Show That The Specific Heat At Constant Volume Cv Of An Ideal Gas Does Not Depend Upon Specific Volume.

Show That The Specific Heat At Constant Volume Cv Of An Ideal Gas Does Not Depend Upon Specific Volume.

Understanding the thermodynamic properties of gases is fundamental in the study of physical sciences and engineering. Among these properties, the specific heats at constant volume (Cv) and constant pressure (Cp) are crucial in analyzing energy transfer during heating and cooling processes. A key question often posed is whether the specific heat at constant volume, Cv, of an ideal gas depends on the specific volume (v). This article provides a comprehensive explanation demonstrating that Cv for an ideal gas remains constant regardless of the specific volume, grounded in the fundamental principles of thermodynamics and kinetic theory.

Introduction to Specific Heats and Ideal Gases

Before delving into the proof, it is important to understand what specific heats are and the defining characteristics of an ideal gas.

What Are Specific Heats?

Specific heats refer to the amount of heat required to raise the temperature of a substance per unit mass under specific conditions:
- Specific heat at constant volume (Cv): Heat needed to increase temperature by 1°C (or 1 K) at constant volume.
- Specific heat at constant pressure (Cp): Heat needed to increase temperature by 1°C (or 1 K) at constant pressure.

Mathematically,
$$ C_v = \left(\frac{\partial Q}{\partial T} \right)_V, \quad C_p = \left(\frac{\partial Q}{\partial T} \right)_P $$

For gases, the relation between these two specific heats is expressed via Mayer’s relation:
$$ C_p - C_v = R $$
where R is the universal gas constant.

Characteristics of an Ideal Gas

An ideal gas is a hypothetical gas composed of point particles that do not interact with each other except during elastic collisions. Its behavior is

described accurately by the ideal gas law:
$$ PV = nRT $$
or, in specific form:
$$ P v = R T $$
where:
- P = pressure,
- V = volume,
- n = number of moles,
- R = universal gas constant,
- T = temperature,
- v = specific volume (volume per unit mass).

The simplicity of the ideal gas law and the assumptions underlying it make ideal gases a useful model for understanding fundamental thermodynamic concepts, including the dependence (or independence) of Cv on specific volume.

Understanding the Dependence of Cv on Specific Volume

The core question is whether the specific heat at constant volume, Cv, varies with changes in specific volume v for an ideal gas. To address this, we examine the thermodynamic foundations and kinetic theory explanations.

Theoretical Foundations

The specific heat at constant volume is defined as:
$$ C_v = \left(\frac{\partial u}{\partial T} \right)_V $$
where u is the internal energy per unit mass.

For an ideal gas:
- Internal energy u depends only on temperature T,
- The kinetic theory suggests that internal energy is due solely to the translational motion of molecules.

This dependence can be summarized as:
$$ u = u(T) $$

Since internal energy u is independent of volume or pressure for an ideal gas, it follows that:
$$ \frac{\partial u}{\partial v} = 0 $$
implying that Cv, which is related to the change of internal energy with temperature at constant volume, does not depend on the specific volume.

Empirical and Theoretical Evidence

Empirical measurements of Cv for various gases at different specific volumes (or densities) show negligible variation, supporting the theoretical conclusion. Theoretically, the independence stems from the kinetic theory, which attributes internal energy solely to molecular motion, unaffected by the volume at constant temperature.

Mathematical Demonstration

To rigorously demonstrate that Cv does not depend on specific volume for an ideal gas, consider the thermodynamic relations and the properties of ideal gases.

Internal Energy and Its Relation to Temperature

For an ideal gas, the internal energy per unit mass u is a function solely of temperature:

$$ u = u(T) $$

and the differential change in internal energy is:

$$ du = C_v dT $$

Since u depends only on T, then:

$$ \left(\frac{\partial u}{\partial v} \right)_T = 0 $$

This indicates that at constant temperature, changing the specific volume does not alter the internal energy.

Thermodynamic Identity and Cv

The fundamental thermodynamic relation for internal energy per unit mass u is:

$$ du = T ds - P dv $$

At constant volume (dv = 0), this simplifies to:

$$ du = T ds $$

Differentiating u with respect to T at constant volume:

$$ C_v = \left(\frac{\partial u}{\partial T} \right)_V $$

Since u depends only on T, and not on v, Cv remains unaffected by the specific volume.

Using the Ideal Gas Law

From the ideal gas law:

$$ P v = R T $$
or equivalently,
$$ P = \frac{R T}{v} $$

The differential of pressure at constant T:
$$ dP = - \frac{R T}{v^2} dv $$

At constant volume, the pressure changes with temperature, but the internal energy remains solely a function of T. Therefore, variations in v do not influence u, reinforcing that Cv is independent of specific volume.

Kinetic Theory Perspective

The kinetic theory of gases offers a microscopic explanation for the independence of Cv from specific volume.

Translational Motion and Internal Energy

In an ideal gas:
- Internal energy arises from the translational kinetic energy of molecules.
- The average kinetic energy per molecule is:
$$ \frac{3}{2} k T $$
where k is Boltzmann’s constant.

Total internal energy per unit mass is then proportional to temperature:
$$ u = \frac{3}{2} R T $$
since R is the gas constant.

This relation confirms that internal energy depends solely on temperature, not on volume or pressure.

Implications for Cv

Because the microscopic model shows that the energy associated with molecular motion does not depend on the specific volume, Cv, which measures how internal energy changes with temperature, must also be independent of specific volume.

Practical Implications and Applications

Understanding that Cv does not depend on specific volume simplifies thermodynamic calculations involving ideal gases. It allows engineers and scientists to:
- Calculate energy changes during heating at constant volume without concern for volume variations.

- Use constant Cv values across different states, provided the gas behaves ideally.
- Develop models for thermodynamic cycles like Otto, Diesel, and Stirling engines with reliable assumptions.

Limitations and Real Gases

While the above analysis holds true for ideal gases, real gases exhibit interactions and molecular sizes that cause deviations:
- Cv may vary with specific volume at high pressures or low temperatures.
- Van der Waals gases and other real gas models account for intermolecular forces, leading to dependence of Cv on volume.

However, within the ideal gas approximation, the independence of Cv from specific volume remains a fundamental principle.

Conclusion

In conclusion, the specific heat at constant volume, Cv, of an ideal gas does not depend upon the specific volume. This conclusion follows from fundamental thermodynamic principles and the kinetic theory of gases:
- Internal energy in an ideal gas depends solely on temperature.
- Variations in specific volume do not alter the internal energy at a given temperature.
- The differential relations show no dependence of Cv on volume.

This understanding simplifies thermodynamic analysis and underpins many practical applications involving ideal gases. Recognizing the independence of Cv from specific volume is a cornerstone in thermodynamics, helping to differentiate idealized models from real-world complexities.

References:
- Moran, M. J., & Shapiro, H. N. (2010). Fundamentals of Engineering Thermodynamics. Wiley.
- Sonntag, R. E., Borgnakke, C., & Van Wylen, G. (2003). Fundamentals of Thermodynamics. Wiley.
- Çengel, Y. A., & Boles, M. A. (2015). Thermodynamics: An Engineering Approach. McGraw-Hill Education.
- Kinetic Theory of Gases - Physics LibreTexts

Frequently Asked Questions

Why does the specific heat at constant volume (Cv)

of an ideal gas not depend on specific volume?

Because in an ideal gas, the internal energy depends only on temperature and not on volume, making Cv independent of specific volume.

How is the specific heat at constant volume (Cv) related to the internal energy of an ideal gas?

For an ideal gas, Cv is defined as the amount of heat required to raise the temperature by one degree at constant volume, and it is directly related to changes in internal energy, which depends solely on temperature.

Can the specific volume affect the internal energy of an ideal gas?

No, in an ideal gas, the internal energy is independent of specific volume and depends only on temperature, which is why Cv remains unaffected by volume changes.

What assumptions about ideal gases lead to Cv being independent of specific volume?

The assumption that particles do not interact and have negligible volume ensures the internal energy depends only on temperature, making Cv independent of specific volume.

Is the specific heat at constant volume (Cv) the same for all ideal gases?

No, Cv varies among different ideal gases depending on their degrees of freedom, but importantly, it remains independent of specific volume.

What role does the kinetic theory of gases play in understanding why Cv does not depend on specific volume?

The kinetic theory shows that internal energy depends only on the average kinetic energy of molecules, which depends on temperature, not volume, explaining why Cv is volume-independent.

How does the relationship between Cv and Cp support the idea that Cv does not depend on specific volume?

Since Cp and Cv are related through the gas's degrees of freedom and both are functions of temperature only, this supports the fact that Cv does not depend on specific volume.

Does the ideal gas law (PV = nRT) imply that Cv depends on volume?

No, the ideal gas law relates pressure, volume, and temperature, but Cv pertains to energy change at constant volume and is independent of volume itself.

Why is it important to understand that Cv does not depend on specific volume when studying thermodynamics of ideal gases?

Because it simplifies calculations and models, allowing us to focus on temperature dependence of internal energy and heat capacities without considering volume changes.

Can the specific heat at constant volume (Cv) vary with temperature for an ideal gas?

Yes, Cv can vary with temperature for real gases, but for an ideal gas, it is often considered constant over a range of temperatures, and still independent of specific volume.

Additional Resources

Show That The Specific Heat At Constant Volume C_v Of An Ideal Gas Does Not Depend Upon Specific Volume

---

Introduction

The specific heat capacities of gases—particularly at constant volume (C_v) and constant pressure (C_p)—are fundamental concepts in thermodynamics, underpinning our understanding of heat transfer, energy exchange, and the behavior of gases under various conditions. Among these, the specific heat at constant volume, C_v, holds particular significance because it characterizes the amount of heat required to raise the temperature of a unit mass of gas without allowing volume change.

A key question often posed in thermodynamics is whether C_v varies with the specific volume of the gas. For an ideal gas, the answer is a resounding no: C_v is independent of specific volume. This property is not only theoretically elegant but also practically useful, simplifying calculations and models involving ideal gases. This article provides a comprehensive, detailed, and analytical exploration of why and how the specific heat at constant volume for an ideal gas does not depend upon the specific volume, delving into the fundamental thermodynamic principles, mathematical derivations, and physical interpretations that underpin this assertion.

Understanding the Concept of Specific Heat at Constant Volume

Definition and Significance

The specific heat at constant volume, C_v, is defined as:

$$
C_v = \left(\frac{\partial Q}{\partial T} \right)_V
$$

where:

- Q is the heat added to the system,
- T is the temperature,
- The subscript V indicates that the volume remains constant during the process.

In essence, C_v quantifies how much heat energy must be supplied to increase the temperature of the gas by a unit degree while keeping the volume fixed.

For an ideal gas, the internal energy U depends solely on temperature, not on volume or pressure, which is a cornerstone of the ideal gas model. This independence forms the basis for understanding why C_v does not depend on specific volume.

Theoretical Foundations: Thermodynamics of an Ideal Gas

Internal Energy and Its Dependence

The internal energy U of an ideal gas is a function only of temperature:

$$
U = U(T)
$$

This stems from the assumption that the particles do not interact except during elastic collisions, and that the potential energy associated with particle interactions is negligible. Consequently, the heat capacity at constant volume relates directly to the change in internal energy:

$$
C_v = \left(\frac{\partial U}{\partial T} \right)_V
$$

Since U is a function solely of T, C_v must also be independent of the volume or specific volume.

Equation of State for an Ideal Gas

The ideal gas law relates pressure P, specific volume v, and temperature T:

$$
PV = RT
$$

or, in terms of specific volume:

$$
P v = R T
$$

where:

- P is the pressure,
- v is the specific volume (volume per unit mass),
- R is the specific gas constant,
- T is the absolute temperature.

This law implies that for a given temperature, the product $P v$ remains constant, reinforcing the idea that volume variations do not alter the fundamental energetic properties of the gas, provided the process is ideal.

Derivation and Explanation: Why C_v Is Independent of Specific Volume

1. The Internal Energy of an Ideal Gas Depends Only on Temperature

The crux of the argument lies in the nature of an ideal gas's internal energy:

$$
U = U(T)
$$

This indicates that the microscopic energy states contributing to the internal energy are functions solely of temperature, not of the spatial configuration (volume or specific volume). Therefore, any change in the internal energy caused by a temperature change is unaffected by the specific volume.

Mathematically, this can be expressed as:

\[

$$
\left(\frac{\partial U}{\partial v} \right)_T = 0
$$

which signifies that at constant temperature, variations in specific volume do not influence the internal energy.

2. Mathematical Derivation Using Thermodynamic Identities

The first law of thermodynamics states:

$$
dQ = dU + P\,dV
$$

At constant volume ($dV = 0$), this simplifies to:

$$
dQ_V = dU
$$

Since $U = U(T)$, then:

$$
dQ_V = \frac{\partial U}{\partial T} dT
$$

which yields:

$$
C_v = \left(\frac{\partial U}{\partial T} \right)_V
$$

Because U depends only on T, C_v is independent of specific volume v.

This derivation underscores that, in an ideal gas, the heat capacity at constant volume is a function only of the temperature and intrinsic properties of the gas, not of the specific volume.

3. Physical Interpretation

From a physical standpoint, the internal energy of an ideal gas is primarily due to the kinetic energy of its molecules. Since kinetic energy depends solely on temperature, and not on the spatial arrangement or specific volume, the energy required to raise the temperature remains unaffected by volume changes, provided the process occurs at constant volume.

In other words, increasing the specific volume at constant temperature does not alter the molecular kinetic energies or the internal energy, hence C_v remains constant regardless of the specific volume.

Empirical and Theoretical Evidence Supporting the Independence

1. Experimental Data

Experimental measurements of C_v for various gases, such as nitrogen, oxygen, and noble gases, demonstrate that at a given temperature, C_v remains essentially unchanged over a broad range of specific volumes. Any minor variations observed at high pressures or low temperatures are attributed to deviations from ideal behavior, not the ideal gas assumptions.

2. Theoretical Models

The kinetic theory of gases provides a microscopic explanation: the internal energy per molecule depends on the average kinetic energy, which is directly proportional to temperature. Since the molecules are considered point particles with no intermolecular interactions, the energy per molecule—and consequently the internal energy—is independent of the volume per se.

The equipartition theorem further asserts that each quadratic degree of freedom contributes equally to the internal energy, emphasizing its dependence on temperature alone.

Implications and Practical Significance

1. Simplification of Thermodynamic Calculations

The independence of C_v from specific volume simplifies the analysis of thermodynamic processes involving ideal gases. It allows engineers and scientists to model heat transfer and energy changes without needing to account for volume-dependent variations in C_v.

2. Design of Engines and Thermodynamic Cycles

In thermodynamic cycles like Carnot, Otto, and Diesel engines, the assumption that C_v is constant and independent of volume makes it easier to derive cycle efficiencies and work outputs, assuming ideal gas behavior.

3. Limitations and Real Gas Behavior

While the ideal gas model predicts volume independence of C_v, real gases exhibit deviations at high pressures and low temperatures due to intermolecular forces and finite particle sizes. In such cases, C_v may show dependence on specific volume. However, these effects are negligible under ideal conditions.

Summary and Conclusion

The assertion that the specific heat at constant volume C_v of an ideal gas does not depend on specific volume is rooted deeply in the fundamental principles of thermodynamics and kinetic theory. The primary reasons are:

- The internal energy U of an ideal gas is a function solely of temperature and not of volume.
- The molecular kinetic energies, which dominate the internal energy, are unaffected by volume changes at constant temperature.
- Mathematical derivations from thermodynamic identities confirm that C_v depends only on temperature for an ideal gas.
- Empirical data and microscopic models support the theoretical conclusion, reinforcing its validity across a broad range of conditions.

This property simplifies the analysis and understanding of ideal gases, making it a cornerstone concept in thermodynamics. Recognizing that C_v remains constant with respect to specific volume underpins many practical applications, from designing engines to understanding atmospheric phenomena, and highlights the elegance of the ideal gas model in capturing the essence of gaseous behavior.

Final Remarks

While real gases deviate from this idealized behavior under certain conditions, the fundamental principle remains an important theoretical foundation. It underscores the importance of idealized models in physics and engineering, providing clarity and simplicity in the study of thermodynamic processes. The independence of C_v from specific volume exemplifies how fundamental assumptions lead to elegant and powerful results in science.

Show That The Specific Heat At Constant Volume Cv Of An Ideal Gas Does Not Depend Upon Specific Volume

Show That The Specific Heat At Constant Volume Cv Of An Ideal Gas Does Not Depend Upon Specific Volume

Related Articles

- Slides: Are There Striations Across The Width Of The Muscle Cells? Number Of Nuclei? What Are The Size,
- Three Friends Ordered Personal-sized Pizzas For Lunch. Lucia Ate 2/3 Of Her Pizza, Allie Ate 1/4 Of Her
- The First Argument In A Python Method Inside An Object Is Self. By Convention, You Call It Called Self,

Back to Home