Problem 3 (10pts). (1) (5pts) Please Solve The Trigonometric Equation Tan2 (2) Sec(x) Tan? (x) = 1. (2)

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Introduction to the Trigonometric Equation

Solving trigonometric equations is a fundamental skill for students studying mathematics, especially in calculus and analytical geometry. The problem presented involves multiple trigonometric functions: tangent (tan) and secant (sec). Specifically, the equation is:

$$
\tan^2 (2) \cdot \sec(x) \cdot \tan(x) = 1
$$

At first glance, this appears to be a complex equation involving multiple functions and possibly some notation ambiguities. To interpret and solve it effectively, we'll analyze each component carefully.

Understanding the Components of the Equation

Deciphering the Notation

The provided equation reads:

$$
\tan^2 (2) \sec(x) \tan(x) = 1
$$

- The notation $\tan^2(2)$ indicates $\left(\tan(2)\right)^2$. Since no units are specified, we assume the angle is in radians.
- The $\sec(x)$ is the secant function, reciprocal of cosine: $\sec(x) = \frac{1}{\cos(x)}$.
- The $\tan(x)$ is the tangent function.

The presence of $\tan^2(2)$ suggests that the constant $\tan^2(2)$ can be calculated explicitly, so the equation becomes a relation involving $\sec(x)$ and $\tan(x)$.

Rearranged Equation

Expressed explicitly:

$$
\left(\tan(2)\right)^2 \cdot \sec(x) \cdot \tan(x) = 1
$$

or equivalently,

$$
\left(\tan(2)\right)^2 \cdot \tan(x) \cdot \sec(x) = 1
$$

Step-by-Step Solution Strategy

To solve this equation, we will:

1. Calculate the constant $\tan^2(2)$.
2. Express $\sec(x)$ in terms of $\tan(x)$ or vice versa.
3. Simplify the equation to a manageable form.
4. Find the general solutions for x.
5. Determine the domain restrictions based on the functions involved.

Calculating $\tan^2(2)$

Since 2 is in radians, we need to compute $\tan(2)$.

Using a calculator:

$$
\tan(2) \approx \tan(2\, \text{radians}) \approx -2.1850
$$

(approximate to four decimal places).

Therefore,

\[

$$
\tan^2(2) \approx (-2.1850)^2 \approx 4.7753
$$

This constant simplifies our original equation to:

$$
4.7753 \cdot \sec(x) \cdot \tan(x) = 1
$$

which can be rewritten as:

$$
\sec(x) \cdot \tan(x) = \frac{1}{4.7753}
$$

Numerically,

$$
\sec(x) \cdot \tan(x) \approx 0.2094
$$

Expressing $\sec(x)$ in Terms of $\tan(x)$

Recall the fundamental identity:

$$
\sec^2(x) = 1 + \tan^2(x)
$$

which implies:

$$
\sec(x) = \pm \sqrt{1 + \tan^2(x)}
$$

However, because $\sec(x)$ appears multiplied by $\tan(x)$, and the goal is to find an expression involving only $\tan(x)$, it’s often more convenient to write the entire equation in terms of $\tan(x)$.

So, the equation:

$$
\sec(x) \cdot \tan(x) = 0.2094
$$

becomes:

$$
\pm \sqrt{1 + \tan^2(x)} \cdot \tan(x) = 0.2094
$$

Solving for $\tan(x)$

Let’s consider both cases for the sign of $\sec(x)$:

Case 1: $\sec(x) = + \sqrt{1 + \tan^2(x)}$

$$
\sqrt{1 + \tan^2(x)} \cdot \tan(x) = 0.2094
$$

Case 2: $\sec(x) = - \sqrt{1 + \tan^2(x)}$

$$
- \sqrt{1 + \tan^2(x)} \cdot \tan(x) = 0.2094
$$

But note the left side in the second case is negative, and the right side is positive, so:

$$
- \sqrt{1 + \tan^2(x)} \cdot \tan(x) = 0.2094
\Rightarrow \sqrt{1 + \tan^2(x)} \cdot \tan(x) = -0.2094
$$

Therefore, the equations reduce to:

$$
\sqrt{1 + t^2} \cdot t = \pm 0.2094
$$

where $t = \tan(x)$.

Formulating a Quadratic Equation in $\tan(x)$

Squaring both sides to eliminate the square root:

$$
(\sqrt{1 + t^2} \cdot t)^2 = (0.2094)^2
$$

which simplifies to:

$$
(1 + t^2) \cdot t^2 = 0.0439
$$

or

$$
t^2 (1 + t^2) = 0.0439
$$

Expanding:

$$
t^2 + t^4 = 0.0439
$$

Rearranged as:

$$
t^4 + t^2 - 0.0439 = 0
$$

Let’s set $u = t^2$, transforming the quartic into a quadratic in u:

$$
u^2 + u - 0.0439 = 0
$$

Solving the Quadratic in u

Apply the quadratic formula:

$$
u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

where $a = 1$, $b = 1$, and $c = -0.0439$:

$$
u = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (-0.0439)}}{2}
$$

Calculate the discriminant:

$$

D = 1 + 4 \times 0.0439 = 1 + 0.1756 = 1.1756
\]

Square root:

\[
\sqrt{1.1756} \approx 1.0843
\]

Thus,

\[
u = \frac{-1 \pm 1.0843}{2}
\]

Calculating the two roots:

- For the positive root:

\[
u = \frac{-1 + 1.0843}{2} = \frac{0.0843}{2} = 0.04215
\]

- For the negative root:

\[
u = \frac{-1 - 1.0843}{2} = \frac{-2.0843}{2} = -1.04215
\]

Since \(u = t^2\), it must be non-negative:

\[
t^2 = u \geq 0
\]

Therefore, discard the negative root \(u = -1.04215\). The valid solution is:

\[
t^2 = 0.04215
\]

which gives:

\[
t = \pm \sqrt{0.04215} \approx \pm 0.2053
\]

Finding x from $\tan(x) = t$

Recall:

$$
\tan(x) = t \approx \pm 0.2053
$$

The general solutions for x are:

$$
x = \arctan(t) + k\pi, \quad k \in \mathbb{Z}
$$

Calculating the principal values:

- For $t \approx 0.2053$:

$$
x \approx \arctan(0.2053) \approx 0.202\, \text{radians}
$$

- For $t \approx -0.2053$:

$$
x \approx \arctan(-0.2053) \approx -0.202\, \text{radians}
$$

Thus, the solutions are:

$$
x \approx 0.202 + k\pi,\quad x \approx -0.202 + k\pi, \quad k \in \mathbb{Z}
$$

Domain Restrictions and Final Solution Set

Frequently Asked Questions

How do I solve the trigonometric equation tan^2(2x)

sec(x) tan^2(x) = 1?

To solve the equation, first express all functions in terms of sine and cosine, then simplify using trigonometric identities. Recognize that tan^2(2x) = (sin 2x / cos 2x)^2 and tan^2(x) = (sin x / cos x)^2. The sec(x) = 1 / cos x. Substitute these into the equation and simplify step by step to find the solutions for x.

What identities are useful for solving tan^2(2x) sec(x) tan^2(x) = 1?

Key identities include: tan 2x = 2 tan x / (1 - tan^2 x), sec x = 1 / cos x, and tan^2 x = sin^2 x / cos^2 x. Using these can help rewrite the equation entirely in terms of tan x or sin and cos, making it easier to solve.

Are there specific domain considerations when solving tan^2(2x) sec(x) tan^2(x) = 1?

Yes, since sec x = 1 / cos x, we must ensure cos x ≠ 0 to avoid undefined expressions. Also, solutions for tan x and tan 2x should be checked within the principal domains of the tangent and secant functions to find valid solutions.

Can I use substitution to simplify the equation tan^2(2x) sec(x) tan^2(x) = 1?

Absolutely. Let t = tan x. Then tan 2x = 2t / (1 - t^2). Substituting these into the equation transforms it into an algebraic equation in terms of t, which can then be solved using algebraic methods.

What are the steps to verify solutions obtained from solving tan^2(2x) sec(x) tan^2(x) = 1?

After finding potential solutions, substitute them back into the original equation to check for extraneous solutions, especially considering domain restrictions like cos x ≠ 0. This ensures all solutions are valid within the original equation's context.

Additional Resources

Understanding and Solving the Trigonometric Equation: $\tan^2(2x) \cdot \sec(x) \cdot \tan(x) = 1$

Trigonometry, a fundamental branch of mathematics, plays an essential role in various

scientific and engineering disciplines, including physics, navigation, signal processing, and more. Among the myriad of trigonometric equations encountered by students and professionals alike, those involving multiple functions and complex expressions often pose significant challenges. One such intriguing problem is the equation:

$$\tan^2(2x) \cdot \sec(x) \cdot \tan(x) = 1$$

This problem not only tests one's grasp of fundamental identities but also demands strategic manipulation and a thorough analytical approach. In this article, we delve into the structure of this equation, explore potential pathways for its solution, and provide a comprehensive, step-by-step analysis that illuminates the underlying principles of trigonometry involved.

Decomposition and Initial Observations

Understanding the Components of the Equation

The given equation involves three primary trigonometric functions:

1. Tan(2x) — the tangent of double the angle
2. Sec(x) — the secant of x
3. Tan(x) — the tangent of x

The equation reads:

$$\tan^2(2x) \cdot \sec(x) \cdot \tan(x) = 1$$

Breaking down its components:

- **Tan(2x): Known to be related to Tan(x) via the double-angle formula**
- **Sec(x): The reciprocal of cosine, i.e., 1/cos(x)**
- **Tan(x): The ratio sin(x)/cos(x)**

The expression involves the square of the double-angle tangent, multiplied by secant and tangent of the same variable x.

Applying Fundamental Trigonometric Identities

To simplify and solve the equation, we need to

recall and utilize key identities:

Double-Angle Identity for Tan(2x)

The double-angle formula for tangent is:

$$
\tan(2x) = \frac{2 \tan x}{1 - \tan^2 x}
$$

Squaring both sides gives:

$$
\tan^2(2x) = \left(\frac{2 \tan x}{1 - \tan^2 x} \right)^2 = \frac{4 \tan^2 x}{(1 - \tan^2 x)^2}
$$

This expression will be central in rewriting the original equation.

Expressing Sec(x) and Tan(x)

Since sec(x) = 1 / cos(x), and Tan(x) = sin(x)/cos(x), we can express all functions in terms of sine and cosine, but it’s often more efficient to work with tangent directly, especially

because the identities are more straightforward.

Rewriting the Equation in Terms of Tan(x)

To streamline the problem, let’s define:

$$
t = \tan x
$$

Our goal is to express the entire equation in terms of t and then solve for t, which will give solutions for x.

Using the identities:

- $\tan(2x) = \frac{2t}{1 - t^2}$
- $\sec x = \frac{1}{\cos x}$

But since $\sec x = \sqrt{1 + t^2}$, because:

$$
\sec^2 x = 1 + \tan^2 x \implies \sec x = \pm \sqrt{1 + t^2}
$$

Similarly, $\tan x = t$.

Note: The $\pm$ indicates that secant can be positive or negative depending on the quadrant.

Transforming the Equation

Substituting the identities into the original equation:

$$
\left(\frac{2t}{1 - t^2} \right)^2 \times \sec x \times t = 1
$$

which simplifies to:

$$
\frac{4t^2}{(1 - t^2)^2} \times \sec x \times t = 1
$$

Replacing $\sec x$ with $\pm \sqrt{1 + t^2}$:

$$
\frac{4t^2}{(1 - t^2)^2} \times \pm \sqrt{1 +
$$

t^2} \times t = 1
\]

Multiplying through:

\[
\pm \frac{4 t^3}{(1 - t^2)^2} \sqrt{1 + t^2} = 1
\]

At this point, the presence of the square root complicates the equation. To proceed systematically, it's advisable to analyze the positive and negative cases separately.

Addressing the Sign Ambiguity and Domain Constraints

Key considerations:

- **The sign of \(\sec x\) depends on the quadrant where \(x\) is located.**
- **The tangent function \(t = \tan x\) can take any real value, but the domain restrictions of the original functions must be respected.**
- **To avoid ambiguity, analyze the equation for**

positive and negative values separately.

Deriving a Polynomial Equation in t

To make the equation more manageable, square both sides to eliminate the square root, keeping in mind that squaring introduces extraneous solutions that must be checked later.

Starting from:

$$
\pm \frac{4 t^3}{(1 - t^2)^2} \sqrt{1 + t^2} = 1
$$

Squaring both sides:

$$
\left(\frac{4 t^3}{(1 - t^2)^2} \sqrt{1 + t^2} \right)^2 = 1^2
$$

which simplifies to:

\[

\frac{16 t^6}{(1 - t^2)^4} (1 + t^2) = 1
\]

Multiplying both sides by \((1 - t^2)^4\):

\[
16 t^6 (1 + t^2) = (1 - t^2)^4
\]

This is a polynomial equation in \(t\), and solving it will yield potential solutions for \(\tan x\).

Solving the Polynomial Equation

Equation:

\[
16 t^6 (1 + t^2) = (1 - t^2)^4
\]

Expanding the right side:

\[
(1 - t^2)^4 = \left((1 - t^2)^2 \right)^2
\]

First, find $(1 - t^2)^2$:

$$
(1 - t^2)^2 = 1 - 2 t^2 + t^4
$$

Then, square again to get $(1 - t^2)^4$:

$$
(1 - 2 t^2 + t^4)^2
$$

Expanding:

$$
= 1^2 - 2 \times 1 \times 2 t^2 + 2 \times 1 \times t^4 + (2 t^2)^2 - 2 \times 2 t^2 \times t^4 + t^8
$$

But it’s clearer to expand systematically:

$$
(1 - 2 t^2 + t^4)^2 = 1^2 + (-2 t^2)^2 + (t^4)^2 + 2 \times 1 \times (-2 t^2) + 2 \times 1 \times t^4 + 2 \times (-2 t^2) \times t^4
$$

Calculations:

- $1^2 = 1$
- $(-2 t^2)^2 = 4 t^4$
- $(t^4)^2 = t^8$
- $2 \times 1 \times (-2 t^2) = -4 t^2$
- $2 \times 1 \times t^4 = 2 t^4$
- $2 \times (-2 t^2) \times t^4 = -4 t^6$

Adding all:

$$
(1 - 2 t^2 + t^4)^2 = 1 - 4 t^2 + 4 t^4 + 2 t^4 - 4 t^6 + t^8
$$

Combine like terms:

$$
= 1 - 4 t^2 + (4 t^4 + 2 t^4) - 4 t^6 + t^8
$$
$$
= 1 - 4 t^2 + 6 t^4 - 4 t^6 + t^8
$$

Now, the original polynomial equation becomes:

$$
16 t^6 (1 + t^2) = 1 - 4 t^2 + 6 t^4 - 4 t^6 + t^8
$$

Expand the left side:

$$
16 t^6 + 16 t^8 = 1 - 4 t^2 + 6 t^4 - 4 t^6 + t^8
$$

Bring all terms to one side:

$$
16 t^6 + 16 t^8 -
$$

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