \sin kx \, dx = -\frac{\cos kx}{k}
\]

where \(k = \frac{\pi n}{4}\).

Calculating \(a_n\):

\[
a_n = \frac{1}{4} \left[7 \left(\frac{\sin \frac{\pi n x}{4}}{\frac{\pi n}{4}} \bigg|_{x=-1}^{x=0} \right) - \left(\frac{\sin \frac{\pi n x}{4}}{\frac{\pi n}{4}} \bigg|_{x=0}^{x=7} \right) \right]
\]

which simplifies to:

\[
a_n = \frac{1}{4} \left[7 \left(\frac{\sin 0 - \sin \left(- \frac{\pi n}{4}\right)}{\frac{\pi n}{4}} \right) - \left(\frac{\sin \left(\frac{7 \pi n}{4}\right) - \sin 0}{\frac{\pi n}{4}} \right) \right]
\]

Since \(\sin 0 = 0\), and \(\sin \left(- \frac{\pi n}{4}\right) = - \sin \frac{\pi n}{4}\), this becomes:

\[
a_n = \frac{1}{4} \left[7 \left(\frac{0 + \sin \frac{\pi n}{4}}{\frac{\pi n}{4}} \right) - \left(\frac{\sin \frac{7 \pi n}{4} - 0}{\frac{\pi n}{4}} \right) \right]
\]

\[
a_n = \frac{1}{4} \left[7 \frac{\sin \frac{\pi n}{4}}{\frac{\pi n}{4}} - \frac{\sin \frac{7 \pi n}{4}}{\frac{\pi n}{4}} \right]
\]

Similarly, \(\sin \frac{7 \pi n}{4} = - \sin \frac{\pi n}{4}\) because \(\sin (2\pi - x) = - \sin x\), and \(\frac{7 \pi n}{4} = 2\pi - \frac{\pi n}{4}\).

Thus,

\[
a_n = \frac{1}{4} \left[\frac{7 \sin \frac{\pi n}{4} - (- \sin \frac{\pi n}{4})}{\frac{\pi n}{4}} \right] = \frac{1}{4} \left[\frac{8 \sin \frac{\pi n}{4}}{\frac{\pi n}{4}} \

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