# One Mole Of An Ideal Gas, Cp = (7/2) R And Cv = (5/2) R, Is Expanded Adiabatically In A Piston/cylinder

**One Mole Of An Ideal Gas, Cp = (7/2) R And Cv = (5/2) R, Is Expanded Adiabatically In A Piston/cylinder**

Understanding the thermodynamic behavior of gases during various processes is fundamental in physics and engineering. Specifically, analyzing how an ideal gas responds to adiabatic expansion offers insights into energy transfer, work done, and temperature changes in thermodynamic systems. This article explores the scenario where one mole of an ideal gas, characterized by specific heat capacities $C_p = \frac{7}{2} R$ and $C_v = \frac{5}{2} R$, undergoes an adiabatic expansion within a piston-cylinder assembly. We will delve into the underlying principles, mathematical formulations, and practical implications of this process, providing a comprehensive understanding suitable for students, educators, and engineers alike.

---

## Fundamentals of Ideal Gases and Specific Heats

### Ideal Gas Law

An ideal gas obeys the equation:

$$
PV = nRT
$$

where:
- $P$ = pressure,
- $V$ = volume,
- $n$ = number of moles,
- $R$ = universal gas constant ($8.314\, \mathrm{J\,mol^{-1}\,K^{-1}}$),
- $T$ = temperature in Kelvin.

In the context of a single mole of gas ($n=1$), the relation simplifies to:

$$
PV = RT
$$

This fundamental equation links pressure, volume, and temperature during any process involving an ideal gas.

## Specific Heats: $C_v$ and $C_p$

The specific heats at constant volume and constant pressure define how much heat energy is required to change the temperature of the gas:

- Heat capacity at constant volume, $C_v$: Energy needed to raise the temperature by 1 K when volume is held fixed.
- Heat capacity at constant pressure, $C_p$: Energy needed to raise the temperature by 1 K when pressure is held fixed.

The relation between the two for an ideal gas is:

$$
C_p = C_v + R
$$

Given values:

$$
C_v = \frac{5}{2} R,\quad C_p = \frac{7}{2} R
$$

These values indicate a monatomic ideal gas, as typical for such gases, where the degrees of freedom contribute to the specific heat capacities.

---

# Adiabatic Processes: Basic Concepts

## Definition of an Adiabatic Process

An adiabatic process is one in which no heat exchange occurs between the system and its surroundings:

$$
Q = 0
$$

This implies that all changes in the internal energy of the gas are due solely to work done during expansion or compression.

## Adiabatic Equations for Ideal Gases

For adiabatic processes involving ideal gases, the following relations hold:

1. Poisson's Equation:

\[

$$
PV^\gamma = \text{constant}
$$

where $\gamma = \frac{C_p}{C_v}$ is the heat capacity ratio.

2. Temperature-Volume Relation:

$$
TV^{\gamma - 1} = \text{constant}
$$

3. Temperature-Pressure Relation:

$$
\frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{\gamma -1}{\gamma}}
$$

---

# Calculating Key Parameters in the Adiabatic Expansion

Given:
$$
C_v = \frac{5}{2} R,\quad C_p = \frac{7}{2} R
$$

we find:
$$
\gamma = \frac{C_p}{C_v} = \frac{\frac{7}{2} R}{\frac{5}{2} R} = \frac{7}{5} = 1.4
$$

This ratio, $\gamma = 1.4$, is typical for diatomic gases or gases with similar degrees of freedom.

---

## Initial Conditions and Assumptions

Suppose the initial state of the gas is characterized by:
- Initial pressure: $P_1$
- Initial volume: $V_1$
- Initial temperature: $T_1$

The process involves the gas expanding adiabatically to a final state with:
- Final pressure: $P_2$
- Final volume: $V_2$
- Final temperature: $T_2$

---

# Step-by-Step Analysis of the Adiabatic Expansion

## 1. Relating Initial and Final States

Using the adiabatic relation:

$$
PV^\gamma = \text{constant}
$$

we get:

$$
P_1 V_1^\gamma = P_2 V_2^\gamma
$$

Similarly, for temperature and volume:

$$
T_1 V_1^{\gamma - 1} = T_2 V_2^{\gamma - 1}
$$

and for pressure and temperature:

$$
\frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}}
$$

Note: These relations are vital in calculating the thermodynamic state after expansion.

---

## 2. Work Done During Expansion

The work done by the gas during adiabatic expansion from state 1 to 2 is:

$$
W_{1 \to 2} = \int_{V_1}^{V_2} P\, dV
$$

Using the adiabatic relation $PV^\gamma = \text{constant}$, we express $P$:

$$
P = \frac{P_1 V_1^\gamma}{V^\gamma}
$$

Thus,

$$
W_{1 \to 2} = \int_{V_1}^{V_2} \frac{P_1 V_1^\gamma}{V^\gamma} dV
$$

Evaluating the integral:

$$
W_{1 \to 2} = \frac{P_1 V_1^\gamma}{1 - \gamma} \left( V_2^{1 - \gamma} - V_1^{1 - \gamma} \right)
$$

Alternatively, using the relation between initial and final states:

$$
W_{1 \to 2} = \frac{P_2 V_2 - P_1 V_1}{\gamma - 1}
$$

which simplifies calculations when $P_1, V_1, P_2, V_2$ are known.

---

## 3. Change in Internal Energy

Since the process is adiabatic, the internal energy change corresponds directly to the work performed:

$$
\Delta U = C_v (T_2 - T_1)
$$

The internal energy change can also be expressed as:

$$
\Delta U = \frac{5}{2} R (T_2 - T_1)
$$

indicating that the internal energy decreases during expansion, as the temperature drops.

---

## 4. Final Temperature Calculation

From the temperature-volume relation:

$$
T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{\gamma - 1}
$$

or, using pressures:

$$
T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}}
$$

which allows calculating the final temperature once the pressure or volume ratios are known.

---

# Practical Example and Numerical Illustration

Suppose:
- Initial pressure $P_1 = 1\, \mathrm{atm}$,
- Initial volume $V_1 = 1\, \mathrm{m^3}$,
- Initial temperature $T_1 = 300\, \mathrm{K}$,
- The gas expands until the pressure drops to $P_2 = 0.5\, \mathrm{atm}$.

Calculate:
- Final temperature $T_2$,
- Work done $W$,
- Change in internal energy $\Delta U$.

Step 1: Convert pressures to SI units:

$$
1\, \mathrm{atm} = 101.3\, \mathrm{kPa}
$$

Thus:

$$
P_1 = 101.3\, \mathrm{kPa}
$$

$$
P_2 = 50.65\, \mathrm{kPa}
$$

Step 2: Calculate $T_2$:

$$
T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} = 300 \times \left( \frac{50.65}{101.3} \right)^{\frac{0.4}{1.4}}
$$

$$
T_2 = 300 \times (0.5)^{0.2857} \approx 300 \times 0.823 \
$$

# Frequently Asked Questions

## What are the specific heat capacities (Cp and Cv) of the ideal gas in this scenario?

In this scenario, the molar heat capacity at constant pressure is $C_p = (7/2) R$, and at constant volume is $C_v = (5/2) R$.

## What is the significance of an adiabatic process for the expansion of an ideal gas?

An adiabatic process involves no heat exchange with the surroundings, meaning the gas expands or compresses without gaining or losing heat, and the process is thermally insulated.

## How do the values of Cp and Cv relate to the degrees of freedom of the gas molecules?

For an ideal gas, Cp and Cv are related to the degrees of freedom (f) by the relations $C_p = (f/2 + 1) R$ and $C_v = (f/2) R$. Given $C_p = (7/2) R$ and $C_v = (5/2) R$, the degrees of freedom are $f = 3$.

## What is the relation between pressure, volume, and temperature during an adiabatic process?

During an adiabatic process for an ideal gas, the relation is $PV^\gamma$ = constant, and $TV^{\gamma-1}$ = constant, where $\gamma = C_p/C_v$.

## Calculate the adiabatic index (γ) for the gas based on the given Cp and Cv values.

$\gamma = C_p / C_v = (7/2 R) / (5/2 R) = 7/5 = 1.4$.

## How can we determine the change in temperature during adiabatic expansion or compression in this case?

Using the relation $T_2 / T_1 = (V_1 / V_2)^{\gamma - 1}$, where $V_1$ and $V_2$ are initial and final volumes, and $\gamma = 1.4$ for this gas.

## What is the work done during the adiabatic expansion of an ideal gas?

The work done can be calculated using $W = (C_v)(T_1 - T_2)$, or more generally, $W = (P_1V_1 - P_2V_2)/ (\gamma - 1)$, depending on initial and final states.

## Why is it important to know the specific heat capacities when analyzing adiabatic processes?

Knowing Cp and Cv allows us to calculate γ, the temperature and pressure changes during expansion or compression, and the work done, providing a complete thermodynamic description of the process.

# Additional Resources

**One Mole Of An Ideal Gas, Cp = (7/2) R And Cv = (5/2) R, Is Expanded Adiabatically In A Piston/cylinder**

---

Introduction

The study of thermodynamic processes involving ideal gases remains foundational in understanding energy transfer, work, and heat within physical systems. When analyzing such processes, especially adiabatic expansion or compression, the specific heat capacities at constant pressure (Cp) and constant volume (Cv) are crucial parameters that govern the behavior of the gas. This article delves into the thermodynamics of a single mole of an ideal gas characterized by Cp = (7/2) R and Cv = (5/2) R, undergoing an adiabatic expansion within a piston-cylinder assembly. The examination encompasses the fundamental principles, derivation of key equations, and the physical insights gleaned from such a process.

---

Background and Significance

The Ideal Gas Model and Specific Heats

An ideal gas is a theoretical construct where particles are considered point masses that do not interact except during elastic collisions. This simplification allows for precise mathematical modeling of thermodynamic processes, making it an essential pedagogical and practical tool.

The specific heats, Cp and Cv, denote the amount of heat required to raise the temperature of one mole of a substance by one Kelvin at constant pressure and volume, respectively. For an ideal gas, these are related through the gas constant R:

- At constant volume:

$$
C_v = \frac{f}{2} R
$$

- At constant pressure:

$$
C_p = C_v + R = \left(\frac{f}{2} + 1\right) R
$$

\]

where \(f\) is the degrees of freedom of the gas molecules.

Significance of the Given Values

In this particular scenario, the gas has:

\[
C_v = \frac{5}{2} R \quad \text{and} \quad C_p = \frac{7}{2} R
\]

This indicates the gas has \(f=5\) degrees of freedom, which is characteristic of polyatomic gases with internal rotational degrees of freedom active (such as certain molecules with complex structures). Understanding how this specific heat configuration influences the thermodynamic behavior during adiabatic expansion is vital for applications ranging from engine cycles to atmospheric physics.

---

Thermodynamic Foundations

Fundamental Relationships for Ideal Gases

The internal energy \(U\) of an ideal gas depends solely on temperature:

\[
U = n C_v T
\]

where \(n\) is the number of moles and \(T\) is the temperature.

The work \(W\) done during a process, particularly in a piston-cylinder arrangement, is associated with volume change:

\[
W = \int P dV
\]

for pressure \(P\) and volume \(V\).

The ideal gas law links pressure, volume, and temperature:

\[
PV = nRT
\]

which will serve as the basis for deriving relations during the adiabatic process.

Adiabatic Process: Key Principles

An adiabatic process is characterized by the absence of heat exchange between the system and its surroundings:

$$
Q = 0
$$

According to the first law of thermodynamics:

$$
\Delta U = Q - W
$$

which simplifies to:

$$
\Delta U = -W
$$

for adiabatic expansion or compression.

Since $U$ depends only on temperature, the process involves changes in temperature and volume, with the relation between these variables governed by the adiabatic equation.

---

## Analytical Derivation of the Adiabatic Expansion

### The Adiabatic Equation for an Ideal Gas

For an ideal gas undergoing an adiabatic process, the relation between pressure and volume is given by:

$$
PV^{\gamma} = \text{constant}
$$

where $\gamma$ is the heat capacity ratio:

$$
\gamma = \frac{C_p}{C_v}
$$

Given $C_p = \frac{7}{2} R$ and $C_v = \frac{5}{2} R$:

$$
\gamma = \frac{\frac{7}{2} R}{\frac{5}{2} R} = \frac{7/2}{5/2} = \frac{7}{5} = 1.4
$$

This ratio is key in predicting how the pressure, temperature, and volume change during adiabatic expansion.

### Deriving Pressure-Volume and Temperature-Volume Relations

1. Pressure-Volume Relation:

$$
PV^{\gamma} = \text{constant}
$$

2. Temperature-Volume Relation:

Using the ideal gas law $PV = nRT$:

$$
T V^{\gamma -1} = \text{constant}
$$

or equivalently:

$$
T V^{\frac{7}{5} - 1} = T V^{\frac{2}{5}} = \text{constant}
$$

3. Temperature-Pressure Relation:

$$
T P^{\frac{\gamma -1}{\gamma}} = \text{constant}
$$

which can be derived from the above relations.

---

### Physical Insights and Process Characteristics

#### Behavior of the Gas During Adiabatic Expansion

- Temperature Drop: Since the gas expands adiabatically, work is performed on the surroundings, resulting in a decrease in the internal energy and, consequently, the temperature. The relation:

$$
T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma -1}
$$

indicates that as volume increases ($V_2 > V_1$), the temperature decreases.

- Pressure Reduction: The pressure also diminishes during expansion:

$$
P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{\gamma}
$$

- Work Done by the Gas:

$$
W = C_v (T_1 - T_2)
$$

which quantifies the energy transferred as work during the process.

Efficiency and Practical Implications

The adiabatic expansion illustrates how gases can perform work with minimal heat exchange—an aspect exploited in engines like the Otto cycle and other thermodynamic cycles. The specific heats influence the efficiency, as the ratio $\gamma$ determines the thermal response of the gas, affecting the work output and temperature profiles.

Reversibility and Irreversibility

The idealized adiabatic process discussed assumes reversibility, meaning no entropy change ($\Delta S = 0$). In real-world systems, irreversibilities introduce entropy production, slightly altering these relationships. Nevertheless, the ideal model provides a baseline for understanding actual processes.

---

Numerical Example and Application

Suppose a mole of this ideal gas is initially at:

- Pressure, $P_1 = 1\, \text{atm}$
- Volume, $V_1 = 22.4\, \text{L}$ (standard molar volume at STP)
- Temperature, $T_1 = 273\, \text{K}$

The gas expands adiabatically to a final volume $V_2 = 44.8\, \text{L}$, doubling the volume.

Calculations:

1. Final Temperature:

$$
T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma -1} = 273 \times \left(\frac{22.4}{44.8}\right)^{0.4} = 273 \times (0.5)^{0.4}
$$

$$
\approx 273 \times 0.7579 \approx 207\, \text{K}
$$

2. Final Pressure:

$$
P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{\gamma} = 1 \times (0.5)^{1.4} \approx 1 \times 0.378
$$

\approx 0.378\, \text{atm}
\]

3. Work Done:

\[
W = C_v (T_1 - T_2) = \frac{5}{2} R (273 - 207) = \frac{5}{2} \times 8.314 \times 66 \approx 2.5 \times 8.314 \times 66
\]

\[
\approx 2.5 \times 548.3 \approx 1370.8\, \text{J}
\]

This numerical illustration demonstrates the energy transfer involved and the thermodynamic changes during adiabatic expansion.

---

Broader Implications and Applications

Thermodynamic Cycles

The adiabatic process is integral to several thermodynamic cycles like the Carnot and Otto cycles. Understanding the specific heat ratios allows engineers to optimize engines, refrigerators, and turbines for maximum efficiency.

Atmospheric and Environmental Physics

The adiabatic expansion or compression of air affects weather phenomena, such as cloud formation and atmospheric convection. The specific heats determine the lapse rate—the rate at which temperature decreases with altitude.

Material Science and Chemical Engineering

The behavior of gases under adiabatic conditions influences the design of reactors, compressors, and pipelines, especially where rapid expansions or compressions are involved.

---

Conclusion

The adiabatic expansion of one mole of an ideal gas with specific heats \(C_p = \frac{7

## One Mole Of An Ideal Gas Cp 7 2 R And Cv 5 2 R Is Expanded Adiabatically In A Piston Cylinder

One Mole Of An Ideal Gas Cp 7 2 R And Cv 5 2 R Is Expanded Adiabatically In A Piston Cylinder

## Related Articles

- Millions Of Peasants Were Uprooted And Deported To Siberian Towns Due To The Policy Of Collectivization
- S.T.E.A.L Characterization Of Crooks From "Of Mice And Men"Speech- What Does The Character Say?how Does
- Please Help ASAP! Correct Answer Only! This Assignment Is Due Today.A Student Has To Choose An Elective