Select The Correct Answer. The Function G(x) = X2 Is Transformed To Obtain Function H: H(x) = G(x + 7).

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Understanding the Transformation of Functions: G(x) to H(x)

When analyzing functions in algebra, understanding how transformations affect the graph is fundamental. The transition from a basic function such as $G(x) = x^2$ to a transformed version $H(x) = G(x + 7)$ illustrates core concepts in function transformations, including shifts and translations. This article delves into the nature of this transformation, explaining the concept step-by-step and providing clarity on how the original function G(x) is modified to produce H(x).

The Basic Function $G(x) = x^2$

What is G(x)?

The function $G(x) = x^2$ is a quadratic function, representing a parabola that opens upward. Its key features include:

- Vertex: Located at the origin (0,0).
- Axis of symmetry: The vertical line $x = 0$.
- Range: $y \geq 0$.
- Domain: All real numbers, $\mathbb{R}$.

Graphically, G(x) creates a symmetric U-shaped curve centered at the origin.

Graph of $G(x) = x^2$

Understanding the basic graph helps visualize the effect of transformations:

- For $x = 0$, $G(0) = 0$.
- For $x = 1$, $G(1) = 1$.
- For $x = -1$, $G(-1) = 1$.

- The parabola extends infinitely upward on both sides.

Transformation of $G(x)$ to $H(x)$: The Role of Horizontal Shifts

Definition of the Transformation

Given the function:

$$
H(x) = G(x + 7),
$$

we are asked to understand how this modifies the original function $G(x) = x^2$.

Key Concept: Horizontal Shift

The transformation from $G(x)$ to $H(x)$ involves a horizontal shift of the graph:

- Replacing x with $(x + 7)$ shifts the graph horizontally.
- The sign of the addition determines the direction:
- If the argument is $(x + k)$, the graph shifts to the left by k units.
- If the argument is $(x - k)$, the graph shifts to the right by k units.

In our case, since the expression inside the function is $(x + 7)$, the graph of $G(x)$ shifts 7 units to the left.

Why a Shift to the Left?

Mathematically, the transformation:

$$
H(x) = G(x + 7),
$$

means for each x-value, the output is determined by evaluating G at $(x + 7)$. To find the corresponding original x-value, set:

$$
x + 7 = t,
$$

where t is the original input for $G(t)$. So,

$$
t = x + 7,
$$

which indicates that for a given x, the parabola's point corresponds to the point on G at $t = x + 7$. As x increases, t increases as well, but the entire graph is shifted leftward because the input is effectively being offset by +7.

Visualizing the Transformation

Graphical Explanation

- The original graph of $G(x) = x^2$ is centered at (0,0).
- The transformed graph $H(x) = G(x + 7)$ shifts this parabola 7 units to the left.
- The vertex, originally at (0,0), moves to (-7, 0).

Impact on Key Features

Original G(x)	Transformed H(x) = G(x + 7)
Vertex at (0, 0)	Vertex at (-7, 0)
Axis of symmetry at x = 0	Axis of symmetry at x = -7
Graph shifted left by 7 units	Graph's position changed accordingly

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Step-by-Step Analysis of the Transformation

1. Identify the original function: $G(x) = x^2$.
2. Apply the transformation: Replace x with (x + 7).
3. Interpret the effect:
- The (+7) inside the argument causes a shift to the left.
- The shape of the parabola remains unchanged; only its position shifts.
4. Determine the new vertex:
- Original vertex at (0, 0).
- New vertex at (-7, 0).
5. Understand the symmetry:
- Symmetry line shifts from x = 0 to x = -7.

Mathematical Explanation of Horizontal Shifts

General Rule for Horizontal Shifts

For a function $f(x)$:

- Shift left by k units: $f(x + k)$.
- Shift right by k units: $f(x - k)$.

Application to $G(x) = x^2$

- The transformation:

$$
H(x) = G(x + 7) = (x + 7)^2,
$$

shifts the parabola 7 units to the left.

Why does this work?

- The input x in $H(x)$ corresponds to the input t in $G(t)$, where:

$$
t = x + 7.
$$

- When x increases by 1, t increases by 1, maintaining the shape but shifting the graph.

Real-World Examples of Horizontal Shifts

Understanding the practical implications of such transformations helps grasp their importance:

- Physics: Position functions shifting in time.
- Economics: Cost or revenue functions shifted due to external factors.
- Engineering: Signal processing involving shifting signals.

In all cases, the core idea is that changing the input variable shifts the graph horizontally, affecting the location but not the shape.

Practice Problems: Applying the Concept

1. Identify the transformation:

Given $G(x) = x^2$, what is the effect of $H(x) = G(x - 4)$?

- Answer: Shift the parabola 4 units to the right.

2. Graph interpretation:

Where is the vertex of $H(x) = (x + 3)^2$?

- Answer: At (-3, 0).

3. Transformation description:

Describe the transformation from $G(x) = x^2$ to $H(x) = (x - 5)^2$.

- Answer: Shift the parabola 5 units to the right.

Summary and Key Takeaways

Aspect	Description
Original function	$G(x) = x^2$
Transformed function	$H(x) = G(x + 7)$
Transformation type	Horizontal shift (translation)
Direction of shift	7 units to the left
Vertex movement	From (0, 0) to (-7, 0)
Effect on graph	Same shape, shifted left

Understanding how the transformation $H(x) = G(x + 7)$ modifies the original quadratic function is essential for mastering function transformations. Recognizing the shift to the left by 7 units allows for accurate graphing and analysis of quadratic functions in various contexts.

Additional Resources for Mastery

- Graphing Quadratic Functions: Practice plotting $G(x) = x^2$ and $H(x) = (x + 7)^2$.
- Transformation Rules: Memorize the rules for shifting, reflecting, and stretching functions.

- Interactive Tools: Use graphing calculators or online graphing tools to visualize transformations dynamically.

Conclusion

Transforming functions by modifying their input arguments, such as replacing x with (x + 7), results in horizontal shifts of the graph. In the case of $G(x) = x^2$, the transformation to $H(x) = G(x + 7)$ shifts the parabola 7 units to the left, moving its vertex from (0, 0) to (-7, 0). Recognizing these transformations enhances understanding of function behavior and graphing skills, which are fundamental in algebra and calculus.

Remember: Whenever you see a function of the form $f(x + k)$, think shift to the left by k units; for $f(x - k)$, think shift to the right by k units. This simple rule is a cornerstone in understanding and visualizing function transformations effectively.

Frequently Asked Questions

What is the original function G(x) given in the problem?

G(x) = x^2

How is the transformed function H(x) related to G(x)?

H(x) is obtained by replacing x with (x + 7) in G(x), so H(x) = G(x + 7).

What type of transformation is applied to G(x) to get H(x)?

A horizontal shift (or translation) of the graph to the left by 7 units.

What is the explicit expression for H(x)?

H(x) = (x + 7)^2

Does the transformation involve a vertical or horizontal shift?

A horizontal shift.

In the function $H(x) = G(x + 7)$, what does adding 7 inside the argument do to the graph of G?

It shifts the graph to the left by 7 units.

If $G(x)$ is a parabola opening upwards, how does the graph of $H(x)$ compare?

It is the same parabola shifted 7 units to the left.

What is the key concept demonstrated by transforming $G(x)$ to $H(x)$ in this problem?

Understanding horizontal shifts in function transformations.

Additional Resources

Select The Correct Answer. The Function $G(x) = X^2$ Is Transformed To Obtain Function H: $H(x) = G(x + 7)$.

Understanding how functions transform is a fundamental aspect of algebra that enables students and professionals alike to manipulate and interpret mathematical models effectively. In particular, transformations involving shifts, stretches, and reflections are core concepts that reveal how the graph of a basic function can be modified to produce new functions with desired properties. The problem at hand involves transforming the basic quadratic function $G(x) = x^2$ into a new function $H(x)$, defined as $H(x) = G(x + 7)$. This seemingly straightforward substitution opens a window into the mechanics of function transformations, specifically horizontal shifts.

In this article, we will delve into the details of this transformation, explain the underlying principles, and guide you through identifying the correct form of the transformed function. By the end, you'll have a comprehensive understanding of how the transformation $G(x) \to H(x) = G(x + 7)$ affects the graph of the original quadratic function and how to determine the resulting function accurately.

Understanding the Basic Function $G(x) = x^2$

Before exploring the transformation, it is essential to understand the fundamental characteristics of the base function $G(x) = x^2$.

The Graph of $G(x) = x^2$

- Shape and Features: The graph is a parabola opening upwards with its vertex at the origin (0, 0).
- Symmetry: It is symmetric with respect to the y-axis.
- Domain and Range: The domain is all real numbers ($-\infty, \infty$), and the range is $[0, \infty)$.
- Key points: For example, $G(0) = 0$, $G(1) = 1$, $G(-1) = 1$.

This basic understanding allows us to visualize how transformations will modify this fundamental shape.

Deciphering the Transformation $H(x) = G(x + 7)$

The transformation involves replacing the input x in $G(x)$ with $x + 7$. This substitution is more than algebraic; it signifies a shift in the graph's position.

What Does the $x + 7$ Inside the Function Mean?

- The expression $G(x + 7)$ indicates that for each value of x, instead of plugging in x, we're plugging in $x + 7$ into the original function.
- This is a standard horizontal transformation, often called a horizontal shift or translation.

Horizontal Shift Explained

- When the function is $G(x + c)$, the graph shifts horizontally.
- Direction of shift:
- If $c > 0$, the graph shifts to the left by c units.
- If $c < 0$, the graph shifts to the right by $|c|$ units.

In our case, $c = 7$, which is positive, so the graph shifts to the left by 7 units.

Visualizing the Transformation

Imagine the original parabola centered at the origin. When we replace x by $x + 7$, each point on the original parabola moves:

- The point (x, x^2) on the original graph becomes $(x, (x)^2)$.
- On the transformed graph, the point corresponding to x now occurs at $(x, (x + 7)^2)$.

This means that the entire parabola shifts leftward, so the vertex at $(0, 0)$ moves to $(-7, 0)$.

Key observations:
- The shape of the parabola remains unchanged.
- The vertex moves from $(0, 0)$ to $(-7, 0)$.
- The parabola is still symmetric about the vertical line $x = -7$.

Mathematical Representation of the Transformed Function

Given the original function:

$$G(x) = x^2$$

and the transformation:

$$H(x) = G(x + 7)$$

we substitute directly:

$$H(x) = (x + 7)^2$$

This quadratic expression fully captures the transformed function.

Expanded Form of $H(x)$

Expanding the squared binomial:

$$H(x) = (x + 7)^2 = x^2 + 14x + 49$$

This expanded form is often useful for graphing or analyzing properties such as the vertex, axis of symmetry, and intercepts.

Key Properties of the Transformed Function $H(x)$

Understanding the properties of the transformed function helps in visualizing and verifying the transformation.

Vertex

- The vertex of $H(x) = (x + 7)^2$ is at $x = -7$.
- To find the y-coordinate, plug $x = -7$ into $H(x)$:

$$H(-7) = (-7 + 7)^2 = 0$$

- Vertex: $(-7, 0)$.

Axis of Symmetry

- The parabola's axis of symmetry is the vertical line passing through the vertex:

$$x = -7$$

Intercepts

- Y-intercept: Set $x = 0$:

$$H(0) = (0 + 7)^2 = 49$$

- X-intercepts: Set $H(x) = 0$:

$$(x + 7)^2 = 0 \Rightarrow x + 7 = 0 \Rightarrow x = -7$$

- Since the quadratic is a perfect square, it touches the x-axis at $x = -7$ only, indicating a repeated root.

Implications for Graphing and Applications

The understanding of this transformation allows for precise graphing and application in real-world scenarios. For example:

- Graphing: Knowing the vertex and axis of symmetry, along with intercepts, enables quick sketching.
- Problem-solving: Recognizing shifts helps in solving equations, optimization problems, and modeling real-life phenomena where position adjustments are necessary.

Summary: The Correct Form of $H(x)$

Based on the transformation, the correct expression for the function \(H(x) \) is:

\[\boxed{H(x) = (x + 7)^2} \]

or, in expanded form,

\[H(x) = x^2 + 14x + 49. \]

This expression reflects a parabola shifted 7 units to the left of the original \(G(x) = x^2 \).

Choosing the Correct Answer in Multiple-Choice Contexts

If faced with multiple options, the key is to identify the form that:

- Represents a horizontal shift to the left by 7 units.
- Is obtained by replacing \(x \) with \(x + 7 \) in the original function \(G(x) = x^2 \).
- Matches the algebraic form \((x + 7)^2 \).

Common incorrect choices might include:

- \((x - 7)^2 \), which would shift the parabola to the right.
- \(x^2 + 7 \), which would be a vertical shift.
- \((x + 7)^2 + c \), which would include a vertical translation as well.

In summary:

Option	Description	Correctness
\((x + 7)^2 \)	Horizontal shift 7 units to the left	Correct
\((x - 7)^2 \)	Shifted right	Incorrect
\(x^2 + 7 \)	Vertical translation	Incorrect
\((x + 7)^2 + c \)	Horizontal and vertical shifts	Depends on \(c \), but not the given transformation	Incorrect unless specified

Conclusion:

The precise application of transformations is crucial in both academic and practical contexts. Recognizing that \(H(x) = G(x + 7) \) corresponds to a horizontal shift left by 7 units, and expressing it as \((x + 7)^2 \), allows for correct graphing and problem-solving. This understanding forms a foundational skill in algebra, enabling learners and professionals to manipulate functions confidently and accurately.

Final note: Always verify transformations by considering how key points move and by analyzing the algebraic form of the transformed function. This approach ensures a clear, accurate understanding of how base functions are modified through shifts and other transformations.

Select The Correct Answer The Function G X X2 Is Transformed To Obtain Function H H X G X 7

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