QUESTION 17 1 POINT What Is The Horizontal Asymptote Of The Graph Of F(x) = 4x +3 /9x8xGive Your Answer

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Understanding the concept of horizontal asymptotes is a fundamental part of analyzing rational functions in algebra. In this article, we will explore what horizontal asymptotes are, how to determine them, and specifically analyze the function $F(x) = \frac{4x + 3}{9x8x}$ to identify its horizontal asymptote. Whether you're a student seeking clarity on this topic or someone brushing up on rational functions, this comprehensive guide aims to provide clear explanations, step-by-step methods, and practical examples.

What Is a Horizontal Asymptote?

Definition of Horizontal Asymptote

A horizontal asymptote of a graph is a horizontal line that the graph approaches but does not necessarily touch as the input variable x approaches infinity ($+\infty$) or negative infinity ($-\infty$). It indicates the long-term behavior of the function.

Importance of Horizontal Asymptotes in Graphing

- They help predict the end behavior of the graph.
- They assist in sketching the overall shape of the graph.

- They are essential in understanding limits involving infinity.

Examples of Horizontal Asymptotes

- The function $f(x) = \frac{1}{x}$ has a horizontal asymptote at $y=0$.
- The function $f(x) = \frac{2x + 1}{x}$ has a horizontal asymptote at $y=2$.

Analyzing Rational Functions to Find Horizontal Asymptotes

General Approach

For rational functions of the form $f(x) = \frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are polynomials, the horizontal asymptote depends on the degrees of these polynomials.

Degree Comparison Method

1. Degree of numerator (n): The highest power of x in $P(x)$.
2. Degree of denominator (m): The highest power of x in $Q(x)$.

Based on the degrees, the rules are:
- If $n < m$, the horizontal asymptote is $y=0$.
- If $n = m$, the horizontal asymptote is $y=\frac{\text{leading coefficient of } P}{\text{leading coefficient of } Q}$.
- If $n > m$, there is no horizontal asymptote (the graph may tend toward infinity or negative infinity).

Special Considerations

- Sometimes, functions have oblique (slant) asymptotes when $n = m + 1$.

- For functions with vertical asymptotes, the horizontal asymptote describes behavior at large x, not near vertical asymptotes.

Understanding the Given Function $F(x) = \frac{4x + 3}{9x8x}$

Clarifying the Function's Form

The provided function appears as $F(x) = \frac{4x + 3}{9x8x}$. It seems there might be a formatting issue or typographical error. The likely intended function is:

$$
F(x) = \frac{4x + 3}{9x \times 8x}
$$

or

$$
F(x) = \frac{4x + 3}{(9x)(8x)}
$$

which simplifies to:

$$
F(x) = \frac{4x + 3}{72x^2}
$$

assuming the intention was to write $9x \times 8x$.

For clarity, we'll proceed with this interpretation:

$$
F(x) = \frac{4x + 3}{72x^2}
$$

Breakdown of the Function

- Numerator: $4x + 3$ (degree 1 polynomial).
- Denominator: $72x^2$ (degree 2 polynomial).

Determining the Horizontal Asymptote of $F(x) = \frac{4x + 3}{72x^2}$

Degree Comparison

- Numerator degree: $n = 1$.
- Denominator degree: $m = 2$.

Since $n < m$, according to the rules, the horizontal asymptote is:

$$
y = 0
$$

This means the graph of $F(x)$ approaches $y=0$ as $x \to \pm\infty$.

Mathematical Justification Using Limits

To confirm, we can evaluate the limits:

$$
\lim_{x \to \pm \infty} F(x) = \lim_{x \to \pm \infty} \frac{4x + 3}{72x^2}
$$

Dividing numerator and denominator by x^2:

$$
\lim_{x \to \pm \infty} \frac{\frac{4x}{x^2} + \frac{3}{x^2}}{72} = \lim_{x \to \pm \infty} \frac{\frac{4}{x} + \frac{3}{x^2}}{72}
$$

Since $\lim_{x \to \pm \infty} \frac{1}{x} = 0$ and $\lim_{x \to \pm \infty} \frac{1}{x^2} = 0$:

$$
\lim_{x \to \pm \infty} F(x) = \frac{0 + 0}{72} = 0
$$

Thus, confirming the horizontal asymptote at $y=0$.

Summary of the Key Concepts

Horizontal Asymptote Identification Steps

1. Express the function in simplified form, identifying numerator and denominator degrees.

2. Compare the degrees:

    - If numerator degree < denominator degree: asymptote at $y=0$.
    - If degrees are equal: asymptote at ratio of leading coefficients.
    - If numerator degree > denominator degree: no horizontal asymptote.

3. Use limits at infinity to confirm the behavior.

Importance of Accurate Function Format

Ensuring the function is written correctly is crucial for analysis. Misinterpretations can lead to incorrect conclusions about asymptotes.

Additional Examples and Practice Problems

Example 1: Find the horizontal asymptote of $f(x) = \frac{5x^3 + 2}{2x^3 - 7}$

- Both numerator and denominator have degree 3.
- Leading coefficients: 5 (numerator), 2 (denominator).
- Horizontal asymptote: $y = \frac{5}{2}$.

Example 2: Find the horizontal asymptote of $g(x) = \frac{7x + 1}{3x^2 + 4}$

- Numerator degree: 1.
- Denominator degree: 2.
- Since numerator degree < denominator degree: asymptote at $y=0$.

Conclusion

The key to understanding and identifying horizontal asymptotes lies in analyzing the degrees of polynomials in the numerator and denominator. For the function $F(x) = \frac{4x + 3}{72x^2}$, the numerator is degree 1, and the denominator is degree 2, leading to the conclusion that:

$$
\boxed{
\text{Horizontal asymptote at } y=0
}
$$

This aligns with the limit calculations and general rules for rational functions. Recognizing the pattern and applying these principles allows for quick and accurate determination of the end behavior of a wide variety of functions.

Final Thoughts

Understanding horizontal asymptotes is not just about memorizing rules but also about grasping the behavior of functions as they extend towards infinity. Practicing with different functions, verifying with limits, and ensuring correct function formatting are essential skills in mastering this topic. Whether analyzing simple rational functions or more complex expressions, the principles remain the same, providing a solid foundation for advanced calculus and graphing techniques.

Remember: Always verify the function's form carefully. Clarify ambiguous expressions, and use limits to confirm your findings. With consistent practice, identifying horizontal asymptotes will become an intuitive part of your mathematical toolkit.

Frequently Asked Questions

What is the horizontal asymptote of the function f(x) = (4x + 3) / (9x + 8)?

The horizontal asymptote is y = 4/9.

How do you determine the horizontal asymptote of a rational function like f(x) = (4x + 3) / (9x + 8)?

For rational functions, compare the degrees of numerator and denominator; if they are equal, the asymptote is the ratio of leading coefficients, which is 4/9 in this case.

What happens to the function f(x) = (4x + 3) / (9x + 8) as x approaches infinity?

As x approaches infinity, f(x) approaches 4/9, which is the horizontal asymptote.

Does the function f(x) = (4x + 3) / (9x + 8) have any other

asymptotes?

No, it only has a horizontal asymptote at y = 4/9; vertical asymptotes may exist where the denominator equals zero.

How can I verify the horizontal asymptote of f(x) = (4x + 3) / (9x + 8)?

You can analyze the end behavior by dividing numerator and denominator by x, then taking the limit as x approaches infinity.

What is the significance of the horizontal asymptote y = 4/9 in the graph of f(x) = (4x + 3) / (9x + 8)?

It indicates the value that the graph approaches but does not necessarily reach as x becomes very large or very small.

Is the horizontal asymptote y = 4/9 relevant for all x-values in the function f(x) = (4x + 3) / (9x + 8)?

No, the asymptote describes the behavior as x approaches infinity or negative infinity, not the function's value at finite x.

If the degree of numerator and denominator are equal, how do you find the horizontal asymptote?

Divide the leading coefficient of the numerator by that of the denominator; here, 4/9.

Additional Resources

Understanding the Horizontal Asymptote of the Function $f(x) = \frac{4x + 3}{9x - 8x}$

In the realm of mathematical analysis, especially within the domain of rational functions, the concept of asymptotes plays a crucial role in understanding the behavior of graphs at extreme values of x. Among these, the horizontal asymptote provides insight into the end behavior of a function as $x \to \pm \infty$. This article aims to thoroughly examine the specific function:

$$
f(x) = \frac{4x + 3}{9x - 8x}
$$

and determine its horizontal asymptote, while exploring the broader principles that underpin such analysis.

Understanding the Function: Simplification and Basic Characteristics

Before diving into the asymptotic behavior, it is essential to comprehend the structure of the given function.

Simplification of $f(x)$

The original function appears as:

\[

$$
f(x) = \frac{4x + 3}{9x - 8x}
$$

Notice that the denominator simplifies:

$$
9x - 8x = (9 - 8) x = 1x
$$

Therefore, the simplified form of the function is:

$$
f(x) = \frac{4x + 3}{x}
$$

This simplification is pivotal because it transforms the function into a more manageable form, enabling straightforward analysis of its end behavior.

Domain Considerations

The domain of $f(x)$ excludes any x that makes the denominator zero:

$$
x \neq 0
$$

Thus, the domain is:

$$
\{ x \in \mathbb{R} \mid x \neq 0 \}
$$

Analyzing End Behavior: Horizontal Asymptote Fundamentals

What Is a Horizontal Asymptote?

A horizontal asymptote of a function describes a horizontal line $y = L$ that the graph approaches as $x \to \pm \infty$. It indicates the behavior of the function at the far right or far left on the Cartesian plane.

How To Find the Horizontal Asymptote

The process depends on the degrees of the numerator and denominator:

- If the degree of the numerator is less than the degree of the denominator, the horizontal asymptote is $y=0$.
- If the degrees are equal, the asymptote is the ratio of the leading coefficients.
- If the degree of the numerator exceeds that of the denominator, the function does not have a horizontal asymptote (but may have an oblique/slant asymptote).

In our case, after simplification:

$$
f(x) = \frac{4x + 3}{x}
$$

the numerator and denominator are both degree 1 polynomials, making the degrees equal.

Calculating the Horizontal Asymptote for $f(x) = \frac{4x + 3}{x}$

Step 1: Express as a Polynomial Division

To analyze the behavior as $x \to \pm \infty$, it's common to perform polynomial division:

$$
\frac{4x + 3}{x}
$$

Dividing numerator by denominator:

$$
\frac{4x}{x} + \frac{3}{x} = 4 + \frac{3}{x}
$$

Step 2: Examine the Limit as $x \to \pm \infty$

The dominant term as $x \to \pm \infty$ is 4. The term $\frac{3}{x}$ approaches zero because:

$$
\lim_{x \to \pm \infty} \frac{3}{x} = 0
$$

Therefore, the entire function approaches:

$$
\lim_{x \to \pm \infty} f(x) = \lim_{x \to \pm \infty} \left(4 + \frac{3}{x} \right) = 4 + 0 = 4
$$

Conclusion: The horizontal asymptote is $y = 4$.

Graphical Interpretation and Significance of the Asymptote

Visualizing the Function

Graphing the function $f(x) = \frac{4x + 3}{x}$ reveals that as x moves towards large positive or negative values, the graph approaches the line $y=4$. The graph will have a vertical asymptote at $x=0$ (since the original denominator was zero there, and the function is undefined at that point), but the key focus here is the horizontal asymptote.

Significance in Real-World Contexts

Understanding the horizontal asymptote provides insights into the long-term behavior of the modeled phenomena. For instance, in economics, a rational function might model supply-demand equilibrium, and the asymptote could represent a limiting value or a steady-state level that the system approaches but never exceeds.

Broader Implications and Related Concepts

Asymptotes in Rational Functions

- Vertical asymptotes occur where the denominator is zero, and the function tends toward infinity or negative infinity.

- Oblique (slant) asymptotes appear when the degree of the numerator exceeds that of the denominator by exactly one.
- Horizontal asymptotes (as in this case) describe the behavior at the extremes of x.

Application in Calculus and Limit Analysis

The process of finding asymptotes hinges on limits, an essential concept in calculus. The limit approach allows mathematicians to describe the behavior of functions at points where they might otherwise be undefined or unbounded.

Limitations and Cautions

While asymptotes provide valuable insights, they do not depict the entire graph. For example, the function may have local maxima or minima, inflection points, or other features that are not captured solely by asymptotic analysis.

Summary and Final Remarks

The function $f(x) = \frac{4x + 3}{9x - 8x}$, which simplifies to $f(x) = \frac{4x + 3}{x}$, exhibits a clear horizontal asymptote at $y=4$. This conclusion stems from the degree comparison of numerator and denominator and the application of limit analysis, revealing that as $x \to \pm \infty$, the function approaches the line $y=4$. Recognizing such asymptotic behavior is fundamental in understanding the long-term trends of rational functions, which find applications across mathematics, physics, economics, and engineering.

In essence, the horizontal asymptote provides a window into the function's ultimate behavior, serving as a vital tool for mathematicians and practitioners alike to interpret and predict the behavior of complex systems modeled by rational functions.

Final Note: When analyzing any rational function, always consider simplifying the expression, examine the degrees of numerator and denominator, and utilize limits to accurately identify asymptotic behavior. This methodical approach ensures precise understanding and interpretation of the function's graph and real-world implications.

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