Question 4 1 Pts A Hydrogen Atom Has An Electron In The N =10 State. What Is The Speed Of This Electron

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Understanding the behavior of electrons in hydrogen atoms is a fundamental aspect of quantum mechanics and atomic physics. When an electron occupies a specific energy level, such as the N=10 state, its properties, including speed, can be derived using principles from Bohr's model and quantum theory. This article explores how to determine the speed of an electron in the N=10 orbital of a hydrogen atom, providing a comprehensive explanation suitable for students and enthusiasts alike.

Introduction to Hydrogen Atom Energy Levels

The hydrogen atom, being the simplest atomic system, serves as a foundational model in understanding atomic structures. It consists of one proton in the nucleus and one electron orbiting around it. The electron's energy levels are quantized, meaning it can only occupy specific discrete states characterized by the principal quantum number, N.

Principal Quantum Number (N)

- Defines the energy level of the electron.
- N=1 represents the ground state.
- Higher values of N correspond to higher energy, larger orbital radius, and lower electron binding energy.
- For N=10, the electron is in a relatively high-energy state, far from the nucleus.

Understanding Electron Speed in the Bohr Model

The Bohr model provides a simplified approach to calculating the properties of electrons in hydrogen-like atoms. It assumes the electron moves in circular orbits around the nucleus, with quantized angular momentum.

Key Assumptions of the Bohr Model

- Electron orbits are stable and quantized.
- The angular momentum of the electron is an integer multiple of $\hbar$ (reduced Planck's constant).
- The electrostatic force provides the necessary centripetal force for circular motion.

Formula for Electron Speed in Nth Orbit

The velocity v of an electron in the Nth orbit of a hydrogen atom is given by:

$$
v = \frac{e^2}{2 \varepsilon_0 h} \times \frac{1}{N}
$$

Alternatively, a more directly applicable formula derived from Bohr's theory is:

$$
v_N = \frac{Z e^2}{2 \varepsilon_0 h} \times \frac{1}{N}
$$

where:

- Z is the atomic number (for hydrogen, $Z=1$)
- e is the elementary charge ($1.602 \times 10^{-19} \, C$)
- ε_0 is the vacuum permittivity ($8.854 \times 10^{-12} \, C^2/(N \cdot m^2)$)
- h is Planck's constant ($6.626 \times 10^{-34} \, Js$)
- N is the principal quantum number

However, a more straightforward and common formula used is:

$$
v_N = \frac{Z e^2}{2 \varepsilon_0 h} \times \frac{1}{N}
$$

But to simplify calculations, physicists often use the known velocity in the first orbit and scale accordingly.

Calculating Electron Speed at N=10

To find the speed of an electron in the N=10 state, it's easiest to start from the known value of the electron's speed in the ground state (N=1) and scale it by $1/N$.

Known Electron Speed in the Ground State

The speed of the electron in the ground state (N=1) of hydrogen is approximately:

$$
v_1 \approx 2.18 \times 10^6 \, \text{m/s}
$$

This value is obtained from the Bohr model and experimental data.

Scaling to N=10

Since the electron's speed in the Bohr model decreases with increasing N as:

$$
v_N = \frac{v_1}{N}
$$

we can calculate the speed at N=10 as:

$$
v_{10} = \frac{2.18 \times 10^6 \, \text{m/s}}{10} = 2.18 \times 10^5 \, \text{m/s}
$$

Therefore, the electron in the N=10 state of a hydrogen atom moves at approximately 218,000 meters per second.

Implications of Electron Speed at High Energy States

Understanding the speed of electrons at high quantum numbers like N=10 has several significant implications:

- Quantum Behavior: Despite the high energy, the electron still exhibits wave-like properties.
- Orbital Size: The radius of the orbit increases with N, approximately proportional to N^2. For N=10, the orbital radius is much larger than the ground state.
- Electron Dynamics: The electron moves at a fraction of the speed of light, validating non-relativistic approximations at these energy levels.

Additional Considerations in Electron Dynamics

While the Bohr model provides a useful approximation, modern quantum mechanics describes electrons as wavefunctions rather than particles moving in well-defined orbits.

Quantum Mechanical Perspective

- The electron's position is described by a probability cloud.
- The concept of a "speed" becomes less precise, but the expectation value of velocity can be estimated.
- For high N states, the classical analogy remains useful for understanding average behavior.

Limitations of the Bohr Model

- Does not account for electron spin or relativistic effects.
- Not accurate for multi-electron systems.
- Serves primarily as an introductory model for understanding hydrogen-like atoms.

Summary and Key Takeaways

- The electron in a hydrogen atom's N=10 state moves at approximately 218,000 m/s.
- The velocity scales inversely with the principal quantum number N.
- The value is derived from Bohr's model, which, despite its limitations, offers valuable insights into atomic structure.
- Understanding electron speed helps in grasping atomic behavior, spectral lines, and quantum mechanics fundamentals.

Conclusion

Determining the speed of an electron in a high-energy state like N=10 is crucial for understanding atomic physics and quantum mechanics. Using the principles established by Bohr, we find that the electron's speed decreases with increasing energy levels, resulting in a velocity of roughly 2.18 x 10^5 meters per second at N=10. While modern quantum mechanics offers a more nuanced picture, the Bohr model remains a vital educational tool to conceptualize atomic behavior.

FAQs

Q1: Is the electron in the N=10 state moving fast enough to be relativistic?
No, at approximately 218,000 m/s, the electron's speed is much less than the speed of light, so relativistic effects are negligible.

Q2: How does the orbital radius change at N=10?
The radius increases proportional to \(N^2\), so the N=10 orbit is about 100 times larger than the ground state orbit.

Q3: Can we observe electrons moving at this speed directly?
No, electrons are quantum particles, and their wave-like nature makes direct observation of speed at this scale impossible. However, their energy levels and spectral lines provide indirect evidence.

Q4: How accurate is the Bohr model for calculating electron speed?
While useful for basic estimates and hydrogen-like systems, the Bohr model has limitations. Quantum mechanics provides more precise descriptions, especially for complex atoms.

By comprehending the principles behind electron motion at various energy states, students and researchers can deepen their understanding of atomic structure, spectral phenomena, and the quantum nature of matter.

Frequently Asked Questions

What is the main goal of Question 4 regarding the hydrogen atom's electron?

To determine the speed of an electron in the N=10 energy level of a hydrogen atom.

Which physical principle is used to find the electron's speed in the N=10 state?

The Bohr model of the hydrogen atom, which relates energy levels to radius and velocity, is used.

What is the formula to calculate the speed of an electron in a given energy level in a hydrogen atom?

$v = (e^2) / (2\varepsilon_0 h) (1/n)$, derived from Bohr's model, where n is the energy level.

What constants are needed to calculate the electron's speed in this problem?

The elementary charge (e), permittivity of free space (ε_0), Planck's constant (h), and the principal quantum number (n=10).

Why does the energy level N=10 matter when calculating the electron's speed?

Because the speed depends on the orbital radius, which increases with N, affecting the electron's velocity.

Is the calculated speed of the electron in N=10 within a measurable range?

Yes, it is typically on the order of hundreds of kilometers per second, measurable with precise instruments.

How does increasing the principal quantum number N affect the electron's speed?

As N increases, the electron's orbital radius increases, and its speed decreases accordingly.

Can you provide the approximate numerical value for

the electron's speed at N=10?

Approximately 4.7 × 10^5 meters per second (470 km/s), based on calculations using Bohr's model.

Additional Resources

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Understanding the behavior of electrons within atoms is fundamental to quantum mechanics and atomic physics. When examining a hydrogen atom with its electron in a specific energy state, such as the principal quantum number $n=10$, it becomes intriguing to determine the characteristics of the electron, particularly its speed. This question invites us to delve into the quantum mechanical model of the atom, analyze the relevant equations, and interpret the physical significance of these calculations. This article provides a comprehensive review of how to compute the electron's speed in the $n=10$ state of a hydrogen atom, including the underlying principles, calculations, implications, and broader context.

Understanding the Quantum State of the Electron in Hydrogen

Principal Quantum Number n

In quantum mechanics, electrons in atoms are described by wavefunctions characterized by a set of quantum numbers. The principal quantum number n primarily determines the energy level and average distance of the electron from the nucleus. For hydrogen, which is a one-electron system, the energy levels are given by:

$$
E_n = - \frac{13.6\, \text{eV}}{n^2}
$$

where $13.6\, \text{eV}$ is the ionization energy of hydrogen.

Given $n=10$, the electron resides in a relatively high energy state, farther from the nucleus, with a correspondingly larger orbital radius and different dynamical properties compared to lower-energy states.

Wavefunction and Orbital Characteristics

The wavefunction for the hydrogen atom's electron in the $n=10$ state can be described using solutions to the Schrödinger equation. These solutions involve radial and angular parts, with the radial part dominantly influencing the electron's average distance from the nucleus.

While the full wavefunction is complex, for classical approximations such as calculating the average speed of the electron, a simplified model assuming a Bohr-like orbit can be employed, especially considering the hydrogen atom's solvable nature and the correspondence principle.

Classical Model Approximation: The Bohr Model

Bohr Model Fundamentals

The Bohr model, developed in the early 20th century, combines classical physics with quantization rules to describe the hydrogen atom. Despite its limitations, it provides valuable estimates of the electron's velocity in different energy levels.

In the Bohr model, the electron orbits the nucleus in a circular path with a quantized angular momentum:

$$
L = n \hbar
$$

where $\hbar$ is the reduced Planck's constant.

The radius of the orbit (the Bohr radius for $n=1$) scales with n^2:

$$
r_n = n^2 a_0
$$

where $a_0 \approx 0.529 \times 10^{-10}\,\text{m}$ is the Bohr radius.

The velocity of the electron in the n-th orbit is then:

$$
v_n = \frac{e^2}{2 \varepsilon_0 h n}
$$

or more simply,

\[

v_n = \frac{Z e^2}{2 \varepsilon_0 h n}
\]

which for hydrogen simplifies to:

\[
v_n = \frac{e^2}{2 \varepsilon_0 h n}
\]

Alternatively, using known constants, the velocity can be expressed as:

\[
v_n = \frac{13.6\, \text{eV}}{n} \times \text{constant}
\]

but a more precise calculation involves detailed constants.

Pros of the Bohr Model:
- Provides a straightforward way to estimate electron speeds.
- Offers intuitive insight into the quantization of energy levels.
- Useful for high-level approximations and educational purposes.

Cons of the Bohr Model:
- Cannot accurately predict phenomena like fine structure or electron probability distributions.
- Oversimplifies the electron's wave nature and quantum behavior.
- Not valid for multi-electron atoms or relativistic scenarios.

Calculating the Electron Speed at \(n=10 \)

Using the Bohr Model Formula

For hydrogen, the speed of the electron in the \(n \)-th orbit can be estimated by the relation:

\[
v_n = \frac{v_1}{n}
\]

where the ground state velocity \(v_1 \) is approximately:

\[
v_1 \approx 2.18 \times 10^6\, \text{m/s}
\]

Therefore, for \(n=10 \):

\[

$$
v_{10} = \frac{2.18 \times 10^6\, \text{m/s}}{10} = 2.18 \times 10^5\, \text{m/s}
$$

This approximation suggests that the electron moves at about 218,000 meters per second in the $n=10$ state.

Key Points:
- The higher the quantum number, the slower the electron's average speed.
- The speed decreases proportionally with increasing n.

Quantum Mechanical Perspective

From a purely quantum mechanical standpoint, the electron doesn't traverse a classical orbit but exists as a probability cloud. However, the expectation value of the electron's momentum and related kinetic energy can be calculated, which ties into its "average speed."

The kinetic energy KE for the electron in the n state is:

$$
KE = -E_n = \frac{13.6\, \text{eV}}{n^2}
$$

which for $n=10$:

$$
KE = \frac{13.6\, \text{eV}}{100} = 0.136\, \text{eV}
$$

Converting this to joules:

$$
KE = 0.136 \times 1.602 \times 10^{-19}\, \text{J} \approx 2.18 \times 10^{-20}\, \text{J}
$$

Since $KE = \frac{1}{2} m v^2$, solving for v:

$$
v = \sqrt{\frac{2 KE}{m_e}}
$$

where $m_e = 9.11 \times 10^{-31}\, \text{kg}$:

$$
v = \sqrt{\frac{2 \times 2.18 \times 10^{-20}}{9.11 \times 10^{-31}}} \approx \sqrt{4.78 \times 10^{10}} \approx 2.18 \times 10^5\, \text{m/s}
$$

which corroborates the Bohr model approximation.

Implications:
- Both classical and quantum approaches converge to similar velocity estimates.
- These velocities are a small fraction of the speed of light (~0.07%), confirming non-relativistic treatment suffices.

Physical Significance and Broader Context

Relevance of Electron Speed

Knowing the electron's speed in a given energy state helps in understanding:
- The size of the electron's orbit or probability cloud.
- The energy distribution within the atom.
- Transition probabilities during excitation or emission.

For the \(n=10 \) state, the electron's large orbital radius (~\(100 a_0 \)) and relatively slow speed reflect a more diffuse electron probability distribution compared to lower states like \(n=1 \).

Limitations of the Approximations

While these calculations provide useful estimates, they are based on models that simplify the true quantum nature of electrons:
- The Bohr model neglects wave-particle duality.
- The quantum mechanical description involves probability densities, not definite orbits.
- Real electron velocities fluctuate, and the concept of a precise "speed" is a classical approximation.

Conclusion

The calculation of the electron's speed in the hydrogen atom at the \(n=10 \) energy level illustrates the interplay between classical models and quantum mechanics. Using fundamental constants and well-established formulas, the estimated speed is approximately 218,000 m/s. This value highlights that electrons in higher energy states orbit or, more accurately, exist with lower average velocities and larger spatial distributions. Understanding these dynamics is crucial in atomic physics, spectroscopy, and quantum chemistry, providing insights into atomic structure and electron behavior.

Summary of Key Points:
- The electron's speed at \(n=10 \) is roughly \(2.18 \times 10^5\, \text{m/s} \).
- Both classical Bohr model and quantum kinetic energy calculations agree.

- The electron is relatively slow compared to the speed of light, validating non-relativistic assumptions.
- These insights are essential for interpreting atomic spectra and understanding quantum states.

Final Thoughts:
While simplified models like the Bohr model serve as useful tools for estimation, the true nature of electrons in atoms is governed by quantum mechanics, where they are described by wavefunctions and probability densities. Nevertheless, such calculations remain integral to foundational physics education and provide a bridge between classical intuition and quantum reality.

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