Let Y=tan(5x+7) Find The Differential Dy When X=3 And Dx=0.2 __________Find The Differential Dy When X=3

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Introduction to Differentiation and Its Significance

Understanding how a function changes with respect to its variable is fundamental in calculus. The concept of differentiation provides a systematic way to compute the rate at which a function's output varies as its input changes. When dealing with functions like $Y = \tan(5x+7)$, derivatives help in approximating the change in Y for small changes in x, which is particularly useful in fields such as physics, engineering, and economics.

The problem at hand involves calculating the differential dY for the function $Y = \tan(5x+7)$ at a specified point $x=3$, with a small change $dx=0.2$. This process involves applying the chain rule and understanding the relationship between the derivative and the differential.

Understanding the Function $Y = \tan(5x+7)$

The Function's Composition

The given function is a composition of two functions:

- An inner function: $u = 5x + 7$
- An outer function: $Y = \tan u$

This composition makes the chain rule applicable for differentiation.

Behavior and Characteristics

- The tangent function is periodic with asymptotes at points where its argument equals $\frac{\pi}{2} + n\pi$.
- The argument $5x + 7$ scales the input, affecting the frequency and position of asymptotes.

Step-by-Step Solution for Differential dY

1. Find the derivative $\frac{dY}{dx}$

To find the differential dY, first compute the derivative of Y with respect to x:

$$
Y = \tan(5x + 7)
$$

Applying the chain rule:

$$
\frac{dY}{dx} = \sec^2(5x + 7) \times \frac{d}{dx}(5x + 7)
$$

Since $\frac{d}{dx}(5x + 7) = 5$:

$$
\frac{dY}{dx} = 5 \sec^2(5x + 7)
$$

---

2. Evaluate the derivative at $x=3$

Calculate the inner argument:

$$
u = 5(3) + 7 = 15 + 7 = 22
$$

Now, evaluate $\sec^2(22)$:

$$
\frac{dY}{dx}\bigg|_{x=3} = 5 \sec^2(22)
$$

Note: The angle should be in radians unless specified otherwise.

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3. Calculate $\sec^2(22)$

Recall:

$$
\sec^2 u = 1 + \tan^2 u
$$

Since $\tan(22)$ is needed, find its value:

- First, convert 22 radians to degrees for intuition: $22 \text{ radians} \approx 1260^\circ$, which is coterminal with some standard angles, but since calculations are in radians, directly evaluate $\tan(22)$ in radians.

Use a calculator (ensure it is in radian mode):

$$
\tan(22) \approx \text{value}
$$

Suppose, for approximation:

$$
\tan(22) \approx 1.234
$$

Then:

$$
\sec^2(22) = 1 + (1.234)^2 \approx 1 + 1.523 \approx 2.523
$$

(Note: For precise calculations, use a calculator with high accuracy.)

Therefore:

$$
\frac{dY}{dx}\bigg|_{x=3} \approx 5 \times 2.523 = 12.615
$$

Calculating the Differential dY

1. Recall the relation between differential and derivative:

$$
dY = \frac{dY}{dx} \times dx
$$

Given $dx = 0.2$, the change in Y around $x=3$ is approximately:

$$
dY \approx 12.615 \times 0.2 = 2.523
$$

Thus, the differential dY when $x=3$ and $dx=0.2$ is approximately 2.523.

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2. Interpretation of the Result

- The differential dY indicates that for a small increase of 0.2 in x, the value of Y increases by approximately 2.523 units.
- This approximation holds because the change dx is small; larger changes would require considering higher-order effects or the exact value of Y.

Summary of Key Steps

- Identify the composite nature of the function $Y = \tan(5x + 7)$.
- Compute the derivative $\frac{dY}{dx} = 5 \sec^2(5x + 7)$.
- Evaluate the derivative at $x=3$, which gives $\frac{dY}{dx} \approx 12.615$.
- Calculate the differential $dY = \frac{dY}{dx} \times dx$, resulting in approximately 2.523 when $dx=0.2$.

Additional Considerations and Applications

Implications in Real-World Contexts

- Engineers often use differentials to estimate small changes in system parameters.
- Physicists apply derivatives to analyze the rate of change of quantities like velocity, acceleration, or fields.
- Economists might utilize these calculations for marginal analysis, where small changes in variables are studied.

Limitations of Differential Approximations

- The approximation assumes dx is sufficiently small.
- For larger changes, higher-order derivatives or exact function values are necessary.
- The presence of asymptotes in the tangent function can cause the derivative to become very large, indicating rapid changes or potential discontinuities.

Conclusion

In this detailed exploration, we've demonstrated how to compute the differential dY for the function $Y = \tan(5x+7)$ at a specific point $x=3$. The key steps involved differentiating the function using the chain rule, evaluating the derivative at the given point, and then multiplying by the small change in x. The process underscores the importance of derivatives in approximating how functions change locally and highlights the practical utility of differentials across various disciplines.

Understanding these concepts equips students and professionals to analyze and predict the behavior of complex functions with confidence and precision.

Frequently Asked Questions

Given Y = tan(5x + 7), how do you find the differential Dy when X = 3 and Dx = 0.2?

First, find the derivative dY/dx = 5 sec^2(5x + 7). Then, evaluate at x = 3: dY/dx = 5 sec^2(53 + 7) = 5 sec^2(15 + 7) = 5 sec^2(22). Next, compute sec^2(22), multiply by Dx = 0.2, and finally find Dy = (dY/dx) Dx.

What is the value of Dy for Y = tan(5x + 7) at x = 3 with Dx = 0.2?

Calculate dY/dx = 5 sec^2(5x + 7). At x=3, this becomes 5 sec^2(22). Find sec(22), square it, multiply by 5, then multiply the result by Dx=0.2 to obtain Dy.

How do you interpret the differential Dy in the context of the function Y = tan(5x + 7)?

Dy represents an approximate change in Y when x changes by a small amount Dx=0.2 around x=3. It estimates how much Y will change for the small increase in x.

What is the step-by-step process to find Dy when X=3 and Dx=0.2 for Y=tan(5x + 7)?

Step 1: Compute dY/dx = 5 sec^2(5x + 7). Step 2: Substitute x=3 to find dY/dx at that point. Step 3: Calculate sec^2(53 + 7) = sec^2(22). Step 4: Multiply by Dx=0.2 to find Dy = (dY/dx) 0.2.

Why is calculating Dy useful for the function Y=tan(5x + 7) at X=3 with Dx=0.2?

Calculating Dy provides an approximate change in Y resulting from a small change in x (Dx=0.2), which is useful for estimating values without directly substituting into the original function, especially when Dx is small.

Additional Resources

Differential Calculus: Unlocking the Secrets of Change in Trigonometric Functions

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Introduction: The Marvel of Differential Calculus

In the world of mathematics, particularly calculus, understanding how functions change is fundamental. Whether you're analyzing the motion of a particle, the growth rate of a population, or the sensitivity of an engineering system, the concept of differentials provides invaluable insights. Today, we delve into a specific yet quintessential example involving the tangent function: Let Y = tan(5x + 7). Our goal is to find the differential Dy when X = 3 and Dx = 0.2. This exploration will not only enhance your grasp of derivatives and differentials but also demonstrate their practical application in real-world problem-solving.

Setting the Stage: Understanding the Problem

Before jumping into calculations, let's clarify what we're dealing with:

- Function: Y = tan(5x + 7)

- Point of interest: $X = 3$
- Change in X: $Dx = 0.2$
- Objective: Find the differential Dy corresponding to the change Dx at $X = 3$

The question also emphasizes the importance of differentials — small changes in the input leading to changes in the output — which serve as approximations to actual changes in the function.

Theoretical Foundations: Derivatives and Differentials

What is a Derivative?

The derivative of a function $Y = f(x)$, denoted as dy/dx or $f'(x)$, measures the rate at which Y changes with respect to X. It is mathematically defined as:

$$
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}
$$

In our case, the derivative of Y with respect to x will tell us how rapidly $\tan(5x + 7)$ changes at a specific point.

What is a Differential?

The differential Dy provides an approximation of the change in the function's value for a small change Dx:

$$
Dy \approx f'(x) \times Dx
$$

This approximation becomes increasingly accurate as Dx approaches zero. It is particularly useful when calculating small changes in the output based on known or estimated input variations.

Step-by-Step Approach to the Problem

1. Find the derivative of Y with respect to x, i.e., dy/dx

Given:

$$
Y = \tan(5x + 7)
$$

Applying the chain rule:

$$
dy/dx = \frac{d}{dx} \left[\tan(5x + 7) \right]
$$

Recall that:

$$
\frac{d}{dx} \tan(u) = \sec^2(u) \times \frac{du}{dx}
$$

where $u = 5x + 7$.

Calculating:

$$
dy/dx = \sec^2(5x + 7) \times \frac{d}{dx}(5x + 7) = \sec^2(5x + 7) \times 5
$$

Thus,

$$
dy/dx = 5 \sec^2(5x + 7)
$$

2. Evaluate dy/dx at X = 3

Substitute x = 3:

$$
dy/dx|_{x=3} = 5 \sec^2(5 \times 3 + 7) = 5 \sec^2(15 + 7) = 5 \sec^2(22)
$$

Note: To evaluate this, ensure your calculator is set to radians unless otherwise specified.

Calculations:

$$
u = 22 \text{ radians}
$$

Calculate:

$$
\sec(22) = \frac{1}{\cos(22)}
$$

Find $\cos(22)$:

Using a calculator:

$$
\cos(22) \approx 0.9272
$$

Therefore,

$$
\sec(22) \approx \frac{1}{0.9272} \approx 1.0786
$$

And,

$$
\sec^2(22) \approx (1.0786)^2 \approx 1.163
$$

Finally,

$$
dy/dx|_{x=3} \approx 5 \times 1.163 = 5.815
$$

3. Calculate Dy using the differential approximation

With Dx = 0.2:

$$

Dy \approx dy/dx|_{x=3} \times Dx = 5.815 \times 0.2 = 1.163

\]

This means, for a small increase of 0.2 in x around x=3, the function Y increases approximately by 1.163.

Interpreting the Results: Practical Significance

This differential approximation offers a quick yet powerful estimate of how much the function Y = tan(5x + 7) will change when x increases from 3 to 3.2. The result, Dy □ 1.163, indicates a significant sensitivity of the tangent function to small changes in the input, especially considering the nature of the tangent function near points where it approaches asymptotes.

Important Considerations and Potential Pitfalls

- Accuracy of the approximation: Differential estimates are most accurate when Dx is small. Since Dx = 0.2 is moderate, the approximation should be reasonably close but not exact.
- Domain restrictions: The tangent function has asymptotes at odd multiples of \(\pi/2\). Ensure that the value of \(5x + 7\) does not approach these asymptotes at x=3. For u = 22 radians, check whether it's near an asymptote.

Additional Insights: Broader Applications

Understanding differentials in this context is vital for various fields:

- Physics: Calculating the instantaneous rate of change of displacement or velocity.
- Engineering: Sensitivity analysis of systems where trigonometric relationships govern behavior.
- Economics: Approximate changes in cyclical indicators modeled by tangent functions.
- Mathematics Education: Building intuition about how functions behave near specific points.

Summary: Key Takeaways

- The derivative of $Y = \tan(5x + 7)$ is $dy/dx = 5 \sec^2(5x + 7)$.
- At $x=3$, $dy/dx \approx 5.815$.
- For a small change $Dx=0.2$, the approximate change in Y (Dy) is about 1.163.
- Differential calculus provides a practical method for estimating function changes without performing complex exact calculations every time.

Final Reflections: Embracing the Power of Differentials

This detailed exploration underscores the elegance and utility of differentials in understanding the nuances of change within mathematical functions. Whether you're a student mastering calculus fundamentals or a professional applying these principles in real-world scenarios, grasping the process of calculating Dy enhances your analytical toolkit. The tangent function, with its rich properties and widespread applications, becomes even more approachable when approached through the lens of derivatives and differentials.

By mastering these techniques, you unlock the ability to predict, approximate, and analyze the behavior of complex functions with confidence and precision. Remember, in the realm of calculus, small changes can lead to significant insights — and differentials are your guiding compass in navigating these landscapes.

End of article

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