One Number Is Five More Than Another Number. If The Square Of The Smaller Number Is 3 More Than 3 Times

One Number Is Five More Than Another Number. If The Square Of The Smaller Number Is 3 More Than 3 Times is a classic algebra problem that combines simple relationships with quadratic equations. Such problems are fundamental in developing algebraic thinking and problem-solving skills. In this comprehensive guide, we will explore the steps to formulate, solve, and understand this type of problem thoroughly. By the end, you'll be equipped with the necessary knowledge to approach similar problems confidently.

Understanding the Problem Statement

Before diving into solving the problem, it’s essential to understand what is being asked and the information provided.

Breaking Down the Statement

- "One Number Is Five More Than Another Number": This suggests a relationship between two numbers, which we can denote as variables.

- "If The Square Of The Smaller Number Is 3 More Than 3 Times": This provides an additional condition involving the square of the smaller number and a multiple of the other number.

Defining Variables

To translate the problem into solvable equations, assign variables:

Choosing Variables

- Let x = the smaller number
- Let y = the larger number

Expressing the Relationships

- From the first statement:

$y = x + 5$

- From the second statement:

The square of the smaller number (x^2) is 3 more than 3 times the larger number ($3y$):

$x^2 = 3y + 3$

Formulating the Equations

Using the variables and relationships identified:

Equation 1

$$y = x + 5$$

Equation 2

$$x^2 = 3y + 3$$

Substituting y from Equation 1 into Equation 2:

$$x^2 = 3(x + 5) + 3$$

Solve for the Variables

Now, focus on solving the equation for x.

Step 1: Expand and Simplify

$$x^2 = 3x + 15 + 3$$

$$x^2 = 3x + 18$$

Step 2: Rearrange into Standard Quadratic Form

$$ x^2 - 3x - 18 = 0 $$

This quadratic equation can be solved using factoring, completing the square, or the quadratic formula.

Solving the Quadratic Equation

Method 1: Factoring (if possible)

We look for two numbers that multiply to -18 and add to -3.

Factors of -18:

- (-6, 3): sum = -3

- 6, -3: sum = 3

Check:

- (-6) 3 = -18

- (-6) + 3 = -3

Yes, so the factors are (-6) and 3.

Rewrite the quadratic:

$$ x^2 - 6x + 3x - 18 = 0 $$

Group terms:

$$ (x^2 - 6x) + (3x - 18) = 0 $$

Factor each group:

$$ x(x - 6) + 3(x - 6) = 0 $$

Factor out common binomial:

$$ (x - 6)(x + 3) = 0 $$

Set each factor equal to zero:

- $x - 6 = 0 \Rightarrow x = 6$
- $x + 3 = 0 \Rightarrow x = -3$

Method 2: Quadratic Formula (alternative)

Quadratic formula:

$$ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} $$

Here, $a = 1$, $b = -3$, $c = -18$:

$$ x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-18)}}{2(1)} $$

$$ x = \frac{3 \pm \sqrt{9 + 72}}{2} $$

$$ x = \frac{3 \pm \sqrt{81}}{2} $$

$$ x = \frac{3 \pm 9}{2} $$

Solutions:

- $x = \frac{3 + 9}{2} = \frac{12}{2} = 6$
- $x = \frac{3 - 9}{2} = \frac{-6}{2} = -3$

Both methods give the same solutions.

Finding Corresponding Values of the Larger Number

Recall:

$$ y = x + 5 $$

Substitute each x value:

- For $x = 6$:

$$ y = 6 + 5 = 11 $$

- For $x = -3$:

$$ y = -3 + 5 = 2 $$

Verification of Solutions

Always verify the solutions in the original statements to ensure correctness.

Check for $x = 6, y = 11$:

- "One Number Is Five More Than Another":

$$11 = 6 + 5 \quad \checkmark$$

- "Square Of The Smaller Number Is 3 More Than 3 Times":

$$x^2 = 6^2 = 36$$

$$3y + 3 = 3 \times 11 + 3 = 33 + 3 = 36 \quad \checkmark$$

Both conditions are satisfied.

Check for $x = -3, y = 2$:

- "One Number Is Five More Than Another":

$$2 = -3 + 5 \quad \checkmark$$

- "Square Of The Smaller Number Is 3 More Than 3 Times":

$$ x^2 = (-3)^2 = 9 $$

$$ 3y + 3 = 3 \times 2 + 3 = 6 + 3 = 9 \quad \checkmark $$

Both conditions are satisfied here as well.

Interpreting the Results

The solutions reveal that the problem admits two valid pairs of numbers:

1. $(x, y) = (6, 11)$
2. $(x, y) = (-3, 2)$

Both pairs satisfy the original conditions, illustrating that some algebraic problems have multiple solutions. It's important to verify each solution rather than assuming a unique answer.

Real-World Applications of Such Algebraic Problems

Understanding and solving problems like these have practical applications in various fields:

Financial Calculations

- Calculating investments, loans, or savings where relationships between amounts are linear or quadratic.

Engineering and Physics

- Modeling relationships involving distances, velocities, or forces where quadratic equations naturally arise.

Business and Economics

- Analyzing profit functions, cost functions, or market equilibrium scenarios.

Educational Development

- Building foundational skills in mathematics, critical thinking, and analytical reasoning.

Tips for Solving Similar Algebraic Word Problems

To efficiently approach algebra problems involving relationships:

1. **Read the problem carefully:** Identify what is being asked and what information is given.
2. **Define variables:** Assign symbols to unknown quantities.

3. **Translate statements into equations:** Convert worded relationships into algebraic expressions.

4. **Solve systematically:** Use substitution, factoring, completing the square, or the quadratic formula as appropriate.

5. **Verify solutions:** Always substitute back into original statements to confirm correctness.

6. **Interpret solutions:** Consider the context to determine which solutions are meaningful.

Conclusion

Problems involving relationships such as "One number is five more than another," combined with conditions involving squares and multiples, are excellent exercises to develop algebraic reasoning. By carefully defining variables, translating conditions into equations, and applying systematic solving techniques, you can confidently tackle similar problems. Remember to verify each solution thoroughly and consider the real-world implications of your findings.

Mastering these concepts not only enhances mathematical skills but also prepares you for more complex problem-solving scenarios in academics, careers, and everyday life. Keep practicing with different variations to strengthen your understanding and become proficient in algebraic reasoning.

Frequently Asked Questions

How can we set up equations to solve for the two numbers given that one number is five more than the other and their squares relate as described?

Let the smaller number be x. Then the larger number is $x + 5$. The given condition states that the square of the smaller number is 3 more than 3 times the larger number, so we set up the equation: $x^2 = 3(x + 5) + 3$.

What steps are involved in solving for the smaller number when given these conditions?

First, substitute the larger number into the equation, then expand and simplify to form a quadratic: $x^2 = 3x + 15 + 3$, which simplifies to $x^2 - 3x - 18 = 0$. Next, solve this quadratic to find the possible values of the smaller number.

What are the solutions for the smaller number in this problem?

Solving the quadratic $x^2 - 3x - 18 = 0$ using the quadratic formula yields $x = (3 \pm \sqrt{(9 + 72)})/2 = (3 \pm \sqrt{81})/2 = (3 \pm 9)/2$. Therefore, the solutions are $x = (3 + 9)/2 = 6$ or $x = (3 - 9)/2 = -3$.

What are the corresponding larger numbers for each solution of the smaller number?

If the smaller number is 6, the larger is $6 + 5 = 11$. If the smaller number is -3, the larger is $-3 + 5 = 2$.

How can we verify that these solutions satisfy the original conditions?

Substitute each pair back into the original conditions: for $x=6$, check if $6^2 = 3(11) + 3$; for $x=-3$, check if $(-3)^2 = 3(2) + 3$. Both pairs satisfy the equations, confirming they are valid solutions.

Additional Resources

Mathematical Relationships: Decoding the Puzzle of Two Numbers

---

Introduction

Mathematics often presents intriguing puzzles that challenge our problem-solving skills and deepen our understanding of relationships between quantities. One such classic problem involves two numbers with a specific relationship: One number is five more than another number, and the square of the smaller number is three more than three times the larger number. While it might sound straightforward at first glance, unpacking these conditions requires careful analysis, algebraic manipulation, and logical reasoning.

Imagine this problem as a finely crafted tool—much like a precision instrument—designed to reveal the hidden connections between numbers. By examining the problem step-by-step, we not only solve the puzzle but also reinforce fundamental algebraic principles that serve as essential tools for more complex mathematical explorations.

This article takes an in-depth look at this problem, breaking down each condition, exploring the relationships, and providing a comprehensive guide to arrive at the solution. Whether you're a student brushing up on algebra or a math enthusiast seeking a challenging problem, this detailed analysis aims to shed light on the elegance and logic embedded in the problem.

Understanding the Problem Statement

The problem can be summarized in two primary conditions:

1. One number is five more than another number.

2. The square of the smaller number is three more than three times the larger number.

Let's define variables to represent the two numbers:

- Let x be the smaller number.
- Let y be the larger number.

The goal is to find the values of x and y that satisfy both conditions simultaneously.

Breaking Down the Conditions

Condition 1: One number is five more than another number

This can be expressed mathematically as:

$$
\text{Either } y = x + 5 \quad \text{or} \quad x = y + 5
$$

Since the problem states "One number is five more than another number," it implies the difference is positive, but it doesn't specify which is larger. To be thorough, we will analyze both cases:

- Case 1: $y = x + 5$
- Case 2: $x = y + 5$

The analysis for each case will help us determine all possible solutions.

Condition 2: The square of the smaller number is three more than three times the larger number

Expressed mathematically as:

$$
x^2 = 3y + 3
$$

This equation links the two numbers' values and will serve as the main equation to solve in conjunction with the first condition.

Analyzing Case 1: $y = x + 5$

Step 1: Substitute $y = x + 5$ into the second condition:

$$
x^2 = 3(x + 5) + 3
$$

Step 2: Simplify the right side:

$$
x^2 = 3x + 15 + 3
$$

$$
x^2 = 3x + 18
$$

Step 3: Rearrange to form a quadratic equation:

\[

x^2 - 3x - 18 = 0

\]

Step 4: Solve the quadratic:

Using the quadratic formula:

\[

x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}

\]

where \(a = 1 \), \(b = -3 \), and \(c = -18 \),

\[

x = \frac{3 \pm \sqrt{(-3)^2 - 4 \times 1 \times (-18)}}{2}

\]

\[

x = \frac{3 \pm \sqrt{9 + 72}}{2}

\]

\[

x = \frac{3 \pm \sqrt{81}}{2}

\]

\[

x = \frac{3 \pm 9}{2}

\]

This gives two solutions:

- Solution 1:

\[

$$
x = \frac{3 + 9}{2} = \frac{12}{2} = 6
$$

- Solution 2:

$$
x = \frac{3 - 9}{2} = \frac{-6}{2} = -3
$$

Step 5: Find corresponding y for each x:

- For $x = 6$:

$$
y = x + 5 = 6 + 5 = 11
$$

- For $x = -3$:

$$
y = -3 + 5 = 2
$$

Results for Case 1:

x	y	Check $x^2 = 3y + 3$
6	11	$36 \stackrel{?}{=} 3 \times 11 + 3 = 33 + 3 = 36$ □
-3	2	$9 \stackrel{?}{=} 3 \times 2 + 3 = 6 + 3 = 9$ □

Both solutions satisfy the second condition.

Analyzing Case 2: $x = y + 5$

Step 1: Substitute $x = y + 5$ into the second condition:

$$
(y + 5)^2 = 3y + 3
$$

Step 2: Expand the left side:

$$
y^2 + 10y + 25 = 3y + 3
$$

Step 3: Rearrange into quadratic form:

$$
y^2 + 10y + 25 - 3y - 3 = 0
$$

$$
y^2 + 7y + 22 = 0
$$

Step 4: Solve the quadratic:

Discriminant:

$$
\Delta = 7^2 - 4 \times 1 \times 22 = 49 - 88 = -39
$$

\]

Since the discriminant is negative, there are no real solutions for \(y \) in this case.

Conclusion: The second case yields no real solutions, so only the solutions from Case 1 are valid in the real number system.

Summary of Findings

From the analysis above, the valid solutions are:

Smaller Number (\(x \))	Larger Number (\(y \))
6	11
-3	2

Both pairs satisfy the initial conditions:

- Pair 1: \(x = 6 \), \(y = 11 \)

\[
\text{Check: } 6^2 = 36 \quad \text{and} \quad 3 \times 11 + 3 = 33 + 3 = 36
\]
which aligns perfectly.

- Pair 2: \(x = -3 \), \(y = 2 \)

\[
\text{Check: } (-3)^2 = 9 \quad \text{and} \quad 3 \times 2 + 3 = 6 + 3 = 9

\]
again, a perfect match.

Additional Considerations and Context

Interpreting the Solutions:

- The solution $(6, 11)$ aligns with the natural understanding of the problem where the smaller number is less than the larger, and the difference is exactly five.
- The solution $(-3, 2)$ involves negative numbers, which are mathematically valid but may or may not be contextually relevant depending on the problem's real-world application.

Implications for Problem Solvers:

This analysis demonstrates the importance of considering multiple cases and verifying solutions within the context of the original conditions. Algebraic substitution and quadratic solving are fundamental tools in this process.

Conclusion

The problem of two numbers with the given relationships reveals the elegance and power of algebraic reasoning. By systematically translating conditions into equations, analyzing multiple cases, and solving quadratic equations, we uncovered all possible solutions that satisfy the criteria.

Final solutions:

- Smaller number: 6, Larger number: 11

- Smaller number: -3, Larger number: 2

This exploration underscores the significance of careful problem framing, methodical solution strategies, and validation of results. Whether for academic purposes or practical applications, mastering such problems enhances one's mathematical intuition and problem-solving confidence.

References & Further Reading

- Algebra and Its Applications by David C. Lay
- Elementary Algebra by Harold R. Jacobs
- Online quadratic equation solvers and algebra tutorials for additional practice

---

Mathematics remains a fascinating field where logic, creativity, and analytical skills converge to solve puzzles that underpin much of our understanding of the world. With each problem solved, we sharpen our mental tools and appreciation for the beauty of numbers.

One Number Is Five More Than Another Number If The Square Of The Smaller Number Is 3 More Than 3 Times

One Number Is Five More Than Another Number If The Square Of The Smaller Number Is 3 More Than 3 Times

Related Articles

- Question 9 Of 20:Select The Best Answer For The Question.9. Which One Of The Following Statements About
- PLEASE HELP!!Read This Sentence From Passage 1."I Pray That Our Heavenly Father May Assuage The Anguish
- In Distributed Storage Models Of Memory Like TODAM And CHARM, Memory Traces Arerepresented By ________