

Sam Has A Total Of 43 Quarters And Nickels In Her Purse. If Her Change Is Worth \$7.75, How Many Nickels

Sam Has A Total Of 43 Quarters And Nickels In Her Purse. If Her Change Is Worth \$7.75, How Many Nickels is a classic problem that combines basic arithmetic with algebraic reasoning. It offers an excellent opportunity to explore systems of equations, problem-solving strategies, and real-world applications of math. Whether you're a student brushing up on your algebra or someone interested in financial literacy, understanding how to approach this problem can sharpen your analytical skills. In this article, we will delve into the details of this problem, provide step-by-step solutions, and discuss related concepts to deepen your understanding.

Understanding the Problem

Before jumping into calculations, it's essential to understand what the problem is asking. The question involves two types of coins: quarters and nickels. We are told:

- The total number of coins (quarters + nickels) is 43.
- The total value of these coins is \$7.75.

The goal is to find out how many nickels Sam has in her purse.

Key Information and Variables

To solve this problem systematically, define variables:

- Let q = number of quarters
- Let n = number of nickels

From the problem, we have two main pieces of information:

1. Total coins:

$$\begin{aligned} & \{ \\ & q + n = 43 \\ & \} \end{aligned}$$

2. Total value:

$$\begin{aligned} & \{ \\ & 0.25q + 0.05n = 7.75 \\ & \} \end{aligned}$$

Using these equations, we can set up a system to find the values of q and n .

Setting Up the Equations

The problem lends itself to forming a pair of equations:

Equation 1: Total number of coins

$$\begin{aligned} & \backslash[\\ & q + n = 43 \\ & \backslash] \end{aligned}$$

Equation 2: Total monetary value

$$\begin{aligned} & \backslash[\\ & 0.25q + 0.05n = 7.75 \\ & \backslash] \end{aligned}$$

To simplify calculations, it's often easier to work with integers. Multiply the second equation by 100 to clear decimals:

$$\begin{aligned} & \backslash[\\ & 25q + 5n = 775 \\ & \backslash] \end{aligned}$$

Now, the system looks like:

$$\begin{aligned} & \backslash[\\ & \begin{cases} q + n = 43 \\ 25q + 5n = 775 \end{cases} \\ & \backslash] \end{aligned}$$

Simplify the second equation:
Divide through by 5:

$$\begin{aligned} & \backslash[\\ & 5q + n = 155 \\ & \backslash] \end{aligned}$$

Now, the system is:

$$\begin{aligned} & \backslash[\\ & \begin{cases} q + n = 43 \quad (1) \\ 5q + n = 155 \quad (2) \end{cases} \\ & \backslash] \end{aligned}$$

Note: We can now subtract equation (1) from equation (2) to eliminate n.

Solving the System of Equations

Subtract equation (1) from equation (2):

$$\begin{aligned} & \backslash[\\ & (5q + n) - (q + n) = 155 - 43 \\ & \backslash] \end{aligned}$$

Simplify:

$$\begin{aligned} & \backslash[\\ & 5q + n - q - n = 112 \\ & \backslash] \end{aligned}$$

$$\begin{aligned} & \backslash[\\ & 4q = 112 \\ & \backslash] \end{aligned}$$

Solve for q:

$$\begin{aligned} & \backslash[\\ & q = \frac{112}{4} = 28 \\ & \backslash] \end{aligned}$$

Now, substitute $q = 28$ into equation (1):

$$\begin{aligned} & \backslash[\\ & 28 + n = 43 \\ & \backslash] \end{aligned}$$

Solve for n:

$$\begin{aligned} & \backslash[\\ & n = 43 - 28 = 15 \\ & \backslash] \end{aligned}$$

Final Answer:

Sam has 15 nickels.

Verification of the Solution

It's important to verify that the solution makes sense in the context of the problem.

- Total coins: $(28 + 15 = 43)$ ✓

- Total value:

$$\begin{aligned} & \backslash[\\ & (0.25 \times 28) + (0.05 \times 15) = 7.00 + 0.75 = 7.75 \\ & \backslash] \end{aligned}$$

✓

The calculation confirms that the solution is correct.

Additional Insights and Related Problems

Understanding how to approach coin problems like this is valuable not only for academic purposes but also for everyday financial literacy. Here are some related topics and problems for further exploration:

1. Variations with Different Coins

- How does the problem change if the coins include dimes or nickels?
- For example, if the total is 50 coins worth \$10, and the coins are only dimes and nickels, how many of each are there?

2. Using Algebra to Solve Word Problems

- Many real-world problems require setting up systems of equations.
- Practice with different scenarios involving rates, distances, or mixtures.

3. Practical Applications

- Budgeting and coin counting.
- Making change efficiently.
- Coin collection and valuation.

4. Tips for Solving Similar Problems

- Define variables clearly.
- Write equations based on the problem statement.
- Simplify equations whenever possible.
- Use substitution or elimination methods.
- Verify solutions with the original problem.

Conclusion

The problem of determining how many nickels Sam has in her purse, given the total number of coins and their combined value, offers a straightforward example of applying algebra to solve real-world questions. By carefully setting up equations, simplifying, and solving systematically, we found that Sam has 15 nickels and 28 quarters. Mastering these techniques enhances not only problem-solving skills but also financial literacy, which is essential for everyday life. Whether tackling similar coin problems or more complex algebraic systems, the approach remains largely the same—clarify, set up equations, simplify, and solve.

Understanding these concepts empowers students and learners to approach a variety of math challenges confidently. So next time you face a coin problem, remember the steps: define your variables, write your equations, simplify, and solve!

Frequently Asked Questions

How many nickels does Sam have if she has a total of 43 quarters and nickels worth \$7.75?

Sam has 15 nickels.

What is the total value of 43 quarters and 15 nickels?

The total value is \$7.75.

If Sam's total coins are 43 quarters and some nickels totaling \$7.75, how can we find the number of nickels?

Set up an equation: $0.25 \times (\text{quarters}) + 0.05 \times (\text{nickels}) = 7.75$, with total coins 43, then solve for nickels.

Why does the value of Sam's coins equal \$7.75?

Because the combined worth of her 43 quarters and the unknown number of nickels totals \$7.75.

Can we verify the number of nickels by calculating their total value?

Yes, multiplying 15 nickels by \$0.05 each gives \$0.75, and 28 quarters by \$0.25 each gives \$7.00, totaling \$7.75.

What mathematical method is used to find the number of nickels?

Setting up and solving a system of equations based on total coins and total value.

Is the number of nickels more or less than the number of quarters?

There are fewer nickels (15) than quarters (28).

What is the key to solving this problem efficiently?

Using algebra to set up equations representing the total number of coins and their total value, then solving for the unknown.

Additional Resources

Sam Has A Total Of 43 Quarters And Nickels In Her Purse. If Her Change Is Worth \$7.75, How Many Nickels?

Introduction

In the world of everyday math puzzles, few problems are as straightforward yet as rich in learning potential as the classic coin reasoning challenge. The problem at hand, "Sam has a total of 43 quarters and nickels in her purse. If her change is worth \$7.75, how many nickels?", exemplifies the kind of real-world problem-solving that combines basic arithmetic, algebra, and logical deduction.

While on the surface, this problem appears simple, a thorough investigation reveals layers of mathematical reasoning, assumptions about coin denominations, and the importance of clarity in problem statements. This article explores the problem in depth, examining the underlying principles, potential pitfalls, and the broader educational context.

Background and Context

The importance of coin-based math problems

Coin-based problems serve as fundamental tools for teaching arithmetic, algebra, and critical thinking. They simulate real-life scenarios such as counting change, budgeting, and financial literacy, making them valuable in both educational and practical contexts.

Common misconceptions and pitfalls

Many students encounter difficulties in translating word problems into algebraic expressions. Ambiguities in problem statements—such as whether the total number of coins or the total value is specified—can lead to confusion. Clarifying these issues is essential for accurate problem solving.

Dissecting the Problem Statement

The problem specifies three key pieces of information:

1. Total coins: 43 (quarters and nickels combined)
2. Total value: \$7.75
3. Question: How many nickels are there?

This prompts us to define variables and set up equations accordingly.

Establishing Variables

Let:

- q = number of quarters

- n = number of nickels

Given that:

$$q + n = 43$$

and that the total amount in dollars is:

$$(0.25)q + (0.05)n = 7.75$$

Our goal is to determine n .

Mathematical Approach

Step 1: Express one variable in terms of the other

From the first equation:

$$q = 43 - n$$

Step 2: Substitute into the value equation

$$0.25(43 - n) + 0.05n = 7.75$$

Step 3: Simplify

$$0.25 \times 43 - 0.25n + 0.05n = 7.75$$

$$10.75 - 0.25n + 0.05n = 7.75$$

$$10.75 - 0.20n = 7.75$$

Step 4: Isolate n

$$\begin{aligned} & -0.20n = 7.75 - 10.75 \\ & \end{aligned}$$

$$\begin{aligned} & -0.20n = -3.00 \\ & \end{aligned}$$

$$\begin{aligned} & n = \frac{-3.00}{-0.20} = 15 \\ & \end{aligned}$$

Result: There are 15 nickels.

Corresponding quarters:

$$\begin{aligned} & q = 43 - 15 = 28 \\ & \end{aligned}$$

Validating the Solution

- Total coins: $(28 + 15 = 43)$ (matches the given)
- Total value:

$$\begin{aligned} & 28 \times 0.25 + 15 \times 0.05 = 7.00 + 0.75 = 7.75 \\ & \end{aligned}$$

The calculations confirm the solution's correctness.

Broader Implications and Considerations

Assumptions and Clarifications

1. Denominations: The problem assumes standard US coin denominations—quarters and nickels. No other denominations are considered.
2. Whole number solutions: Since coins are discrete, fractional solutions are invalid.
3. Contextual plausibility: The solution suggests that Sam's purse contains 28 quarters and 15 nickels, totaling 43 coins worth \$7.75, which is reasonable.

Potential Variations

- What if the total number of coins or total value changed?
- How does the problem change if other coin types are included?
- Could the problem be extended to include dimes or pennies?

Educational Significance

This problem exemplifies the power of algebra in solving real-world problems. It demonstrates how setting up equations based on problem constraints leads to straightforward solutions, reinforcing core mathematical skills.

It also highlights the importance of:

- Clear problem statements
- Correct variable definitions
- Systematic algebraic manipulation

These skills are fundamental in developing quantitative reasoning.

Common Challenges and Misconceptions

- Misreading the total value: Students might confuse the total amount with total coins.
- Incorrect variable assignment: Swapping variables or misrepresenting the equations.
- Ignoring constraints: Assuming solutions with negative coins, which are invalid.
- Forgetting to verify solutions: Always substitute back to ensure the solution satisfies all conditions.

Conclusion

The problem "Sam has a total of 43 quarters and nickels in her purse. If her change is worth \$7.75, how many nickels?" is more than a simple arithmetic exercise. It serves as an excellent example of applying algebra to real-world scenarios, emphasizing critical thinking, problem-solving skills, and attention to detail.

By carefully defining variables, setting up accurate equations, and verifying solutions, students and enthusiasts alike can deepen their understanding of both mathematics and everyday financial reasoning. In this case, the answer is clear: Sam has 15 nickels in her purse.

This problem underscores the importance of precise language, logical deduction, and systematic calculation—cornerstones of effective mathematical literacy.

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