Ind The Maximum And Minimum Values Of The Function ' G(Theta)=7 Theta 9sin(Theta) " On The Interval

Ind The Maximum And Minimum Values Of The Function ' G(Theta)=7 Theta 9sin(Theta) " On The Interval

---

Introduction

Understanding the maximum and minimum values of a function is fundamental in calculus, as these values help in analyzing the behavior of the function over a specified interval. In this article, we delve into the function $G(\theta) = 7\theta + 9\sin(\theta)$, exploring its critical points, the process to find its maximum and minimum values, and interpreting these results within a given interval. This comprehensive analysis will include derivative calculations, critical point identification, and the evaluation of the function at those points to determine the extrema.

Understanding the Function $G(\theta)$

Function Components

The function $G(\theta) = 7\theta + 9\sin(\theta)$ comprises two parts:

- **Linear term:** 7θ, which increases or decreases without bound depending on the sign of θ.
- **Sinusoidal term:** $9\sin(\theta)$, which oscillates between -9 and 9 with a period of 2π.

Understanding how these components interact over the interval is essential for analyzing the function's extrema.

Interval Specification

The problem statement mentions "on the interval" but does not specify the interval. Typically, to analyze maximum and minimum values, an interval must be defined. For this discussion, we will examine the function over a general interval, for example:
- Interval A: $[a, b]$, where a and b are specific real numbers.
- Common case: $[0, 2\pi]$, which covers one full period of the sine function.

If a specific interval is given, such as $[\theta_1, \theta_2]$, the same methodology applies, with the bounds adjusted accordingly.

Calculating the Derivative $G'(\theta)$

Purpose of the Derivative

The first derivative $G'(\theta)$ helps identify critical points where the function's slope is zero, which are potential maxima or minima.

Derivative Calculation

Using basic differentiation rules:
- The derivative of 7θ with respect to θ is 7.
- The derivative of $9\sin(\theta)$ with respect to θ is $9\cos(\theta)$.

Therefore:

$$
G'(\theta) = 7 + 9\cos(\theta)
$$

Finding Critical Points

Setting the Derivative Equal to Zero

Critical points occur where $G'(\theta) = 0$:

$$
7 + 9\cos(\theta) = 0
$$

$$
9\cos(\theta) = -7
$$

$$
\cos(\theta) = -\frac{7}{9}
$$

Since $-1 \leq \cos(\theta) \leq 1$, and $-\frac{7}{9} \approx -0.777...$, this value is valid within the cosine function's range.

Solutions for θ

The solutions are:

$$
\theta = \arccos\left(-\frac{7}{9}\right) \quad \text{and} \quad \theta = 2\pi - \arccos\left(-\frac{7}{9}\right)
$$

Let $\alpha = \arccos\left(-\frac{7}{9}\right)$. Then, the critical points within the interval are:

$$
\theta = \alpha \quad \text{and} \quad \theta = 2\pi - \alpha
$$

If the interval extends beyond 0 to 2π, additional critical points may be considered by adding integer multiples of 2π as needed.

Evaluating the Function at Critical Points and Endpoints

Why Evaluate at Critical Points and Endpoints?

According to the Extreme Value Theorem, continuous functions on closed intervals attain their maximum and minimum values at critical points or at the endpoints of the interval. Therefore, to find the global extrema, evaluate $G(\theta)$ at:

- Critical points within the interval.
- The endpoints of the interval.

Calculating $G(\theta)$ at Critical Points

Using the critical points $\theta = \alpha$ and $\theta = 2\pi - \alpha$:

$$
G(\alpha) = 7\alpha + 9\sin(\alpha)
$$

$$
G(2\pi - \alpha) = 7(2\pi - \alpha) + 9\sin(2\pi - \alpha)
$$

Recall that:

$$
\sin(2\pi - \alpha) = -\sin(\alpha)
$$

Hence:

$$
G(2\pi - \alpha) = 14\pi - 7\alpha - 9\sin(\alpha)
$$

Similarly, evaluate $G(\theta)$ at the endpoints of the interval, say, $\theta = a$ and $\theta = b$.

Determining the Maximum and Minimum Values

Comparison of Values

Once the values of $G(\theta)$ at critical points and endpoints are computed, compare these to identify:

- The maximum value: the largest among these.
- The minimum value: the smallest among these.

Summary of the Process

1. Calculate $G(\theta)$ at critical points:
- $\theta = \alpha$
- $\theta = 2\pi - \alpha$
2. Calculate $G(\theta)$ at the interval endpoints.
3. Compare all these values to find the global maximum and minimum.

Example: Specific Interval $[0, 2\pi]$

Assuming the interval $[0, 2\pi]$:
- Critical points are within the interval.
- Endpoints are at 0 and 2π.

Calculate:
- $\alpha = \arccos(-7/9)$, which is approximately 2.449 radians.
- $G(\alpha)$:

$$
G(\alpha) = 7 \times 2.449 + 9 \times \sin(2.449)
$$

$$
\approx 17.143 + 9 \times 0.637 = 17.143 + 5.733 = 22.876
$$

- $G(2\pi - \alpha)$:

$$
14\pi - 7 \times 2.449 - 9 \times (-0.637) \approx 43.982 - 17.143 + 5.733 = 31.572
$$

- At $\theta = 0$:

$$
G(0) = 0 + 0 = 0
$$

- At $\theta = 2\pi$:

$$
G(2\pi) = 7 \times 2\pi + 9 \times 0 = 14\pi \approx 43.982
$$

Results:
- Maximum value: approximately 43.982 at $\theta = 2\pi$.
- Minimum value: approximately 0 at $\theta = 0$.

Thus, on $[0, 2\pi]$, the maximum of $G(\theta)$ is near 43.982, and the minimum is 0.

Conclusion

The maximum and minimum values of the function $G(\theta) = 7\theta + 9\sin(\theta)$ depend on the interval considered. The key steps involve:
- Differentiating to find critical points.
- Solving for θ where the derivative is zero.
- Evaluating the function at critical points and endpoints.
- Comparing these values to determine the extrema.

This process demonstrates the systematic approach to analyzing the extremal values of a combined linear and sinusoidal function. For specific intervals, perform the calculations accordingly, and the same methodology applies. Understanding these principles is vital in calculus, particularly in optimization problems across various applications such as physics, engineering, and economics.

Note: For other intervals, such as $[a, b]$, simply adjust the critical points and evaluate $G(\theta)$ at those points and at the respective endpoints to find the extrema within that range.

Frequently Asked Questions

What is the main goal when finding the maximum and minimum values of the function $G(\theta) = 7\theta + 9\sin(\theta)$ on an interval?

The main goal is to determine the highest and lowest values that $G(\theta)$ attains within the given interval by analyzing its critical points and endpoints.

How do you find the critical points of $G(\theta) = 7\theta + 9\sin(\theta)$?

Critical points are found by taking the derivative $G'(\theta) = 7 + 9\cos(\theta)$, setting it equal to zero, and solving for θ: $7 + 9\cos(\theta) = 0$.

What are the critical points of $G(\theta) = 7\theta + 9\sin(\theta)$ within the interval?

Critical points occur when $\cos(\theta) = -7/9$, which gives $\theta = \arccos(-7/9)$. Verify if these points lie within the given interval to consider them in the maximum and minimum calculations.

How do the endpoints of the interval affect the maximum and minimum values of $G(\theta)$?

Since the function is continuous on a closed interval, the maximum and minimum values occur either at critical points within the interval or at the endpoints. Therefore, evaluating $G(\theta)$ at these points is essential.

Is $G(\theta) = 7\theta + 9\sin(\theta)$ a periodic function?

No, while $\sin(\theta)$ is periodic, the entire function $G(\theta)$ is not periodic because of the linear term 7θ, which causes $G(\theta)$ to increase or decrease without repeating pattern.

How can I determine whether the critical point corresponds to a maximum or minimum?

Use the second derivative test: compute $G''(\theta) = -9\sin(\theta)$. Evaluate $G''(\theta)$ at the critical points; if $G''(\theta) > 0$, it's a local minimum, if < 0, it's a local maximum.

What is the significance of evaluating $G(\theta)$ at critical points and endpoints?

Evaluating at these points ensures you identify all potential candidates for the absolute maximum and minimum values within the interval, as these points are where the function's slope changes or boundary conditions apply.

Can the maximum and minimum values of $G(\theta)$ be found analytically for any interval?

Yes, for specific intervals, especially those that are finite and well-defined, the maximum and minimum can be found analytically by solving for critical points and evaluating the function at these points and the interval endpoints.

Additional Resources

Ind the Maximum and Minimum Values of the Function G(θ) = 7θ + 9sin(θ) on the Interval

Understanding the maximum and minimum values of functions is a fundamental aspect of calculus, vital for applications across physics, engineering, economics, and beyond. In this article, we focus on analyzing the function G(θ) = 7θ + 9sin(θ) within a specified interval, aiming to find its critical points and determine the absolute maximum and minimum values. Such an analysis not only deepens our grasp of the behavior of this particular function but also exemplifies the general techniques used in optimization problems.

Introduction to the Function G(θ) = 7θ + 9sin(θ)

The function G(θ) = 7θ + 9sin(θ) is a combination of a linear term (7θ) and a sinusoidal component (9sin(θ)). This mixture results in a function that exhibits both linear growth and oscillatory behavior.

Features of the Function:

- Linear component (7θ): Causes the function to increase or decrease steadily depending on the interval's bounds.
- Sinusoidal component (9sin(θ)): Introduces periodic oscillations with amplitude 9, causing the function to fluctuate around the linear trend.
- Overall behavior: The function's shape depends on the interval chosen for θ, which influences the location and number of extrema.

Understanding these features helps in predicting the function's behavior, especially when determining its maximum and minimum values on a given interval.

Calculating the Derivative of G(θ)

To find the critical points where the function attains local maxima or minima, we first compute its derivative:

$$ G'(\theta) = \frac{d}{d\theta} (7\theta + 9\sin(\theta)) $$

Applying basic differentiation rules:

$$ G'(\theta) = 7 + 9\cos(\theta) $$

This derivative gives the rate of change of G(θ) at any point θ.

Interpretation:

- When $G'(\theta) = 0$, the slope of the function is zero, indicating potential extrema.
- The derivative's sign indicates whether the function is increasing or decreasing at a given point.

Finding Critical Points

Set the derivative equal to zero to find critical points:

$$ 7 + 9\cos(\theta) = 0 $$

Solving for $\cos(\theta)$:

$$ 9\cos(\theta) = -7 $$
$$ \cos(\theta) = -\frac{7}{9} $$

Since $-1 \leq \cos(\theta) \leq 1$, the value $-\frac{7}{9}$ is valid within the cosine range, so solutions exist.

Solutions for θ:

$$ \theta = \arccos\left(-\frac{7}{9}\right) \quad \text{and} \quad \theta = 2\pi - \arccos\left(-\frac{7}{9}\right) $$

Let's denote:

$$ a = \arccos\left(-\frac{7}{9}\right) $$

Numerically,

$$ a \approx \arccos(-0.7778) \approx 2.467 \text{ radians} $$

Thus, the critical points are approximately:

$$ \theta_1 \approx 2.467 $$
$$ \theta_2 \approx 2\pi - 2.467 \approx 6.283 - 2.467 \approx 3.816 $$

Determining the Nature of Critical Points

To classify whether these critical points are maxima, minima, or saddle points, analyze the second derivative:

$$ G''(\theta) = \frac{d}{d\theta} G'(\theta) = -9\sin(\theta) $$

Evaluate $G''(\theta)$ at each critical point:

- At $\theta_1 = a$:

$$ G''(a) = -9\sin(a) $$

Calculate $\sin(a)$:

$$ \sin(a) = \sqrt{1 - \cos^2(a)} = \sqrt{1 - \left(-\frac{7}{9}\right)^2} = \sqrt{1 - \frac{49}{81}} = \sqrt{\frac{32}{81}} = \frac{\sqrt{32}}{9} \approx \frac{5.657}{9} \approx 0.629 $$

Since $\sin(a) > 0$,

$$ G''(a) = -9 \times 0.629 \approx -5.661 $$

Negative second derivative indicates θ_1 is a local maximum.

- At $\theta_2 = 3.816$:

Calculate $\sin(\theta_2)$:

$$\sin(\theta_2) = \sin(2\pi - a) = -\sin(a) \approx -0.629$$

Thus,

$$G''(\theta_2) = -9 \times (-0.629) \approx +5.661$$

Positive second derivative indicates θ_2 is a local minimum.

Evaluating the Function at Critical and Boundary Points

To find the maximum and minimum values over a specific interval, say $[a, b]$, evaluate $G(\theta)$ at critical points within the interval as well as at the endpoints.

Suppose the interval is $[0, 2\pi]$:

- At $\theta=0$:

$$G(0) = 7 \times 0 + 9 \times \sin(0) = 0$$

- At $\theta=2\pi$:

$$G(2\pi) = 7 \times 2\pi + 9 \times \sin(2\pi) = 14\pi + 0 \approx 14 \times 3.1416 \approx 43.982$$

- At $\theta=a \approx 2.467$:

$$G(a) = 7 \times 2.467 + 9 \times \sin(2.467)$$

Calculate:

$$7 \times 2.467 \approx 17.269$$

$$\sin(2.467) \approx 0.629$$

$$9 \times 0.629 \approx 5.661$$

$$G(a) \approx 17.269 + 5.661 \approx 22.930$$

- At $\theta=3.816$:

$$G(3.816) = 7 \times 3.816 + 9 \times \sin(3.816)$$

Calculate:

$$7 \times 3.816 \approx 26.712$$

$$\sin(3.816) = -0.629$$

$$9 \times -0.629 \approx -5.661$$

$$G(3.816) \approx 26.712 - 5.661 \approx 21.051$$

Summary of evaluated values:

Point	$G(\theta)$ Value	Nature
0	0	Boundary point
$a \approx 2.467$	22.930	Local maximum
3.816	21.051	Local minimum
$2\pi \approx 6.283$	43.982	Boundary point, likely maximum

Identifying the Absolute Maximum and Minimum

From the evaluations:

- Absolute maximum occurs at $\theta=2\pi$: $G \approx 43.982$
- Absolute minimum at $\theta=0$: $G = 0$

Within the interval $[0, 2\pi]$:

- The maximum value is approximately 43.982, attained at $\theta=2\pi$.
- The minimum value is 0, attained at $\theta=0$.

If the interval changes, these evaluations need to be adjusted accordingly, considering all critical points within the interval.

Graphical Interpretation and Visualization

Visualizing $G(\theta) = 7\theta + 9\sin(\theta)$ over $[0, 2\pi]$ provides an intuitive understanding:

- The linear term causes the overall upward trend.
- The sinusoidal term introduces oscillations, creating local maxima and minima.
- The critical points align with the peaks and troughs of the oscillation superimposed on the linear trend.

A graph would show a steadily increasing function with periodic fluctuations, reaching its highest point near $\theta=2\pi$ and lowest at $\theta=0$.

Extensions and Practical Applications

The techniques demonstrated extend beyond this specific function:

- Optimization in real-world scenarios: such as maximizing profit, minimizing cost, or optimizing resource allocation.
- Engineering applications: analyzing signals, vibrations, or waveforms.
- Physics: studying oscillatory motion with a linear drift.

Understanding how to find maximum and minimum values helps in making informed decisions based on the behavior of functions.

Pros and Cons of the Analysis Approach

Pros:

- Systematic method involving derivatives ensures

Ind The Maximum And Minimum Values Of The Function G Theta 7 Theta 9sin Theta On The Interval

Ind The Maximum And Minimum Values Of The Function G Theta 7 Theta 9sin Theta On The Interval

Related Articles

- In The Early 1800s, Americans Did Not Think Cattle Ranches On The Great Plains Were Practical Becausea.
- Radium-226 (atomic Mass 226.03 Amu) Decays To Radon-224, A Radioactive Gas. The Half-life Of Radium-226
- Process A Has Fixed Costs Of $1000 And Variable Costs Of $5 Per Unit. Process B Has Fixed Costs Of $500

Back to Home