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Is (x + 3) A Factor Of F(x) = 4x^3 + 11x^2 + 75x + 18? Yes; The Remainder Is O So (x + 3) Is A Factor No; The

Understanding whether a polynomial has a specific binomial as a factor is an important concept in algebra, particularly in polynomial factorization and solving equations. The question at hand is whether (x + 3) is a factor of the polynomial $F(x) = 4x^3 + 11x^2 + 75x + 18$. To answer this, we need to explore the Polynomial Remainder Theorem, perform synthetic division or polynomial division, and interpret the results carefully.

What Does It Mean for a Binomial to Be a Factor of a Polynomial?

Definition of Factors in Polynomials

A polynomial $P(x)$ has a factor $(x - a)$ if and only if $P(a) = 0$. This is a direct consequence of the Polynomial Remainder Theorem, which states:

- If $P(a) = 0$, then $(x - a)$ is a factor of $P(x)$.
- Conversely, if $(x - a)$ is a factor of $P(x)$, then $P(a) = 0$.

In our case, the binomial $(x + 3)$ can be rewritten as $(x - (-3))$. Therefore, to determine if $(x + 3)$ is a factor, we evaluate $F(-3)$.

Polynomial Remainder Theorem

This theorem simplifies the process:

- To check if $(x - a)$ divides $P(x)$, evaluate $P(a)$.
- If $P(a) = 0$, then $(x - a)$ is a factor.
- If $P(a) \neq 0$, then $(x - a)$ is not a factor.

Applying this to our problem involves calculating $F(-3)$.

Evaluating F(x) at x = -3

Step-by-Step Calculation

Given:

$$
F(x) = 4x^3 + 11x^2 + 75x + 18
$$

Calculate $F(-3)$:

1. Compute $4(-3)^3$:

$$
4 \times (-3)^3 = 4 \times (-27) = -108

\]

2. Compute \(11(-3)^2 \):

\[

11 \times 9 = 99

\]

3. Compute \(75 \times (-3) \):

\[

75 \times (-3) = -225

\]

4. Sum all parts with the constant term:

\[

-108 + 99 - 225 + 18

\]

Final Calculation

Adding these up:

- \(-108 + 99 = -9 \)

- \(-9 - 225 = -234 \)

- \(-234 + 18 = -216 \)

Thus, \(F(-3) = -216 \).

Interpreting the Result

Since \(F(-3) = -216 \neq 0 \), the Polynomial Remainder Theorem indicates that:

- (x + 3) is not a factor of \(F(x) \).

Because the value of the polynomial at \(x = -3 \) is not zero, the binomial \((x + 3) \) does not divide the polynomial evenly, and the remainder is \(-216\).

Alternative Methods to Confirm the Result

While evaluating \(F(-3) \) provides a quick answer, exploring additional methods can reinforce the conclusion.

Polynomial Division

Performing polynomial division of \(F(x) \) by \((x + 3) \):

- Use synthetic division with \(-3 \):

-3	4	11	75	18
	-12	3	-234	693
4	(11 - 12) = -1	(75 + 3) = 78	(18 - 234) = -216	

Remainder: \(-216\).

Since the remainder is not zero, this confirms that \((x + 3) \) is not a factor.

Factoring the Polynomial

Attempting to factor \(F(x) \) directly:

- Check for rational roots using Rational Root Theorem.
- Possible roots: factors of 18 over factors of 4:

$$
\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}, \pm \frac{18}{2} = \pm 9
$$

Test $x = -3$:

- Already found $F(-3) \neq 0$, so $x = -3$ is not a rational root, reinforcing that $(x + 3)$ is not a factor.

Summary and Conclusion

- To determine if $(x + 3)$ is a factor of $F(x) = 4x^3 + 11x^2 + 75x + 18$, evaluate $F(-3)$.
- The calculation yields $F(-3) = -216$, which is not zero.
- Therefore, $(x + 3)$ is not a factor of $F(x)$.

Why Does This Matter?

Understanding whether a binomial is a factor of a polynomial is crucial for:

- Polynomial factorization.

- Finding roots of equations.
- Simplifying polynomial expressions.
- Solving polynomial equations efficiently.

Using the Polynomial Remainder Theorem simplifies the process, saving time and effort compared to long division.

Additional Tips for Polynomial Factorization

- Always start by checking rational roots using the Rational Root Theorem.
- Use synthetic division for quick testing of potential roots.
- Remember that if $P(a) = 0$, then $(x - a)$ is a factor.
- For higher-degree polynomials, consider factoring by grouping or substitution methods.

Final Thoughts

In conclusion, based on the evaluation of $F(-3)$, the polynomial $4x^3 + 11x^2 + 75x + 18$ does not have $(x + 3)$ as a factor. This example highlights the importance of understanding key algebraic concepts such as the Polynomial Remainder Theorem and synthetic division in analyzing polynomial functions. Always verify potential factors through substitution or division to ensure accurate results.

Meta Description:

Learn how to determine if \((x + 3)\) is a factor of the polynomial \(4x^3 + 11x^2 + 75x + 18\). Step-by-step explanation with the Polynomial Remainder Theorem, synthetic division, and detailed insights.

Frequently Asked Questions

How can we determine if (x + 3) is a factor of the polynomial F(x) = 4x^3 + 11x^2 + 75x + 18?

You can substitute x = -3 into F(x) and check if the result is zero. If it is, then (x + 3) is a factor because the Remainder Theorem states that a polynomial divided by (x - a) has a remainder equal to F(a).

What is the Remainder Theorem and how does it apply to testing factors?

The Remainder Theorem states that when a polynomial F(x) is divided by (x - a), the remainder is F(a). If F(a) equals zero, then (x - a) is a factor of the polynomial. In this case, testing x = -3 helps determine if (x + 3) is a factor.

What steps do you follow to verify if (x + 3) is a factor of the given polynomial?

First, substitute x = -3 into F(x). If the result is zero, then (x + 3) is a factor. If not, then it is not a factor. Calculating F(-3) will confirm this.

What is the value of F(-3) for the polynomial 4x^3 + 11x^2 + 75x +

18?

$F(-3) = 4(-3)^3 + 11(-3)^2 + 75(-3) + 18 = -108 + 99 - 225 + 18 = -216$. Since the result is not zero, (x + 3) is not a factor.

Based on the calculation, is (x + 3) a factor of the polynomial F(x)?

No; because $F(-3) \neq 0$ (it equals -216), (x + 3) is not a factor of the polynomial F(x).

Additional Resources

Is (x + 3) A Factor Of $F(x) = 4x^3 + 11x^2 - 75x + 18$? Yes; The Remainder Is 0 So (x + 3) Is A FactorNo; The

Determining whether a polynomial like $F(x) = 4x^3 + 11x^2 - 75x + 18$ is divisible by a binomial such as (x + 3) is a fundamental skill in algebra. This process involves applying the Polynomial Remainder Theorem and synthetic division to verify if (x + 3) is a factor of the polynomial. The question of whether (x + 3) is a factor hinges on whether the polynomial evaluates to zero when x = -3, given that (x + 3) is zero at x = -3. This article explores this concept in detail, providing a comprehensive analysis of methods to determine factors, the implications of those factors, and the pros and cons of the approaches involved.

Understanding Polynomial Factors and the Remainder Theorem

What Is a Polynomial Factor?

A polynomial factor is a polynomial that divides another polynomial exactly, leaving no remainder. In algebra, if a polynomial $F(x)$ can be expressed as a product of two polynomials, say $G(x)$ and $H(x)$, then $G(x)$ is considered a factor of $F(x)$. For a binomial like $(x + 3)$, being a factor of $F(x)$ means that $F(x) = (x + 3)\,Q(x)$, where $Q(x)$ is some quadratic polynomial in this context.

The Polynomial Remainder Theorem

The Polynomial Remainder Theorem offers a quick way to determine whether a binomial $(x - a)$ is a factor of a polynomial $F(x)$. It states that when you divide $F(x)$ by $(x - a)$, the remainder of this division is equal to $F(a)$. If $F(a) = 0$, then $(x - a)$ is a factor of $F(x)$. Conversely, if $F(a) \neq 0$, then $(x - a)$ is not a factor.

Applying this to our problem, since the binomial in question is $(x + 3)$, which can be rewritten as $(x - (-3))$, the theorem suggests evaluating $F(-3)$. If $F(-3) = 0$, then $(x + 3)$ is a factor of $F(x)$.

Applying the Polynomial Remainder Theorem to $F(x) = 4x^3 + 11x^2 - 75x + 18$

Calculating F(-3)

To determine whether $(x + 3)$ is a factor of the polynomial $F(x)$, substitute $x = -3$ into $F(x)$:

$F(-3) = 4(-3)^3 + 11(-3)^2 - 75(-3) + 18$

Calculating step-by-step:

- $(-3)^3 = -27$
- $4(-27) = -108$

- $(-3)^2 = 9$

- 11 9 = 99

- -75 (-3) = +225

- Now, sum all:

F(-3) = -108 + 99 + 225 + 18

Adding these:

- -108 + 99 = -9

- -9 + 225 = 216

- 216 + 18 = 234

Since F(-3) = 234, which is not zero, the Polynomial Remainder Theorem indicates that (x + 3) is not a factor of F(x).

Implication:
Because F(-3) ≠ 0, (x + 3) does not divide the polynomial evenly. Therefore, the initial assumption that (x + 3) could be a factor is invalid based on this calculation.

Verifying the Result Through Synthetic Division

While the Remainder Theorem provides a quick check, synthetic division offers a step-by-step approach to confirm whether (x + 3) is a factor.

Steps in Synthetic Division:

- Set up the coefficients of F(x): 4 | 11 | -75 | 18
- Use the root x = -3 for synthetic division.
- Proceed with the synthetic division process:

| -3 | 4 | 11 | -75 | 18 |

| | -12 | 3 | :: | :: |

| | 4 | -1 | -72 | :: |

| | | | | |

Calculations:

- Bring down 4
- Multiply 4 -3 = -12; add to 11: 11 + (-12) = -1
- Multiply -1 -3 = 3; add to -75: -75 + 3 = -72
- Multiply -72 -3 = 216; add to 18: 18 + 216 = 234

The final remainder is 234, confirming the earlier calculation that the remainder is not zero. Thus, synthetic division confirms that (x + 3) is not a factor of F(x).

Implications of the Results

What Does It Mean If (x + 3) Is Not a Factor?

Since F(-3) □ 0, the binomial (x + 3) does not divide F(x) evenly. This means the polynomial cannot be factored to include (x + 3) as a factor, and any attempt to factor F(x) must involve other factors.

Alternative Factoring Strategies

When a binomial isn't a factor, other methods can be employed:

- Factoring by grouping: Useful if the polynomial can be grouped into factors.
- Finding rational roots: Using Rational Root Theorem to identify possible roots.
- Quadratic factorization: Once a root is identified, polynomial division can reduce the degree, simplifying the factorization.

In this case, since $(x + 3)$ isn't a factor, further analysis would involve testing other potential roots such as 1, -1, 2, -2, etc., based on factors of the constant term and leading coefficient.

Conclusion: Is (x + 3) a Factor of F(x)?

The comprehensive evaluation using the Polynomial Remainder Theorem and synthetic division confirms that $(x + 3)$ is not a factor of $F(x) = 4x^3 + 11x^2 - 75x + 18$ since $F(-3) \neq 0$ and the remainder is non-zero. This example underscores the importance of these methods in polynomial algebra for quickly assessing factors.

Pros and Cons of Methods Used

Polynomial Remainder Theorem

- Pros:
- Quick and straightforward for initial checks.
- Requires only evaluation of the polynomial at a specific point.

- Cons:

- Doesn't provide the actual factorization if the remainder isn't zero.

- Limited to determining divisibility, not the full factorization.

Synthetic Division

- Pros:

- Offers a detailed process to find factors when the remainder is zero.

- Useful for polynomial division and factorization.

- Cons:

- Slightly more involved than direct evaluation.

- Can be cumbersome for higher-degree polynomials without computational tools.

Overall, these methods are essential tools in algebra for factor analysis and polynomial division, providing both efficiency and accuracy when determining factors.

Final Thoughts

Understanding whether a binomial like (x + 3) is a factor of a polynomial involves mastering the Polynomial Remainder Theorem and synthetic division. In our example, F(x) does not satisfy the necessary condition for (x + 3) being a factor. Recognizing these methods' strengths and limitations equips students and mathematicians with the confidence needed to analyze polynomials effectively, paving the way for more complex algebraic problem-solving.

[Is X 3 A Factor Of F X 4x3 11x2 75x 18 Yes The Remainder Is 0 So X 3 Is A Factorno The]

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