P(x) = 30x - 7x - 7x + 2 (a) Prove That (2x + 1) Is A Factor Of P(x) (b) Factorise P(x) Completely. (c)

P(x) = 30x - 7x - 7x + 2 (a) Prove That (2x + 1) Is A Factor Of P(x) (b) Factorise P(x) Completely. (c)

Understanding how to analyze polynomial expressions is fundamental in algebra. Among the various operations, factoring polynomials and determining their factors are crucial skills that help simplify complex expressions, solve equations, and analyze functions. In this comprehensive guide, we will explore the polynomial P(x) = 30x - 7x - 7x + 2, focusing on proving whether (2x + 1) is a factor, and subsequently, fully factorizing the polynomial.

Understanding the Polynomial P(x) = 30x - 7x - 7x + 2

Before diving into the specific parts of the problem, it is essential to understand the structure of P(x).

Simplifying P(x)

Let’s begin by combining like terms:

- The terms involving x are 30x, -7x, and -7x.
- The constant term is +2.

Adding the x-terms:

30x - 7x - 7x = (30x - 7x) - 7x = 23x - 7x = 16x

Therefore, the simplified form of P(x) is:

P(x) = 16x + 2

This simplification makes the problem more straightforward to analyze and solve.

(a) Prove That (2x + 1) Is A Factor Of P(x)

To determine whether (2x + 1) is a factor of P(x), we use the Factor Theorem.

Understanding the Factor Theorem

The Factor Theorem states:

- A polynomial P(x) has a factor (ax + b) if and only if P(-b/a) = 0.

In our case:

- The potential factor is (2x + 1).
- The root corresponding to this factor is found by setting 2x + 1 = 0, which gives:

x = -1/2

Step-by-step proof:

1. Identify the root associated with the factor: x = -1/2.

2. Evaluate P(x) at x = -1/2: P(-1/2).

3. If P(-1/2) = 0, then (2x + 1) is a factor of P(x).

Calculating P(-1/2)

Recall, P(x) = 16x + 2, so:

P(-1/2) = 16 (-1/2) + 2 = -8 + 2 = -6

Since P(-1/2) ≠ 0 (it equals -6), the polynomial does not satisfy the condition for (2x + 1) to be a factor.

Conclusion:

- Because P(-1/2) ≠ 0, (2x + 1) is not a factor of P(x).

(b) Factorise P(x) Completely

Even though (2x + 1) is not a factor, the polynomial P(x) is already simplified to:

$P(x) = 16x + 2$

This is a linear polynomial, and its factorization is straightforward.

Factorising Linear Polynomials

Any linear polynomial of the form ax + b can be factored as:

$P(x) = d\,(x + b/d)$

where d is a common factor of the coefficients.

Applying this:

- Coefficients are 16 and 2.
- The greatest common divisor (GCD) of 16 and 2 is 2.

Factoring out 2:

$P(x) = 2\,(8x + 1)$

Thus, the complete factorization of P(x) is:

$P(x) = 2(8x + 1)$

Summary and Final Notes

In this comprehensive analysis, we started by simplifying the polynomial $P(x) = 30x - 7x - 7x + 2$ to its simplest form, $P(x) = 16x + 2$. We then examined whether the factor (2x + 1) divides P(x). Using the Factor Theorem, we found that $P(-1/2) = -6 \neq 0$, which indicates (2x + 1) is not a factor.

Finally, recognizing that P(x) is linear, we factorized it completely by extracting the GCD of its coefficients, resulting in:

$P(x) = 2(8x + 1)$

This factorization is useful for solving equations involving P(x) or analyzing its roots.

Additional Tips for Polynomial Factorization

- Always simplify the polynomial first to make factoring easier.
- Use the Factor Theorem to test potential factors efficiently.
- Find the GCD of coefficients for linear polynomials to factor out common factors.
- For higher-degree polynomials, polynomial division and synthetic division are useful techniques for factorization.
- Remember that not all factors will divide the polynomial; always verify by substitution.

Conclusion

Understanding the process of factorization and the application of the Factor Theorem is essential in algebra. While the specific polynomial P(x) = 30x - 7x - 7x + 2 simplifies neatly to 16x + 2, the techniques discussed are broadly applicable to more complex polynomials. Recognizing whether a particular binomial is a factor allows for efficient solving of polynomial equations and deeper insights into the nature of polynomial functions.

Keywords: Polynomial factorization, Factor Theorem, algebra, linear polynomials, algebraic methods, roots, algebraic expressions, mathematical proofs, algebra tutorials

Frequently Asked Questions

How can I verify that (2x + 1) is a factor of P(x) = 30x - 7x - 7x + 2?

To verify if (2x + 1) is a factor of P(x), substitute x = -½ into P(x) (since 2x + 1 = 0 when x = -½). If P(-½) equals zero, then (2x + 1) is a factor. Calculating: P(-½) = 30(-½) - 7(-½) - 7(-½) + 2 = -15 + 3.5 + 3.5 + 2 = -15 + 9 = -6. Since P(-½) ≠ 0, (2x + 1) is not a factor. However, if the problem intends to check divisibility, you can perform polynomial division to confirm.

How do I factorise P(x) = 30x - 7x - 7x + 2 completely?

First, simplify P(x): 30x - 7x - 7x + 2 = (30x - 14x) + 2 = 16x + 2. Next, factor out the common factor 2: P(x) = 2(8x + 1). Since 8x + 1 is linear, the complete factorisation is P(x) = 2(8x + 1).

Is P(x) = 30x - 7x - 7x + 2 a linear polynomial, and what does that imply for its factorisation?

Yes, after simplifying, P(x) = 16x + 2, which is a linear polynomial. This means it can be factored as 2(8x + 1), and no further factorisation is possible over the real numbers.

Can (2x + 1) be a factor of P(x) = 16x + 2? How do I check?

To check if (2x + 1) is a factor, substitute x = -½ into P(x): P(-½) = 16(-½) + 2 = -8 + 2 = -6, which is not zero. Therefore, (2x + 1) is not a factor of P(x).

What is the significance of factorising a polynomial completely, and how does it help in solving equations?

Factorising a polynomial completely simplifies it into irreducible factors, making it easier to find roots or solutions to the equation P(x) = 0. It provides insight into the polynomial's structure and facilitates solving for x efficiently.

Additional Resources

Understanding the Polynomial P(x) = 30x - 7x - 7x + 2: A Comprehensive Guide to Factoring and Roots

In the realm of algebra, polynomials form the backbone of many mathematical concepts and applications. When tackling a polynomial such as P(x) = 30x - 7x - 7x + 2, understanding how to simplify, factor, and determine its roots is essential. This guide aims to walk you through the process step-by-step, demonstrating how to prove that a given binomial is a factor of P(x), and ultimately, how to fully factorize the polynomial. Whether you’re a student brushing up on algebraic techniques or a teacher preparing lesson content, this detailed exploration will enhance your grasp of polynomial factorization.

Breaking Down P(x): Simplification and Initial Observations

Before diving into proofs and factorization, it's important to first simplify the polynomial. The expression:

$P(x) = 30x - 7x - 7x + 2$

contains like terms, which can be combined:

- Combine the x-terms: $30x - 7x - 7x = 30x - 14x = 16x$
- The constant term is +2

Thus, the simplified form of P(x) is:

$P(x) = 16x + 2$

This simplified polynomial is linear, which makes the subsequent steps more straightforward. However, the problem's structure suggests that perhaps the original polynomial was intended to be more complex, or perhaps it's a simplified form for the purpose of the problem. For clarity, we'll proceed with the original expression, understanding that the key is to analyze the polynomial in its combined form.

Part (a): Prove That (2x + 1) Is a Factor of P(x)

Step 1: Understanding Factors of Polynomials

A polynomial P(x) has a factor (2x + 1) if and only if P(x) = 0 when (2x + 1) = 0. This is based on the Factor Theorem, which states:

If (ax + b) is a factor of P(x), then P(-b/a) = 0.

In this case, the binomial (2x + 1) equals zero when:

$2x + 1 = 0$
$\Rightarrow 2x = -1$
$\Rightarrow x = -1/2$

Step 2: Applying the Factor Theorem

To verify that (2x + 1) is indeed a factor of P(x), substitute x = -1/2 into P(x):

$P(-1/2) = 30(-1/2) - 7(-1/2) - 7(-1/2) + 2$

Calculate step-by-step:

- $30(-1/2) = -15$
- $-7(-1/2) = +3.5$

- $-7(-1/2) = +3.5$
- Constant term: $+2$

Sum these:

$$P(-1/2) = -15 + 3.5 + 3.5 + 2 = (-15 + 3.5 + 3.5) + 2$$

Combine:

$$-15 + 3.5 + 3.5 = -15 + 7 = -8$$

Finally:

$$-8 + 2 = -6$$

Since $P(-1/2) \neq 0$ (it equals -6), the polynomial $P(x)$ does not satisfy the condition for $(2x + 1)$ being a factor. Therefore, $(2x + 1)$ is not a factor of $P(x)$.

Conclusion: Based on the Factor Theorem, $(2x + 1)$ is not a factor of $P(x)$, since substituting $x = -1/2$ does not yield zero.

Part (b): Fully Factoring the Polynomial P(x)

Given that the initial polynomial simplifies to $P(x) = 16x + 2$, which is linear, it is already factored as much as possible. However, if we consider the original expression, or in more general cases, the process of factoring involves:

- Finding roots (solutions to $P(x) = 0$)
- Expressing $P(x)$ as a product of its factors

Step 1: Solving for Roots

Set $P(x) = 0$:

$$16x + 2 = 0$$
$$\Rightarrow 16x = -2$$
$$\Rightarrow x = -2/16 = -1/8$$

Step 2: Expressing P(x) as a Factorization

The root $x = -1/8$ indicates that $(x + 1/8)$ is a factor. To write $P(x)$ as a product involving $(x + 1/8)$, rewrite the polynomial:

$$P(x) = 16x + 2$$

Factor out 2:

$P(x) = 2(8x + 1)$

Notice that:

- The factor $(8x + 1)$ corresponds to the root $x = -1/8$, since:

$$8(-1/8) + 1 = -1 + 1 = 0$$

Thus, the fully factored form of $P(x)$ over the real numbers is:

$$P(x) = 2(8x + 1)$$

This is the complete linear factorization of $P(x)$.

Part (c): Summary and Additional Insights

Summary of Findings

- (a) Proof that $(2x + 1)$ is a factor:
Since substituting $x = -1/2$ into $P(x)$ yields -6 (not zero), $(2x + 1)$ is not a factor of $P(x)$.

- (b) Complete factorization of $P(x)$:
The polynomial simplifies to $P(x) = 16x + 2$, which factors as $2(8x + 1)$. The root is $x = -1/8$.

Additional Considerations

- Factor Theorem Application:
The theorem is a powerful tool to determine whether a binomial is a factor by checking roots.

- Complex Roots:
Since $P(x)$ is linear, it has only one real root. For higher-degree polynomials, factoring might involve quadratic or higher-degree factors, possibly requiring quadratic formula or synthetic division.

- Factoring over Different Domains:
While the above factorization is over the real numbers, polynomials can sometimes be factored over complex numbers, revealing additional roots.

Final Tips for Polynomial Factorization

- Always simplify the polynomial first to make factoring easier.
- Use the Factor Theorem to test potential factors.
- Find roots by setting $P(x) = 0$ and solving.
- Express the polynomial as a product of factors corresponding to its roots.

- Remember that linear polynomials are already fully factored, but quadratic or higher-degree polynomials may require further techniques.

Conclusion

Understanding how to factor polynomials and determine their roots is fundamental in algebra. In this case, the initial polynomial simplified neatly to a linear form, making its roots and factors straightforward to identify. The key takeaway is the importance of the Factor Theorem and root-finding in the process of polynomial factorization. While (2x + 1) was tested and found not to be a factor, the process exemplifies essential algebraic reasoning applicable to more complex polynomials. With practice, these techniques become intuitive, empowering you to solve a wide range of polynomial equations efficiently.

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