Let S(t) = 8t^3-24t^2 - 72t Be The Equation Of Motion For A Particle. Find A Function For The Velocity.

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Introduction

In the study of classical mechanics, understanding the motion of particles is fundamental. The position of a particle as a function of time, denoted by S(t), provides critical insights into its behavior, trajectory, and dynamics. When given an equation of motion such as S(t) = 8t^3 - 24t^2 - 72t, the natural next step is to analyze its velocity and acceleration, which are essential for understanding how the particle moves through space over time.

This article explores how to derive the velocity function from the given position function, S(t). We will delve into the mathematical process of differentiation, interpret the physical significance of the resulting velocity function, and discuss how to analyze the particle's motion characteristics such as speed, direction, and critical points like maxima, minima, and points of inflection.

By the end of this discussion, you'll have a comprehensive understanding of how to find the velocity function from a given equation of motion, applying calculus principles to classical mechanics problems. This process is fundamental not only in theoretical physics but also in engineering disciplines, robotics, and any field that involves motion analysis.

Understanding the Equation of Motion S(t)

Before jumping into the derivation of the velocity function, it’s crucial to understand what the given equation of motion represents.

The Position Function S(t)

The position function:

$$ S(t) = 8t^3 - 24t^2 - 72t $$

describes the position of a particle along a line at any time t, where t is typically measured in seconds, and S(t) in units of distance such as meters.

Key points to remember:

- The function is a cubic polynomial, indicating that the particle's motion could involve acceleration changes.
- The coefficients determine the shape and behavior of the motion over time.
- The function is continuous and differentiable, which allows us to perform calculus operations such as differentiation to find velocity and acceleration.

Deriving the Velocity Function from S(t)

The Concept of Velocity

In physics, velocity is defined as the rate of change of position with respect to time. Mathematically, it is the first derivative of the position function:

$$ v(t) = \frac{d}{dt} S(t) $$

where:

- $v(t)$ is the velocity function.
- $S(t)$ is the position function.

Step-by-Step Differentiation Process

Given:

$$ S(t) = 8t^3 - 24t^2 - 72t $$

we differentiate term-by-term:

1. Derivative of $8t^3$:

$$ \frac{d}{dt} (8t^3) = 8 \times 3t^{2} = 24t^{2} $$

2. Derivative of $-24t^2$:

$$ \frac{d}{dt} (-24t^2) = -24 \times 2t = -48t $$

3. Derivative of $-72t$:

$$ \frac{d}{dt} (-72t) = -72 $$

Putting it all together:

$$ v(t) = 24t^{2} - 48t - 72 $$

Final Velocity Function

\[

\boxed{
v(t) = 24t^{2} - 48t - 72
}
\]

This quadratic function describes how the particle’s velocity varies with time.

Analyzing the Velocity Function

Once the velocity function is obtained, it’s important to analyze its behavior to understand the particle’s motion.

Critical Points and Turning Points

Critical points occur where the velocity is zero, meaning the particle momentarily comes to rest or changes direction.

Set \(v(t) = 0 \):

\[
24t^{2} - 48t - 72 = 0
\]

Divide through by 24 to simplify:

\[
t^{2} - 2t - 3 = 0
\]

Use the quadratic formula:

\[
t = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 1 \times (-3)}}{2 \times 1} = \frac{2 \pm \sqrt{4 + 12}}{2} = \frac{2 \pm \sqrt{16}}{2}
\]

\[
t = \frac{2 \pm 4}{2}
\]

So,

- \(t = \frac{2 + 4}{2} = 3 \)
- \(t = \frac{2 - 4}{2} = -1 \)

Since negative time may not be physically meaningful depending on the context, the relevant critical point is at \(t = 3 \) seconds.

Interpreting Critical Points

- At $t = 3$, the velocity is zero, indicating the particle is momentarily at rest.
- For $t < 3$, analyze the sign of $v(t)$ to determine the direction of motion.
- For $t > 3$, the sign of $v(t)$ indicates whether the particle is moving forward or backward along its path.

Sign of Velocity in Different Intervals

Let's evaluate $v(t)$ at specific points:

- At $t = 0$:

$$
v(0) = 24 \times 0 - 48 \times 0 - 72 = -72
$$

- At $t = 2$:

$$
v(2) = 24 \times 4 - 48 \times 2 - 72 = 96 - 96 - 72 = -72
$$

- At $t = 4$:

$$
v(4) = 24 \times 16 - 48 \times 4 - 72 = 384 - 192 - 72 = 120
$$

Summary:

Time t	Velocity $v(t)$	Motion Direction
$t < 3$	Negative (e.g., at $t=0$, $v=-72$)	Moving in negative direction
$t > 3$	Positive (e.g., at $t=4$, $v=120$)	Moving in positive direction

Thus, the particle moves in the negative direction before $t=3$ and in the positive direction after.

Physical Interpretation and Graphical Analysis

Velocity Graph

Plotting $v(t) = 24t^{2} - 48t - 72$:

- The parabola opens upward (since the coefficient of t^2 is positive).
- It crosses zero at $t = -1$ and $t = 3$.
- The minimum point occurs at the vertex of the parabola, which can be found by:

$$
t_{vertex} = -\frac{b}{2a} = -\frac{-48}{2 \times 24} = \frac{48}{48} = 1
$$

At $t=1$:

$$
v(1) = 24 \times 1 - 48 \times 1 - 72 = 24 - 48 - 72 = -96
$$

This value indicates the maximum negative velocity (fastest backward motion) occurs at $t=1$.

Acceleration and Its Role

The acceleration function $a(t)$ is the derivative of velocity:

$$
a(t) = \frac{d}{dt} v(t) = \frac{d}{dt} (24t^{2} - 48t - 72) = 48t - 48
$$

- At $t=0$:

$$
a(0) = -48
$$

- At $t=1$:

$$
a(1) = 48 - 48 = 0
$$

- At $t=2$:

$$
a(2) = 96 - 48 = 48
$$

The acceleration function indicates whether the particle is speeding up or slowing down and when it changes the nature of its motion.

Practical Applications of the Velocity Function

Understanding the velocity function has numerous real-world applications such as:

- Predicting motion: Knowing when the particle is at rest, moving forward, or backward.
- Analyzing maximum speed: Finding points where the particle reaches its maximum or minimum velocity.
- Designing control systems: In robotics, where precise velocity control is necessary.
- Collision detection: To identify moments when the particle might change direction or impact other objects.

Summary and Key Takeaways

- The position function $S(t) = 8t^3 - 24t^2 - 72t$ describes the particle's location over time.
- The velocity function derived from this is:

$$
v(t) = 24t^{2} - 48t - 72
$$

- Critical points occur where $v(t) = 0$, indicating potential changes in direction.
- The particle moves in the negative direction before $t=3$ and in the positive direction afterward.
- The acceleration function:

$$
a(t) = 48t - 48
$$

provides insights into the rate of change of

Frequently Asked Questions

Given the position function $S(t) = 8t^3 - 24t^2 - 72t$, how do you find the velocity function $v(t)$?

To find the velocity function $v(t)$, differentiate the position function $S(t)$ with respect to time t: $v(t) = dS/dt$. So, $v(t) = 24t^2 - 48t - 72$.

What is the importance of differentiating the

position function to find velocity in particle motion?

Differentiating the position function S(t) with respect to time t gives the velocity function v(t), which describes how fast and in which direction the particle is moving at any given moment.

How can we determine the particle's velocity at a specific time t using the given function?

Once you have $v(t) = 24t^2 - 48t - 72$, substitute the specific value of t into this expression to calculate the particle's velocity at that time.

Are there any special points where the particle's velocity is zero? How do you find them?

Yes, points where $v(t) = 0$ are when the particle is momentarily at rest. To find these, solve the quadratic equation $24t^2 - 48t - 72 = 0$ for t.

What is the significance of the critical points obtained from the velocity function?

Critical points where $v(t) = 0$ indicate potential moments where the particle changes direction or momentarily stops, which are important for analyzing the particle's motion.

How does the velocity function relate to acceleration in this context?

The acceleration function a(t) is the derivative of the velocity function v(t). For this problem, $a(t) = d(v(t))/dt = 48t - 48$.

Can the velocity function help determine the particle's motion over time? How?

Yes, analyzing v(t) over intervals allows us to determine when the particle is moving forward or backward and identify turning points, which describe its overall motion.

What is the overall process to find the velocity function from a position function like S(t)?

The process involves differentiating the position function S(t) with respect to time t, applying basic rules of differentiation to obtain $v(t) = dS/dt$.

Additional Resources

Let S(t) = 8t^3 - 24t^2 - 72t Be The Equation Of Motion For A Particle. Find A Function For The Velocity.

Introduction

Understanding the motion of particles is a fundamental aspect of classical mechanics, offering insights into how objects move under various forces and conditions. In the realm of mathematical physics, equations of motion serve as the backbone for analyzing and predicting particle behavior over time. One such equation, given by the function $S(t) = 8t^3 - 24t^2 - 72t$, models the position of a particle as a function of time t.

This investigation aims to delve deeply into this equation, deciphering the particle's velocity profile by deriving an explicit velocity function from the given position function. The process involves applying calculus principles—specifically differentiation—to transition from position to velocity, thereby revealing the particle's instantaneous rate of change of position at any moment in time.

In this article, we will explore the mathematical techniques involved, interpret the physical implications, and analyze the particle's motion characteristics such as velocity, acceleration, and points of interest like turning points and inflection points. Through this comprehensive approach, we aim to provide a thorough understanding of the particle's dynamics dictated by the given equation of motion.

The Mathematical Framework: From Position to Velocity

The Equation of Motion $S(t)$

The position function is specified as:

$$
S(t) = 8t^3 - 24t^2 - 72t,
$$

where t is the time variable, and $S(t)$ provides the position of the particle along a line at any given time.

This cubic polynomial indicates that the particle's motion is nonlinear, potentially involving complex behaviors such as changing directions, acceleration, and possibly oscillatory components depending on the nature of velocity and acceleration functions.

Deriving the Velocity Function

The velocity $v(t)$ of the particle is defined as the instantaneous rate of change of position with respect to time:

$$
v(t) = \frac{dS}{dt}.
$$

Applying basic differentiation rules, specifically the power rule, to each term in $S(t)$:

$$
v(t) = \frac{d}{dt} (8t^3) - \frac{d}{dt}(24t^2) - \frac{d}{dt}(72t).
$$

Calculating each derivative:

- $\frac{d}{dt} (8t^3) = 24t^2$,
- $\frac{d}{dt} (24t^2) = 48t$,
- $\frac{d}{dt} (72t) = 72$.

Therefore, the velocity function is:

$$
\boxed{
v(t) = 24t^2 - 48t - 72.
}
$$

This quadratic function encapsulates how the particle’s velocity varies over time and forms the basis for further analysis.

In-Depth Analysis of the Velocity Function

Simplification and Factoring

To better understand the behavior of $v(t)$, it is helpful to factor the quadratic:

$$
v(t) = 24t^2 - 48t - 72.
$$

Divide through by 24 to simplify:

$$
v(t) = 24(t^2 - 2t - 3).
$$

Factor the quadratic inside the parentheses:

$$
t^2 - 2t - 3 = (t - 3)(t + 1).
$$

Thus, the velocity function in factored form:

$$
\boxed{
v(t) = 24 (t - 3)(t + 1).
}
$$

This factorization reveals critical points where the velocity is zero, indicating potential moments where the particle changes direction.

Critical Points and Turning Points

The zeros of $v(t)$ occur at:

$$
t = -1 \quad \text{and} \quad t = 3.
$$

- At $t = -1$, the particle’s velocity transitions through zero, possibly indicating a turning point.
- At $t = 3$, the velocity also crosses zero, indicating another potential change in motion direction.

To analyze whether these points are maxima, minima, or points of inflection, we examine the acceleration.

Acceleration and Its Implications

Deriving the Acceleration Function

Acceleration $a(t)$ is the derivative of velocity:

$$
a(t) = \frac{dv}{dt} = \frac{d}{dt} (24t^2 - 48t - 72).
$$

Applying the power rule:

$$
a(t) = 48t - 48.
$$

This linear function indicates that acceleration varies directly with time, with the particle experiencing positive or negative acceleration depending on

t.

Significance of Acceleration at Critical Points

Evaluate $a(t)$ at the critical points:

- At $t = -1$:

$$
a(-1) = 48(-1) - 48 = -48 - 48 = -96.
$$

Negative acceleration suggests the particle is decelerating or accelerating in the negative direction at $t = -1$.

- At $t = 3$:

$$
a(3) = 48(3) - 48 = 144 - 48 = 96.
$$

Positive acceleration indicates the particle is speeding up in the positive direction at $t = 3$.

Physical Interpretation of the Motion

Behavior of the Particle over Time

- For $t < -1$:
Since $v(t)$ is positive (as $t - 3 > 0$ and $t + 1 < 0$ for $t < -1$), the particle moves in the positive direction.

- Between $t = -1$ and $t = 3$:
$v(t)$ becomes negative, indicating the particle reverses direction and moves in the negative direction.

- For $t > 3$:
$v(t)$ is positive again, so the particle resumes movement in the positive direction.

This pattern suggests the particle moves forward, then reverses, and finally moves forward again, with the turning points at $t = -1$ and $t = 3$.

Velocity Sign Chart and Motion Summary

Interval	Sign of $v(t)$	Direction of Motion	Notes
$t < -1$	Positive	Moving forward	Approaching $t = -1$
$-1 < t < 3$	Negative	Moving backward	Reversal at $t = -1$

$t > 3$	Positive	Moving forward	Reversal at $t = 3$

Additional Insights: Displacement, Speed, and Energy

While the primary goal is to find the velocity function, understanding the particle's overall motion involves integrating $v(t)$ to find displacement, analyzing the magnitude of $v(t)$ for speed, and considering kinetic energy if mass is involved.

Displacement Calculation

$$
S(t) = 8t^3 - 24t^2 - 72t,
$$

can be evaluated at specific points or over intervals to determine how far the particle travels.

Speed Profile

Speed, the magnitude of velocity, $|v(t)|$, fluctuates based on the quadratic expression, with maxima at points where $v(t)$ reaches extremal values, found via critical points of $v(t)$.

Summary and Conclusions

This investigation into the equation $S(t) = 8t^3 - 24t^2 - 72t$ reveals a complex yet analyzable picture of particle motion. Through calculus techniques, notably differentiation, we obtained the velocity function:

$$
\boxed{
v(t) = 24(t - 3)(t + 1).
}
$$

This quadratic function indicates that the particle experiences direction reversals at $t = -1$ and $t = 3$, with the corresponding acceleration values of -96 and 96, respectively, suggesting significant shifts in motion dynamics.

The analysis underscores the importance of critical points, the interplay between velocity and acceleration, and how mathematical tools illuminate physical phenomena. Such detailed comprehension aids in predicting future positions, understanding energy states, and modeling real-world systems where motion equations resemble the cubic form examined here.

In essence, this case exemplifies the power of calculus in unraveling the intricacies of particle motion, transforming a seemingly simple polynomial into a window of dynamic behavior, movement patterns, and underlying physics principles.

References

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- Marsden, J. E., & Hughes, T. J. R. (1994). Mathematical Foundations of Elasticity. Dover Publications.

Note: The analysis above provides a comprehensive examination of the motion described by the given equation, with mathematical rigor suitable for a scholarly review or publication.

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