Line Y=2x+3 Is Tangent To A Circle With Center (2,-3). Which Of The Length Of The Radius Of The Circle?

Line Y=2x+3 Is Tangent To A Circle With Center (2,-3). Which Of The Length Of The Radius Of The Circle?

When studying the relationship between lines and circles in coordinate geometry, a common problem involves determining whether a given line is tangent to a particular circle, and if so, finding the radius of that circle. In this article, we explore the scenario where the line Y=2x+3 is tangent to a circle with the center at (2, -3). Our goal is to determine the length of the radius of this circle.

Understanding the geometric principles behind tangency, the distance between the line and the circle's center, and how these relate to the radius is essential. We will systematically analyze the problem, apply relevant formulas, and walk through the solution step-by-step.

Understanding the Geometric Relationship: Line and Circle

Before delving into calculations, it is important to clarify some fundamental concepts:

What Does It Mean for a Line to be Tangent to a Circle?

A line is tangent to a circle if it touches the circle at exactly one point. This means:

- The line intersects the circle at exactly one point.
- The shortest distance from the center of the circle to the line equals the radius of the circle.

Key Geometric Principle

The crucial relationship for a tangent line and circle is:

Distance from the circle's center to the line = Radius of the circle

This principle allows us to find the radius if we know the line's equation and the circle's center.

Given Data and Objective

- Line: Y = 2x + 3
- Center of the circle: (2, -3)

Objective: Find the length of the radius of the circle, knowing that the line is tangent to it.

Step-by-Step Solution Approach

To find the radius, we will:

1. Express the line in standard form.
2. Calculate the perpendicular distance from the circle's center to the line.
3. Recognize that this distance equals the radius since the line is tangent.

Let's proceed through each step.

Step 1: Convert the line to standard form

The line is given as:

Y = 2x + 3

Rewrite it as:

2x - Y + 3 = 0

or equivalently:

2x - Y + 3 = 0

This form is suitable for applying the distance formula.

Step 2: Calculate the distance from the center to the line

The perpendicular distance d from a point (x_0, y_0) to a line $ax + by + c = 0$ is given by:

$$
d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}
$$

Plugging in the values:

- $a = 2$
- $b = -1$
- $c = 3$
- $(x_0, y_0) = (2, -3)$

Calculate numerator:

$$
|2 \times 2 + (-1) \times (-3) + 3| = |4 + 3 + 3| = |10| = 10
$$

Calculate denominator:

$$
\sqrt{(2)^2 + (-1)^2} = \sqrt{4 + 1} = \sqrt{5}
$$

Therefore, the distance:

$$
d = \frac{10}{\sqrt{5}} = \frac{10 \sqrt{5}}{5} = 2 \sqrt{5}
$$

Step 3: Determine the radius of the circle

Since the line is tangent to the circle, the distance from the center to the line equals the radius:

$$
r = d = 2 \sqrt{5}
$$

Expressed numerically:

$$
r \approx 2 \times 2.236 = 4.472
$$

Hence, the radius of the circle is $2 \sqrt{5}$ units.

Additional Insights and Related Concepts

Understanding the relationship between lines and circles also involves exploring related concepts such as:

1. Equation of the Circle

- The circle's equation with center (h, k) and radius r:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

- In this scenario:

$$
(x - 2)^2 + (y + 3)^2 = (2 \sqrt{5})^2 = 4 \times 5 = 20
$$

- So, the circle's equation is:

$$
(x - 2)^2 + (y + 3)^2 = 20
$$

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2. Verifying the Tangency Point

- To find the exact point of tangency, substitute the line equation into the circle's equation and analyze the quadratic for a single solution (discriminant zero).

Conclusion and Summary

In summary, when a line is tangent to a circle, the shortest distance from the circle's center to the line equals the radius of the circle. For the line Y=2x+3 and the circle centered at (2, -3), this distance is calculated using the point-to-line distance formula, resulting in a radius of:

\[
\boxed{r = 2 \sqrt{5} \text{ units}}
\]

This value approximately equals 4.472 units, providing a precise measure of the circle's size that is tangent to the given line.

Practical Applications of Tangent and Radius Calculations

Understanding these concepts has practical relevance in numerous fields such as:

- Engineering: Designing circular components that are tangent to other structures.
- Architecture: Calculating space and fitting elements with precise tangency.
- Navigation and Robotics: Path planning involving circular zones and tangent paths.
- Mathematics Education: Building foundational skills in coordinate geometry.

Summary of Key Formulas

- Distance from point to line:

\[
d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}
\]

- Equation of a circle:

\[
(x - h)^2 + (y - k)^2 = r^2
\]

- Tangent condition:

\[

\text{Distance from center to line} = r
\]

Final Remarks

This problem demonstrates the elegant relationship between lines and circles in the coordinate plane. By converting line equations, applying the distance formula, and understanding the geometric principles of tangency, we can accurately determine the radius of a circle given its tangency condition with a line. Mastery of these concepts enhances problem-solving skills in geometry and their applications in real-world scenarios.

Remember: Always verify the conditions for tangency and ensure that calculations are precise, especially when dealing with square roots and algebraic expressions.

Frequently Asked Questions

How do you determine if the line y = 2x + 3 is tangent to a circle centered at (2, -3)?

Calculate the distance from the center (2, -3) to the line y = 2x + 3; if this distance equals the radius, then the line is tangent to the circle.

What is the formula for the distance from a point to a line in coordinate geometry?

The distance from point (x_0, y_0) to line $Ax + By + C = 0$ is $|Ax_0 + By_0 + C| / \sqrt{(A^2 + B^2)}$.

What is the standard form of the line y = 2x + 3?

Rearranged to standard form: 2x - y + 3 = 0.

Calculate the distance from the circle's center (2, -3) to the line 2x - y + 3 = 0.

Distance = $|22 + (-1)(-3) + 3| / \sqrt{(2^2 + (-1)^2)} = |4 + 3 + 3| / \sqrt{(4 + 1)} = 10 / \sqrt{5} \approx 10 / 2.236 \approx 4.472$.

What is the length of the radius if the line y=2x+3 is

tangent to the circle centered at (2, -3)?

The radius is approximately 4.472 units, equal to the distance from the center to the tangent line.

Why does the distance from the center to the tangent line equal the radius of the circle?

Because a tangent line touches the circle at exactly one point, and this point is exactly a radius away from the center, making the perpendicular distance from the center to the line equal to the radius.

Can the length of the radius be different if the line is tangent to the circle?

No, if the line is tangent to the circle, the radius is always equal to the perpendicular distance from the center to the line.

What is the significance of the radius length in relation to a tangent line?

The radius length represents the distance from the circle's center to the point of tangency, which is exactly the perpendicular distance from the center to the tangent line.

How can you verify if a given circle with a specific radius is tangent to the line y=2x+3?

Calculate the distance from the circle's center to the line; if it equals the given radius, then the line is tangent to the circle.

Additional Resources

Circle and Tangent Line: An In-Depth Exploration of Geometric Relationships

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Understanding the Geometric Foundations

The relationship between a line and a circle is a fundamental concept in geometry, offering insights into the properties of shapes and their spatial interactions. When a line is tangent to a circle, it touches the circle at exactly one point, creating a unique set of conditions that can be analyzed both algebraically and geometrically.

In this article, we delve into the specific case where the line Y = 2x + 3 is tangent to a circle centered at (2, -3). Our goal is to determine the length of the radius of this circle, given the tangent condition. This scenario highlights the elegance of coordinate geometry and the power of algebraic methods in solving classical geometric problems.

Core Concepts in Focus

Before diving into calculations, it’s essential to clarify the key concepts involved:

1. Tangent Line to a Circle

- A line that touches a circle at exactly one point.
- The point of contact is called the point of tangency.
- The tangent line is perpendicular to the radius drawn to the point of contact.

2. Circle Equation in Coordinate Plane

- Standard form: $(x - h)^2 + (y - k)^2 = r^2$,
where (h, k) is the center and r is the radius.

3. Distance from a Point to a Line

- The shortest distance from a point (x_0, y_0) to a line $Ax + By + C = 0$ is given by:
$$
d = \frac{|A x_0 + B y_0 + C|}{\sqrt{A^2 + B^2}}
$$
- When the line is tangent to the circle, this distance equals the radius r.

Step-by-Step Analysis of the Problem

Given:
- Line: $Y = 2x + 3$
- Center of circle: $(2, -3)$
- Unknown: radius r

Our task:
- Verify whether the line is tangent to the circle centered at $(2, -3)$.
- Find the length of the radius r of the circle.

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1. Rewriting the Line Equation in Standard Form

To utilize the distance formula, express the line in the standard form $Ax + By + C = 0$.

Starting with:

$$
Y = 2x + 3
$$

Rearranged as:

$$
2x - y + 3 = 0
$$

Here:
- $A = 2$
- $B = -1$
- $C = 3$

---

2. Applying the Distance Formula

The shortest distance from the circle's center $(2, -3)$ to the line $2x - y + 3 = 0$ is:

$$
d = \frac{|A x_0 + B y_0 + C|}{\sqrt{A^2 + B^2}}
$$

Substituting:

$$
x_0 = 2,\quad y_0 = -3
$$

$$
d = \frac{|2 \times 2 + (-1) \times (-3) + 3|}{\sqrt{2^2 + (-1)^2}} = \frac{|4 + 3 + 3|}{\sqrt{4 + 1}} = \frac{|10|}{\sqrt{5}} = \frac{10}{\sqrt{5}}
$$

Simplify numerator and denominator:

$$
d = \frac{10}{\sqrt{5}} = \frac{10 \times \sqrt{5}}{5} = 2 \sqrt{5}
$$

---

3. Determining the Radius r

Since the line is tangent to the circle, the distance from the center to the line must be exactly the radius:

$$
r = d = 2 \sqrt{5}
$$

Therefore, the radius of the circle is $2 \sqrt{5}$ units.

Interpreting the Result: Geometric Significance

This calculation confirms that the line $Y=2x+3$ touches the circle centered at $(2, -3)$ precisely at a single point, and the circle's radius is $2 \sqrt{5}$. This radius value encapsulates the precise distance from the center to the tangent line, reflecting the inherent symmetry and elegance of geometric relationships.

Additional Insights and Contextualization

Why is this important?
Understanding the interplay between lines and circles is essential in multiple fields, from classical geometry to modern computational graphics, physics, and engineering. For example:

- Design and Engineering: Ensuring components fit precisely in mechanical systems often involves tangent constraints.
- Computer Graphics: Rendering curves and objects requires knowledge of tangent lines and radii.
- Navigation and Robotics: Path planning may involve tangent trajectories to circular obstacles.

Mathematical elegance:
The solution illustrates how algebra can be used to analyze geometric configurations swiftly and accurately, transforming a visual problem into manageable calculations.

Summary of Key Takeaways

- The line $Y=2x+3$ is tangent to a circle with center $(2, -3)$ when the perpendicular distance from the center to the line equals the radius.

- Using the distance formula, the radius \(r\) is calculated as \(2 \sqrt{5}\) units.
- This approach highlights the harmony between algebraic and geometric perspectives, reinforcing the importance of coordinate geometry in problem-solving.

Final Thoughts

This detailed examination exemplifies the beauty of geometry combined with algebra, providing a clear pathway from problem statement to solution. Whether you're a student, educator, or professional, mastering these fundamental concepts opens doors to more advanced explorations in mathematics and related disciplines. The precise calculation of the radius not only solves the problem at hand but also underscores the power of mathematical reasoning in understanding the shapes and spaces that surround us.

In essence, the radius of the circle tangent to the line \(Y=2x+3\) with center at \((2, -3)\) is \(2 \sqrt{5}\) units, a result derived through methodical application of the distance formula and geometric principles.

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