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Introduction to LSI System and Impulse Response

Linear Shift-Invariant (LSI) systems are fundamental in signal processing, enabling us to analyze and design filters, communication systems, and various digital signal processing applications. At the core of these systems lies the concept of the unit sample response, often denoted as H(n), which characterizes the system's behavior in response to an impulse input. Understanding how to derive the frequency response from the impulse response is crucial because it reveals how the system modifies signals in the frequency domain.

In this article, we focus on a particular case: when the unit sample response H(n) is given as a linear function, specifically H(n) = 8(n). We will explore how to compute the frequency response, interpret its significance, and understand the implications of this particular impulse response.

Understanding the Unit Sample Response H(n) = 8(n)

What does H(n) = 8(n) represent?

The expression H(n) = 8(n) indicates that the impulse response of the system is proportional to the discrete-time index n. Here, 8(n) suggests that the value of the response increases linearly with n, scaled by a factor of 8. This form is atypical for physical systems, as most real-world systems have impulse responses that are finite or exponentially decaying. Nonetheless, mathematically, it provides an interesting case to analyze.

Key points:
- The response is linearly increasing with n.
- The scaling factor is 8.
- The impulse response is unbounded as n tends to infinity unless restricted to a specific domain.

Note: In practical systems, H(n) is often non-zero for only a finite set of n, but here, we analyze the entire sequence for theoretical insights.

Mathematical Foundation for Frequency Response

Definition of Frequency Response

The frequency response $H(e^{j\omega})$ of an LSI system is obtained by taking the Discrete-Time Fourier Transform (DTFT) of its impulse response H(n):

$$
H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} H(n) e^{-j \omega n}
$$

where:
- ω is the normalized angular frequency, ranging from $-\pi$ to π.
- $H(n)$ is the impulse response.

Purpose of the DTFT:
- To analyze how input signals of different frequencies are scaled and phase-shifted.
- To understand the system's behavior across the frequency spectrum.

Challenges with Infinite or Unbounded H(n)

Since H(n) = 8(n), which increases with n, the summation in the DTFT does not converge for the entire n-axis, implying that the system's response is not absolutely summable and the standard DTFT may diverge.

Implications:
- The system is non-causal and unstable.
- The analysis requires considering a finite or restricted domain for n, or regularization techniques.

Despite these limitations, for the purpose of theoretical analysis, we proceed with the formal mathematical derivation, assuming a finite-length impulse response or considering the response as a generalized function.

Deriving the Frequency Response of H(n) = 8(n)

Formal Calculation for Finite-Length H(n)

Suppose the impulse response is non-zero only for n = 0, 1, 2, ..., N-1, i.e., a finite-length sequence:

$$
H(n) = 8n, \quad 0 \leq n \leq N-1
$$

The DTFT becomes:

$$
H(e^{j\omega}) = \sum_{n=0}^{N-1} 8n e^{-j \omega n}
$$

This is a finite sum that can be computed explicitly.

Evaluating the Sum

The sum:

$$
H(e^{j\omega}) = 8 \sum_{n=0}^{N-1} n e^{-j \omega n}
$$

can be viewed as a weighted geometric series.

Method:

1. Recognize that:

$$
\sum_{n=0}^{N-1} n r^n = r \frac{1 - N r^{N-1} + (N-1) r^N}{(1 - r)^2}
$$

for $r \neq 1$.

2. Here, $r = e^{-j \omega}$.

3. Therefore:

$$
H(e^{j\omega}) = 8 \times \left[\frac{e^{-j \omega} (1 - N e^{-j \omega (N-1)} + (N-1) e^{-j \omega N})}{(1 - e^{-j \omega})^2} \right]
$$

This expression, though complex, provides an explicit form for the frequency response of a finite-length linear-in-n impulse response.

Infinite-Length (Theoretical) Scenario

If we attempt to analyze the infinite-length sequence \(H(n) = 8n \) for \(n \geq 0 \), the sum diverges. However, in a formal sense, the DTFT becomes a distribution (generalized function), akin to the derivative of the Dirac delta function.

Key insight:
- The DTFT of a linear function like n (for $n \geq 0$) relates to derivatives of delta functions.
- This indicates the system's frequency response involves derivatives of delta functions, representing idealized mathematical objects rather than physically realizable systems.

Alternative Approach: Using Z-Transform

Introduction to Z-Transform

The Z-transform of H(n):

\[
H(z) = \sum_{n=0}^{\infty} H(n) z^{-n}
\]

for \(H(n) = 8n \):

\[
H(z) = 8 \sum_{n=0}^{\infty} n z^{-n}
\]

which converges for \(|z| > 1 \).

Known sum:
\[
\sum_{n=0}^\infty n x^n = \frac{x}{(1 - x)^2}
\]

for \(|x| < 1 \).

By substituting \(x = z^{-1} \), we get:

\[

$$
H(z) = 8 \times \frac{z^{-1}}{(1 - z^{-1})^2} = 8 \times \frac{1}{z (1 - \frac{1}{z})^2} = 8 \times \frac{1}{z} \times \frac{z^2}{(z - 1)^2} = 8 \times \frac{z}{(z - 1)^2}
$$

Result:

$$
H(z) = \frac{8z}{(z - 1)^2}
$$

This Z-transform expression is valid for $|z| > 1$.

From Z-Transform to Frequency Response

The frequency response $H(e^{j \omega})$ is obtained by substituting $z = e^{j \omega}$:

$$
H(e^{j \omega}) = \frac{8 e^{j \omega}}{(e^{j \omega} - 1)^2}
$$

This gives a complex frequency response function that describes how the system modifies signals at different frequencies.

Interpretation of the Frequency Response

Magnitude and Phase Response

The magnitude of the frequency response:

$$
|H(e^{j \omega})| = \frac{8 |e^{j \omega}|}{|e^{j \omega} - 1|^2}
$$

Since $|e^{j \omega}| = 1$, the magnitude simplifies to:

$$
|H(e^{j \omega})| = \frac{8}{|e^{j \omega} - 1|^2}
$$

And the phase:

$$
\angle H(e^{j \omega}) = \arg \left(e^{j \omega} \right) - 2 \arg \left(e^{j \omega} - 1 \right)
$$

Behavior at Critical Frequencies

- At $\omega = 0$:

$$
e^{j 0} = 1 \Rightarrow e^{j 0} - 1 = 0
$$

which causes the magnitude to tend to infinity. This indicates a resonance or a pole at DC (zero frequency), characteristic of an integrator-like behavior.

- At $\omega = \pi$:

$$
e^{j \pi} = -1 \Rightarrow e^{j \pi} - 1 = -2
$$

which results in a finite magnitude.

Implication:
- The system acts as an integrator with a gain that becomes unbounded at DC.
- This aligns with the intuition that multiplying n by 8 in the impulse response corresponds to

Frequently Asked Questions

What is the unit sample response H(n) in an LSI system?

The unit sample response H(n) is the system's impulse response, representing its output when the input is a discrete-time unit impulse $\delta(n)$.

How is the frequency response of an LSI system related to its impulse response H(n)?

The frequency response $H(e^{j\omega})$ is the Discrete-Time Fourier Transform (DTFT) of the impulse response H(n).

Given H(n) = 8δ(n), what is the frequency response

$H(e^{j\omega})$?

For $H(n) = 8\delta(n)$, the frequency response $H(e^{j\omega}) = 8$ because the DTFT of $\delta(n)$ is 1, scaled by 8.

What does a delta function $H(n) = 8\delta(n)$ imply about the system's behavior?

It implies the system is an all-pass system with a gain of 8, passing all frequencies without distortion except for amplitude scaling.

How does the frequency response change if H(n) is scaled by a factor of 8?

The frequency response is scaled by the same factor, so $H(e^{j\omega}) = 8$ for all ω, indicating a constant magnitude response.

Is the frequency response of $H(n) = 8\delta(n)$ frequency-dependent?

No, it is frequency-independent; the magnitude is constant (8) for all frequencies.

What is the significance of a constant frequency response in system analysis?

A constant frequency response indicates an ideal system with uniform gain across all frequencies, without any frequency-dependent filtering.

Can the system be considered stable if $H(n) = 8\delta(n)$?

Yes, since the impulse response is finite and bounded, the system is stable.

How would you compute the frequency response if H(n) were a different function, say H(n) = 4(n)?

You would compute the DTFT of the new H(n) = 4(n) to find its frequency response, which would generally be frequency-dependent and require integration or summation.

Why is understanding the frequency response important in signal processing?

It helps analyze how different frequencies are amplified or attenuated by the system, guiding filter design and system behavior prediction.

Additional Resources

Let H(n) Be The Unit Sample Response Of An LSI System. Find The Frequency Response When H(n) = 8(n)

Understanding the frequency response of a linear shift-invariant (LSI) system is fundamental in digital signal processing. The unit sample response (or impulse response) H(n) encapsulates the system's behavior in the time domain, and analyzing its frequency response reveals how the system modifies different frequency components of input signals. In this article, we will explore how to determine the frequency response when the system's unit sample response is given as H(n) = 8(n), providing a comprehensive step-by-step guide suited for students, engineers, or enthusiasts seeking a deeper understanding of the topic.

Introduction to the System's Impulse Response and Frequency Response

In digital systems, the impulse response H(n) characterizes how the system reacts to a discrete-time impulse δ(n). The frequency response $H(e^{j\omega})$ is a complex function that describes how each frequency component of an input signal is scaled and phase-shifted by the system.

The core relationship connecting the impulse response H(n) and the frequency response $H(e^{j\omega})$ is the Discrete-Time Fourier Transform (DTFT):

$$ H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} H(n) e^{-j\omega n} $$

This transform converts the time-domain impulse response into the frequency domain, enabling analysis of the system's behavior across the entire spectrum.

Clarifying the Impulse Response: H(n) = 8(n)

The notation H(n) = 8(n) suggests a specific form of the impulse response. Typically, in DSP, notation like H(n) = 8(n) might mean:

- The impulse response is proportional to n, scaled by 8 (i.e., H(n) = 8n)
- The impulse response is only defined for n ≥ 0, indicating a causal system

Given the context, and common conventions in DSP, the most reasonable interpretation is that:

$$ H(n) = 8n \quad \text{for} \quad n \geq 0 $$
$$ H(n) = 0 \quad \text{for} \quad n < 0 $$

This implies a causal, linearly increasing impulse response scaled by 8.

Note: If the notation was intended differently, such as H(n) = 8 multiplied by the unit step function n, the analysis would be similar, but the causality assumption is typical for physical systems.

Step 1: Expressing the Impulse Response

Assuming the impulse response:

$$ H(n) = \begin{cases}
8n, & n \geq 0 \\
0, & n < 0
\end{cases} $$

This is a linearly increasing sequence, scaled by 8, starting at $n=0$.

Step 2: Computing the Frequency Response $H(e^{j\omega})$

The frequency response is obtained via the DTFT:

$$ H(e^{j\omega}) = \sum_{n=0}^{\infty} 8n e^{-j\omega n} $$

Our task reduces to evaluating this infinite sum:

$$ H(e^{j\omega}) = 8 \sum_{n=0}^{\infty} n e^{-j\omega n} $$

Step 3: Recognize the Sum as a Series

The sum:

$$ S = \sum_{n=0}^{\infty} n r^n $$

is a well-known series in mathematical analysis, converging when $|r| < 1$, with the sum:

$$ \sum_{n=0}^{\infty} n r^n = \frac{r}{(1 - r)^2} $$

In our case, $r = e^{-j\omega}$. Since $|e^{-j\omega}| = 1$, the series converges only for $|r| < 1$, i.e., when the sum is considered as a formal series or under specific convergence conditions (e.g., for $|r|<1$).

Note: For $|r|=1$, the series does not converge in the standard sense. However, in the context of DTFT, the sum is interpreted as a distribution or via the Fourier transform in the sense of generalized functions.

Step 4: Applying the Series Sum Formula

Using the series sum:

$$ \sum_{n=0}^{\infty} n r^n = \frac{r}{(1 - r)^2} $$

with $r = e^{-j\omega}$, we get:

$$ H(e^{jω}) = 8 \times \frac{e^{-jω}}{(1 - e^{-jω})^2} $$

This is the frequency response in a closed form.

Step 5: Simplify the Expression

Express the denominator in a more manageable form to interpret the frequency response:

$$ 1 - e^{-jω} = e^{-jω/2} \left(e^{jω/2} - e^{-jω/2} \right) = 2j e^{-jω/2} \sin(\omega/2) $$

Thus,

$$ (1 - e^{-jω})^2 = [2j e^{-jω/2} \sin(\omega/2)]^2 = 4j^2 e^{-jω} \sin^2(\omega/2) $$

Recall that $j^2 = -1$, so:

$$ (1 - e^{-jω})^2 = -4 e^{-jω} \sin^2(\omega/2) $$

Now, the numerator:

$$ e^{-jω} $$

Putting it all together:

$$ H(e^{jω}) = 8 \times \frac{e^{-jω}}{-4 e^{-jω} \sin^2(\omega/2)} = -2 \times \frac{1}{\sin^2(\omega/2)} $$

Final expression:

$$ \boxed{
H(e^{jω}) = - \frac{2}{\sin^2(\omega/2)} \quad \text{for} \quad \omega \neq 2\pi n
} $$

where n is an integer, corresponding to points where $\sin(\omega/2) = 0$.

Key Insights and Interpretation

- The frequency response has a singularity at $\omega = 2\pi n$, where the denominator becomes zero ($\sin(\omega/2) = 0$), indicating the system's response becomes unbounded at these points.

- The magnitude of the frequency response:

$$ |H(e^{jω})| = \frac{2}{\sin^2(\omega/2)} $$

- The phase (argument) of the response is determined by the complex exponential components, but since our expression is real and negative, the phase shift is approximately π (or 180 degrees), except at the singularities.

Practical Considerations

- Causality: The impulse response $H(n) = 8n$ for $n \geq 0$ indicates a causal system with a linearly increasing impulse response. Such systems are typically non-stable because the energy stored (or response magnitude) increases without bound.

- Stability: For a system to be BIBO (bounded-input bounded-output) stable, the impulse response must be absolutely summable:

$$\sum_{n=0}^{\infty} |H(n)| = 8 \sum_{n=0}^{\infty} n$$

which diverges, indicating the system is not stable.

- Frequency Response Behavior: The magnitude response becomes very large near the frequencies where $\sin(\omega/2)$ approaches zero, which could lead to resonance-like phenomena in physical systems.

Summary and Key Takeaways

- The impulse response $H(n) = 8n$ (for $n \geq 0$) leads to a frequency response:

$$H(e^{j\omega}) = - \frac{2}{\sin^2(\omega/2)}$$

- The response exhibits singularities at multiples of 2π, corresponding to frequencies where $\sin(\omega/2) = 0$.

- The magnitude response is inversely proportional to the square of $\sin(\omega/2)$, indicating sharp peaks and potential instability.

- While the mathematical derivation provides a closed-form expression, physical systems with such impulse responses are generally non-stable and exhibit unbounded behavior.

Final Remarks

Analyzing the frequency response of an LSI system from its impulse response is a cornerstone technique in digital signal processing. When the impulse response is proportional to n, as in $H(n) = 8n$, the resulting frequency response exhibits interesting characteristics, including singularities and potential instability. Understanding these behaviors helps engineers design stable systems or interpret the effects of such responses on signals.

Always consider the physical implications and stability criteria when dealing with impulse responses that grow unboundedly, as such systems, though mathematically intriguing, may not be implementable in real-world applications without modifications.

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