Let U = (-3, 4), V = (2,4) , And W= (4,-1) . Write Each Resulting Vector In Component Form And Find The

Let U = (-3, 4), V = (2,4), And W= (4,-1). Write Each Resulting Vector In Component Form And Find The

---

Understanding vectors is fundamental in various areas of mathematics, physics, engineering, and computer science. Vectors are quantities that have both magnitude and direction, represented mathematically as ordered pairs or tuples in a coordinate plane. In this article, we will explore the vectors U, V, and W with given components, analyze their properties, perform vector operations like addition, subtraction, and scalar multiplication, and interpret the results in component form. This comprehensive exploration aims to deepen your understanding of vector operations and their applications.

Introduction to Vectors and Their Components

A vector in a 2-dimensional space is represented by an ordered pair of numbers, indicating its components along the x-axis and y-axis. For example, a vector A = (a_1, a_2), where:

- a_1 = component along the x-axis
- a_2 = component along the y-axis

Given vectors:

- U = (-3, 4)
- V = (2, 4)
- W = (4, -1)

we will perform various operations to find new vectors, interpret their meaning, and understand how they relate to each other.

Component Form of the Given Vectors

The vectors are already provided in component form:

- U = (-3, 4)
- V = (2, 4)
- W = (4, -1)

This form makes it straightforward to perform vector operations such as addition, subtraction, scalar multiplication, and finding magnitudes or directions.

Vector Operations and Their Results

Let's explore the fundamental vector operations involving U, V, and W, and compute their results in component form.

1. Vector Addition

Definition: The sum of two vectors is obtained by adding their corresponding components:

$$
\mathbf{A} + \mathbf{B} = (a_1 + b_1, a_2 + b_2)
$$

Calculations:

- U + V:

$$
(-3, 4) + (2, 4) = (-3 + 2, 4 + 4) = (-1, 8)
$$

- V + W:

$$
(2, 4) + (4, -1) = (2 + 4, 4 + (-1)) = (6, 3)
$$

- U + W:

$$
(-3, 4) + (4, -1) = (-3 + 4, 4 + (-1)) = (1, 3)
$$

Summary of vector addition results:

Operation	Resulting Vector in Component Form
U + V	(-1, 8)
V + W	(6, 3)
U + W	(1, 3)

---

2. Vector Subtraction

Definition: The difference between two vectors is found by subtracting their corresponding components:

$$
\mathbf{A} - \mathbf{B} = (a_1 - b_1, a_2 - b_2)
$$

Calculations:

- U - V:

$$
(-3, 4) - (2, 4) = (-3 - 2, 4 - 4) = (-5, 0)
$$

- V - W:

$$
(2, 4) - (4, -1) = (2 - 4, 4 - (-1)) = (-2, 5)
$$

- W - U:

$$
(4, -1) - (-3, 4) = (4 - (-3), -1 - 4) = (4 + 3, -1 - 4) = (7, -5)
$$

Summary of vector subtraction results:

Operation	Resulting Vector in Component Form
U - V	(-5, 0)
V - W	(-2, 5)
W - U	(7, -5)

---

3. Scalar Multiplication

Scalar multiplication involves multiplying each component of a vector by a scalar (a real number).

Example:

Suppose we multiply vector U by scalar 3:

\[

$$
3 \times U = 3 \times (-3, 4) = (3 \times -3, 3 \times 4) = (-9, 12)
$$

Similarly, multiplying vector W by scalar -2:

$$
-2 \times W = -2 \times (4, -1) = (-8, 2)
$$

This operation scales the vector's magnitude and may reverse its direction if the scalar is negative.

---

4. Magnitude of Vectors

The magnitude (or length) of a vector $A = (a_1, a_2)$ is given by:

$$
|\mathbf{A}| = \sqrt{a_1^2 + a_2^2}
$$

Calculations:

- |U|:

$$
|\mathbf{U}| = \sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5
$$

- |V|:

$$
|\mathbf{V}| = \sqrt{2^2 + 4^2} = \sqrt{4 + 16} = \sqrt{20} \approx 4.472
$$

- |W|:

$$
|\mathbf{W}| = \sqrt{4^2 + (-1)^2} = \sqrt{16 + 1} = \sqrt{17} \approx 4.123
$$

Applications of Vector Operations

Understanding how to manipulate vectors in component form is essential in many practical applications, such as physics (force and velocity vectors), computer graphics (movement, rotation), and engineering (stress analysis). Let's explore some specific applications relevant to the vectors U, V, and W.

1. Finding the Direction of Resultant Vectors

The direction of a vector is often expressed as an angle θ with respect to the positive x-axis, calculated using:

$$
\theta = \arctan\left(\frac{a_2}{a_1}\right)
$$

For example, the vector U + V = (-1, 8):

$$
\theta_{U+V} = \arctan\left(\frac{8}{-1}\right) = \arctan(-8)
$$

Since the x-component is negative and the y-component is positive, the vector is in the second quadrant. Calculating:

$$
\arctan(-8) \approx -82.87^\circ
$$

Adding 180° to find the angle in the second quadrant:

$$
\theta \approx 180^\circ - 82.87^\circ = 97.13^\circ
$$

Similarly, other vectors' directions can be determined.

2. Computing Dot Product

The dot product of two vectors A = (a_1, a_2) and B = (b_1, b_2) is:

$$
\mathbf{A} \cdot \mathbf{B} = a_1b_1 + a_2b_2
$$

Application:

- U · V:

$$
(-3)(2) + (4)(4) = -6 + 16 = 10

\]

- V · W:

\[
(2)(4) + (4)(-1) = 8 - 4 = 4
\]

- U · W:

\[
(-3)(4) + (4)(-1) = -12 - 4 = -16
\]

The dot product provides insight into the angle between vectors: if zero, vectors are orthogonal; if positive, they point roughly in the same direction; if negative, they point in opposite directions.

3. Cross Product and Its Limitations in 2D

In 2D, the cross product of vectors A and B results in a scalar that indicates the magnitude of the vector perpendicular to the plane, given by:

\[
\mathbf{A} \times \mathbf{B} = a_1b_2 - a_2b_1
\]

Calculations:

- U × V:

\[
(-3)(4) - (4)(2) = -12 - 8 = -20
\]

- V × W:

\[
(2)(-1) - (4)(4) = -2 - 16 = -18
\]

- U × W:

\[
(-3)(-1) - (4)(4) = 3 - 16 = -13
\]

The sign indicates the direction of the perpendicular vector (positive or negative), and the magnitude indicates the area of the parallelogram sp

Frequently Asked Questions

Given vectors U = (-3, 4), V = (2, 4), and W = (4, -1), what is the vector U + V in component form?

U + V = (-3 + 2, 4 + 4) = (-1, 8)

Calculate the vector V - W given V = (2, 4) and W = (4, -1).

V - W = (2 - 4, 4 - (-1)) = (-2, 5)

Find the scalar multiple 3U for the vector U = (-3, 4).

3U = 3 (-3, 4) = (-9, 12)

Determine the dot product of vectors U = (-3, 4) and V = (2, 4).

U · V = (-3)(2) + (4)(4) = -6 + 16 = 10

What is the magnitude of the vector W = (4, -1)?

$||W|| = \sqrt{(4^2 + (-1)^2)} = \sqrt{(16 + 1)} = \sqrt{17} \approx 4.12$

Additional Resources

Understanding Vector Operations: An In-Depth Exploration of Vectors U, V, and W

Vectors are fundamental entities in mathematics and physics, serving as the building blocks for understanding direction, magnitude, and spatial relationships in multi-dimensional space. In this comprehensive review, we focus on three specific vectors: U = (-3, 4), V = (2, 4), and W = (4, -1). Our goal is to analyze these vectors thoroughly, explore their properties, perform various operations, and understand their significance within a coordinate system.

Introduction to Vectors and Their Components

Vectors are quantities characterized by both magnitude and direction. In two-dimensional space, vectors are represented as ordered pairs of real numbers, which describe their components along the x and y axes.

Component Form of a Vector:
- For a vector $\mathbf{A}$, the component form is $\mathbf{A} = (A_x, A_y)$, where:

- A_x is the horizontal component.
- A_y is the vertical component.

Applying this to our vectors:
- U = (-3, 4): points 3 units left along the x-axis and 4 units up along the y-axis.
- V = (2, 4): points 2 units right along x and 4 units up along y.
- W = (4, -1): points 4 units right along x and 1 unit down along y.

Geometric Interpretation of the Vectors

Understanding the geometric placement of these vectors helps visualize their relationships:
- U: Located in the second quadrant, pointing diagonally up-left.
- V: Located in the first quadrant, pointing diagonally up-right.
- W: Located in the fourth quadrant, pointing right and slightly downward.

Plotting these vectors on the Cartesian plane reveals their relative positions:
- U and V are somewhat symmetric in the y-direction but opposite in the x-direction.
- W is distinct, pointing downward and to the right.

Fundamental Vector Operations

To analyze these vectors comprehensively, we need to perform several operations, including addition, subtraction, scalar multiplication, dot product, and cross product (in 2D, the latter is scalar). Each operation sheds light on the relationships between the vectors.

Vector Addition and Subtraction

Addition:
- To add two vectors, sum their corresponding components:

$$
\mathbf{A} + \mathbf{B} = (A_x + B_x, A_y + B_y)
$$

Subtraction:
- To subtract, subtract the corresponding components:

$$
\mathbf{A} - \mathbf{B} = (A_x - B_x, A_y - B_y)

\]

Calculations:

1. U + V
- $(-3, 4) + (2, 4) = (-3 + 2, 4 + 4) = (-1, 8)$

2. V - U
- $(2, 4) - (-3, 4) = (2 + 3, 4 - 4) = (5, 0)$

3. U + W
- $(-3, 4) + (4, -1) = (1, 3)$

4. V - W
- $(2, 4) - (4, -1) = (-2, 5)$

These results highlight how vectors combine and the new directions they create. For instance, U + V = (-1,8) points upward and slightly left, indicating a resultant vector that leans towards the second quadrant.

Scalar Multiplication

Scalar multiplication involves multiplying each component of a vector by a real number (scalar). It changes the magnitude of the vector without altering its direction (except when multiplied by a negative scalar, which reverses direction).

Examples:

- $2 \times \mathbf{U} = 2 \times (-3, 4) = (-6, 8)$
- $-1.5 \times \mathbf{V} = -1.5 \times (2, 4) = (-3, -6)$
- $0.5 \times \mathbf{W} = 0.5 \times (4, -1) = (2, -0.5)$

Scalar multiplication is useful for scaling vectors to desired magnitudes and analyzing directional changes.

Magnitude of Vectors

The magnitude (or length) of a vector $\mathbf{A} = (A_x, A_y)$ is given by:

$$
|\mathbf{A}| = \sqrt{A_x^2 + A_y^2}
$$

Calculations:

1. $|U|$
- $\sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

2. $|V|$
- $\sqrt{2^2 + 4^2} = \sqrt{4 + 16} = \sqrt{20} \approx 4.4721$

3. $|W|$
- $\sqrt{4^2 + (-1)^2} = \sqrt{16 + 1} = \sqrt{17} \approx 4.1231$

The magnitude provides a sense of the vector's length in the plane, essential for normalizing vectors or understanding their relative sizes.

Dot Product and Its Significance

The dot product (scalar product) of two vectors $\mathbf{A} = (A_x, A_y)$ and $\mathbf{B} = (B_x, B_y)$ is:

$$
\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y
$$

Calculations:

1. $U \cdot V$
- $(-3)(2) + 4(4) = -6 + 16 = 10$

2. $U \cdot W$
- $(-3)(4) + 4(-1) = -12 - 4 = -16$

3. $V \cdot W$
- $(2)(4) + 4(-1) = 8 - 4 = 4$

Interpretation:
- If the dot product is positive, vectors point roughly in the same direction.
- Zero indicates orthogonality (perpendicular vectors).
- Negative suggests they point in opposite directions.

In our case:
- $U \cdot V = 10 > 0$: vectors U and V are somewhat aligned.
- $U \cdot W = -16 < 0$: U and W point roughly in opposite directions.
- $V \cdot W = 4 > 0$: V and W have some degree of alignment.

Angle Between Vectors

The angle θ between two vectors can be found via the dot product:

$$
\cos \theta = \frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}| |\mathbf{B}|}
$$

Calculations:

1. Between U and V
- $\cos \theta = \frac{10}{5 \times 4.4721} \approx \frac{10}{22.3607} \approx 0.4472$
- $\theta \approx \arccos(0.4472) \approx 63.4349^\circ$

2. Between U and W
- $\cos \theta = \frac{-16}{5 \times 4.1231} \approx \frac{-16}{20.6155} \approx -0.775$
- $\theta \approx \arccos(-0.775) \approx 139.106^\circ$

3. Between V and W
- $\cos \theta = \frac{4}{4.4721 \times 4.1231} \approx \frac{4}{18.447} \approx 0.2167$
- $\theta \approx \arccos(0.2167) \approx 77.649^\circ$

These angles reveal the relative orientation of the vectors, which is crucial in applications such as physics for calculating work done, in computer graphics for shading and lighting, and in navigation.

Vector Products and Their Roles

In two dimensions, the cross product doesn't produce a vector but a scalar that indicates the area of the parallelogram formed by two vectors and the direction (sign) of the rotation from one to the other.

Cross Product in 2D:
$$
\mathbf{A} \times \mathbf{B} = A_x B_y - A_y B_x
$$

Calculations:

1. U × V
- $(-3)(4) - 4(2) = -12 - 8 = -20$

2. U × W
- $(-3)(-1) - 4(4)$

Let U 3 4 V 2 4 And W 4 1 Write Each Resulting Vector In Component Form And Find The

Let U 3 4 V 2 4 And W 4 1 Write Each Resulting Vector In Component Form And Find The

Related Articles

- John, A Team Member, Has Completed E0 - Agile For Beginners He Wants To Contribute To Tcs Agile Vision.
- Project: A Comparison Of Human Rights Bills Comparing The Bill Of Rights With The Universal Declaration
- List The Pros And Cons Of Amazon's Customer Relationship Managment System From The Company And Customer

Back to Home