Prove That The Points (- 6 - 8) (- 16 12) And(- 26 - 18) Are The Vertices Of An Isosceles Right Angled

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Introduction

In geometry, understanding the properties of triangles and their vertices is fundamental. One common problem involves verifying whether a set of points forms a specific type of triangle, such as an isosceles right-angled triangle. This article aims to rigorously prove that the points (-6, -8), (-16, 12), and (-26, -18) serve as vertices of an isosceles right-angled triangle. Through a systematic approach involving calculations of distances, slopes, and angles, we will establish the triangle's nature and verify its properties step by step.

Understanding the Problem

What is an Isosceles Right-Angled Triangle?

An isosceles right-angled triangle possesses two key properties:

- Right angle: One angle measures exactly 90 degrees.
- Isosceles: The two sides adjacent to the right angle are equal in length.

Proving that three points form such a triangle involves demonstrating:

1. One of the angles at a vertex is 90 degrees.
2. The two sides forming this right angle are equal.

Coordinates of the Given Points

The points provided are:

- Point A: (-6, -8)
- Point B: (-16, 12)
- Point C: (-26, -18)

Our goal is to analyze these points’ positions and distances to confirm the triangle's properties.

Step 1: Calculate the Distances Between Points

Distance calculations between points are fundamental in determining side lengths and identifying right angles.

Distance Formula

The distance d between two points (x_1, y_1) and (x_2, y_2) is:

$$
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

Calculating Side Lengths

Let's compute the three side lengths:

1. AB: between points A (-6, -8) and B (-16, 12)

$$
AB = \sqrt{(-16 - (-6))^2 + (12 - (-8))^2} = \sqrt{(-10)^2 + (20)^2} = \sqrt{100 + 400} = \sqrt{500} \approx 22.36
$$

2. BC: between points B (-16, 12) and C (-26, -18)

$$
BC = \sqrt{(-26 - (-16))^2 + (-18 - 12)^2} = \sqrt{(-10)^2 + (-30)^2} = \sqrt{100 + 900} = \sqrt{1000} \approx 31.62
$$

3. AC: between points A (-6, -8) and C (-26, -18)

$$
AC = \sqrt{(-26 - (-6))^2 + (-18 - (-8))^2} = \sqrt{(-20)^2 + (-10)^2} = \sqrt{400 + 100} = \sqrt{500} \approx 22.36
$$

Summary of Side Lengths

Side	Length	Approximate
AB	22.36	$\sqrt{500}$
BC	31.62	$\sqrt{1000}$
AC	22.36	$\sqrt{500}$

Observation: Sides AB and AC are equal, each approximately 22.36 units long.

Step 2: Determine the Nature of the Triangle

Isosceles Check

Since sides AB and AC are equal, the triangle is isosceles with the equal sides AB and AC. To confirm that it is also right-angled, we need to verify whether the angle between these two sides is 90 degrees.

Step 3: Find the Slopes of the Sides

Calculating the slopes helps determine the potential perpendicularity of sides, which indicates a right angle.

Slope Formula

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

- Slope of AB:

$$
m_{AB} = \frac{12 - (-8)}{-16 - (-6)} = \frac{20}{-10} = -2
$$

- Slope of AC:

$$
m_{AC} = \frac{-18 - (-8)}{-26 - (-6)} = \frac{-10}{-20} = \frac{1}{2}
$$

- Slope of BC:

$$
m_{BC} = \frac{-18 - 12}{-26 - (-16)} = \frac{-30}{-10} = 3
$$

Checking for Perpendicularity

Two lines are perpendicular if the product of their slopes is -1.

- AB and AC:

$$
m_{AB} \times m_{AC} = -2 \times \frac{1}{2} = -1
$$

Since the product is -1, sides AB and AC are perpendicular, and the angle at point A (where these sides meet) is a right angle.

Step 4: Confirming the Triangle is Isosceles and Right-Angled

Summary of Findings:

- Sides AB and AC are equal (~22.36 units).
- Sides AB and AC are perpendicular at point A.

Therefore, the triangle formed by points A, B, and C is an isosceles right-angled triangle with the right angle at point A.

Step 5: Additional Verification (Optional)

To solidify the proof, we can verify that the hypotenuse is the side BC, which is longer (~31.62 units).

- Check the Pythagorean theorem:

$$
AB^2 + AC^2 \stackrel{?}{=} BC^2
$$

Calculations:

$$
(\sqrt{500})^2 + (\sqrt{500})^2 = 500 + 500 = 1000
$$

$$
BC^2 = (\sqrt{1000})^2 = 1000
$$

Since both sides equal 1000, the triangle satisfies the Pythagorean theorem, confirming it is right-angled at A.

Conclusion

Based on the calculations, the triangle with vertices at points (-6, -8), (-16, 12), and (-26, -18) is an isosceles right-angled triangle with:

- Right angle at point A (-6, -8)
- Equal legs AB and AC (~22.36 units)
- Hypotenuse BC (~31.62 units)

This comprehensive analysis confirms that the points form an isosceles right-angled triangle, fulfilling all the defining properties.

Additional Tips for Similar Problems

- Always start by calculating the distances between points to find side lengths.

- Check the slopes of sides meeting at potential right angles to verify perpendicularity.
- Use the Pythagorean theorem as a final confirmation for right-angled triangles.
- Remember that in coordinate geometry, the right angle can be confirmed by the negative reciprocal slopes or by the dot product of vectors being zero.

Final Remarks

Understanding how to analyze points in the coordinate plane to determine the nature of the triangle they form is a vital skill in geometry. This detailed proof not only confirms the specific case but also provides a framework applicable to similar problems involving coordinate points and their geometric relationships.

Frequently Asked Questions

What are the given points to verify if they form an isosceles right-angled triangle?

The points are (-6, -8), (-16, 12), and (-26, -18).

How do you calculate the distance between two points to check for equal sides?

Use the distance formula: Distance = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$.

What is the distance between points (-6, -8) and (-16, 12)?

Distance = $\sqrt{[(-16 + 6)^2 + (12 + 8)^2]} = \sqrt{[(-10)^2 + (20)^2]} = \sqrt{[100 + 400]} = \sqrt{500} \approx 22.36$.

What is the distance between points (-16, 12) and (-26, -18)?

Distance = $\sqrt{[(-26 + 16)^2 + (-18 - 12)^2]} = \sqrt{[(-10)^2 + (-30)^2]} = \sqrt{[100 + 900]} = \sqrt{1000} \approx 31.62$.

What is the distance between points (-6, -8) and (-26, -18)?

Distance = $\sqrt{[(-26 + 6)^2 + (-18 + 8)^2]} = \sqrt{[(-20)^2 + (-10)^2]} = \sqrt{[400 + 100]} = \sqrt{500} \approx 22.36$.

Are any two sides of the triangle formed by these points equal in length?

Yes, the distances between (-6, -8) and (-16, 12), and between (-6, -8) and (-26, -18) are both approximately 22.36, indicating two equal sides.

How do we determine if the triangle is right-angled?

Check if the Pythagorean theorem holds: the sum of the squares of the two shorter sides equals the square of the longest side.

Does the triangle formed by these points satisfy the Pythagorean theorem?

Yes, because the two equal sides are approximately 22.36, and the third side is approximately 31.62, verifying that $22.36^2 + 22.36^2 \approx 31.62^2$.

Is the triangle formed by these points isosceles and right-angled?

Yes, the two equal sides and the Pythagorean relationship confirm that the triangle is both isosceles and right-angled.

What is the conclusion about the points (-6, -8), (-16, 12), and (-26, -18)?

They are the vertices of an isosceles right-angled triangle.

Additional Resources

Prove That The Points (-6, -8), (-16, 12), and (-26, -18) Are The Vertices Of An Isosceles Right-Angled Triangle

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Introduction

In the realm of coordinate geometry, identifying the nature of a triangle formed by three points involves analyzing distances, slopes, and angles. When given three points, determining whether they form an isosceles right-angled triangle requires a systematic approach that combines algebraic calculations with geometric reasoning. The primary objective of this analysis is to prove that the points (-6, -8), (-16, 12), and (-26, -18) are vertices of such a triangle.

This exploration will encompass:
- Calculating the distances between points to identify sides.
- Verifying the right-angle condition through slopes or Pythagoras' theorem.
- Demonstrating the isosceles property by comparing side lengths.
- Summarizing findings to confirm the triangle's specific nature.

Step 1: Understanding the Coordinates and Their Significance

The three points are:

- A (-6, -8)
- B (-16, 12)
- C (-26, -18)

These points are situated in the Cartesian plane, and their relative positions determine the shape formed. Our goal is to analyze the lengths of sides AB, BC, and AC, and the angles at these vertices.

Step 2: Calculating the Distances Between Points

The foundation of this proof lies in computing the side lengths, which are obtained via the distance formula:

$$
\text{Distance between } (x_1, y_1) \text{ and } (x_2, y_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

Applying this:

2.1 Distance AB

$$
AB = \sqrt{(-16 - (-6))^2 + (12 - (-8))^2} = \sqrt{(-16 + 6)^2 + (12 + 8)^2} = \sqrt{(-10)^2 + (20)^2} = \sqrt{100 + 400} = \sqrt{500}
$$
$$
AB = \sqrt{500} = 10 \sqrt{5}
$$

2.2 Distance BC

$$
BC = \sqrt{(-26 - (-16))^2 + (-18 - 12)^2} = \sqrt{(-26 + 16)^2 + (-18 - 12)^2} = \sqrt{(-10)^2 + (-30)^2} = \sqrt{100 + 900} = \sqrt{1000}
$$
$$
BC = \sqrt{1000} = 10 \sqrt{10}
$$

2.3 Distance AC

$$
AC = \sqrt{(-26 - (-6))^2 + (-18 - (-8))^2} = \sqrt{(-26 + 6)^2 + (-18 + 8)^2} = \sqrt{(-20)^2 + (-10)^2} = \sqrt{400 + 100} = \sqrt{500}
$$
$$
AC = \sqrt{500} = 10 \sqrt{5}
$$

Step 3: Analyzing the Side Lengths

From the calculations:

- $AB = AC = 10\sqrt{5}$
- $BC = 10\sqrt{10}$

This immediately indicates that AB and AC are equal, which is a hallmark of an isosceles triangle. The side BC is longer, which suggests that the triangle has at least two equal sides, fulfilling the isosceles criterion.

Step 4: Verifying the Right-Angle Condition

To confirm whether the triangle is right-angled, we need to analyze the angles, specifically at the vertices. There are two approaches:

- Using the Pythagorean theorem: Check if the sum of the squares of the shorter sides equals the square of the longest side.
- Using slopes: Verify if the sides forming the right angle are perpendicular by checking the slopes.

4.1 Pythagorean Theorem Approach

Identify the longest side:

- BC is approximately $10\sqrt{10} \approx 31.62$
- AB and AC are both $10\sqrt{5} \approx 22.36$

Since BC is the longest, the hypotenuse candidate is BC. To confirm:

\[
AB^2 + AC^2 \stackrel{?}{=} BC^2
\]

Calculations:

\[
(10 \sqrt{5})^2 + (10 \sqrt{5})^2 = 2 \times (10^2 \times 5) = 2 \times 100 \times 5 = 2 \times 500 = 1000
\]
\[
BC^2 = (10 \sqrt{10})^2 = 100 \times 10 = 1000
\]

Since:

\[
AB^2 + AC^2 = BC^2 = 1000

\]

This confirms that the triangle satisfies the Pythagorean theorem, and the angle at vertex A is a right angle.

4.2 Slope Method for Perpendicularity

Check the slopes of the sides AB and AC to verify if they are perpendicular (product of slopes = -1):

$$
\text{Slope of AB} = m_{AB} = \frac{12 - (-8)}{-16 - (-6)} = \frac{20}{-10} = -2
$$

$$
\text{Slope of AC} = m_{AC} = \frac{-18 - (-8)}{-26 - (-6)} = \frac{-10}{-20} = \frac{1}{2}
$$

Product:

$$
m_{AB} \times m_{AC} = -2 \times \frac{1}{2} = -1
$$

Since the product is -1, the sides AB and AC are perpendicular, confirming the right angle at vertex A.

Step 5: Confirming the Isosceles Property

From the side lengths:

- AB = AC = $10\sqrt{5}$

This equality confirms that two sides are of equal length. The triangle, therefore, is isosceles, with the equal sides AB and AC meeting at a right angle at vertex A.

Step 6: Geometric Interpretation and Visualization

Understanding the geometric configuration helps in visualizing the triangle:

- Vertices: A (-6, -8), B (-16, 12), C (-26, -18)
- Right Angle at A: Confirmed by the perpendicular slopes and the Pythagorean relation.
- Isosceles: Sides AB and AC are equal, supporting the assertion.

Plotting these points on a coordinate plane would reveal a right-angled isosceles triangle with the right angle at A, sides AB and AC forming the equal legs, and BC as the hypotenuse.

Step 7: Additional Confirmations and Considerations

- Coordinate geometry provides a robust framework for such proofs.
- Distance calculations are precise and leave little room for ambiguity.
- Slope analysis offers an alternative verification of perpendicularity.
- The Pythagorean theorem confirms the right angle with algebraic clarity.
- Symmetry of points about the axes or lines can be explored further for deeper insights.

Conclusion

Through meticulous calculations and geometric reasoning, we have proven that the points (-6, -8), (-16, 12), and (-26, -18) are vertices of an isosceles right-angled triangle. The key findings include:

- Two sides are equal: AB and AC both measure $10\sqrt{5}$.
- The longest side BC measures $10\sqrt{10}$.
- The Pythagorean theorem holds true, confirming a right angle at vertex A.
- The slopes of sides AB and AC are negative reciprocals, confirming perpendicularity.

This comprehensive analysis not only verifies the specific nature of the triangle but also illustrates the power of coordinate geometry in classifying geometric figures based solely on coordinate points. Such methods are invaluable in mathematics, engineering, and physics, where precise spatial analysis is essential.

Final Remarks

Understanding the properties of triangles formed by arbitrary points in the coordinate plane is fundamental in various mathematical applications. The ability to prove the triangle's nature using algebraic and geometric tools enhances problem-solving skills and deepens comprehension of spatial relationships.

This proof underscores the importance of:

- Calculating distances accurately.
- Analyzing slopes for perpendicularity.
- Applying the Pythagorean theorem systematically.
- Recognizing geometric patterns and symmetries.

Armed with these techniques, students and practitioners can confidently analyze and classify complex geometric configurations in coordinate geometry.

End of detailed proof and analysis.

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