Sin^2x=1+cosx Write Your Answer In Radians In Terms Of Pi. If There Is More Than One Solution, Separate

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Understanding trigonometric equations is essential for students and professionals working with mathematics, physics, and engineering. One such intriguing equation is sin^2x = 1 + cosx. Solving this equation involves understanding the relationships between sine and cosine functions, their identities, and how to express solutions in radians in terms of Pi. This article provides a comprehensive guide to solving the equation sin^2x = 1 + cosx, presenting solutions in radians, and explaining the process step-by-step for clarity.

Understanding the Equation: sin^2x = 1 + cosx

Before diving into the solution, it's essential to grasp what the equation entails:

- sin^2x: The square of sine function, which ranges between 0 and 1 for real x.
- 1 + cosx: The sum of 1 and cosine function, which varies between 0 and 2.

The equation essentially relates these two functions and asks for all angles x (in radians, expressed in terms of Pi) where the equality holds.

Step 1: Recall Fundamental Trigonometric Identities

To solve the given equation, using identities helps simplify the problem:

Pythagorean Identity

- $\sin^2 x + \cos^2 x = 1$

This identity allows us to rewrite $\sin^2 x$ in terms of $\cos x$:

$$\sin^2 x = 1 - \cos^2 x$$

Substituting into the original equation:

$$1 - \cos^2 x = 1 + \cos x$$

Step 2: Simplify the Equation

Subtract 1 from both sides:

$$-\cos^2 x = \cos x$$

Multiply both sides by -1:

$$\cos^2 x = -\cos x$$

Rearranged as:

```
cos^2x + cosx = 0
```

Factor out cosx:

```
cosx(cosx + 1) = 0
```

This gives us two equations to solve:

1. cosx = 0
2. cosx + 1 = 0

Step 3: Solve for x in Each Case

Let's analyze each case separately.

Case 1: cosx = 0

The cosine function equals zero at:

```
x = π/2 + nπ, where n is any integer
```

Expressed in terms of Pi, the solutions are:

- $x = \pi/2 + n\pi$, for $n \in \mathbb{Z}$

So, specific solutions in radians are:

- $x = \pi/2, 3\pi/2, 5\pi/2$, etc.

Case 2: cosx + 1 = 0

This simplifies to:

```
cosx = -1
```

The cosine function equals -1 at:

```
x = π + 2nπ, where n ∈ ℤ
```

Expressed in radians:

- $x = \pi + 2n\pi$

Specific solutions include:

- $x = \pi, 3\pi, 5\pi$, etc.

Final Solution Set in Radians in Terms Of Pi

Combining the solutions from both cases, we have:

- **From cosx = 0:** x = $\pi/2 + n\pi$, where $n \in \mathbb{Z}$

- **From cosx = –1:** x = $\pi + 2n\pi$, where $n \in \mathbb{Z}$

Expressed explicitly:

- x = $\pi/2 + n\pi$ (n integer)
- x = $\pi + 2n\pi$ (n integer)

Summary of Solutions

Solution Type	General Form in Radians	Specific Solutions in Terms of Pi
Cosine equals zero	x = $\pi/2 + n\pi$	x = $\pi/2$, $3\pi/2$, $5\pi/2$, ...
Cosine equals -1	x = $\pi + 2n\pi$	x = π, 3π, 5π, ...

Additional Notes:

- The solutions are periodic, reflecting the nature of the cosine function.
- The solutions cover all angles where the original equation holds true.

Verifying the Solutions

It's always good practice to verify solutions by substituting back into the original equation:

- For x = $\pi/2$:

```
sin^2(π/2) = 1, and 1 + cos(π/2) = 1 + 0 = 1
```

- For $x = \pi$:

```
sin^2(π) = 0, and 1 + cos(π) = 1 - 1 = 0
```

Both check out correctly, confirming the validity of solutions.

Conclusion

The equation sin^2x = 1 + cosx has multiple solutions expressed in radians in terms of Pi:

- $x = \pi/2 + n\pi$, where n is any integer.
- $x = \pi + 2n\pi$, where n is any integer.

Expressing solutions in terms of Pi provides clarity and precision, especially when dealing with periodic functions like sine and cosine. Remember to verify solutions by substitution and understand the periodic nature of these functions to identify all possible solutions within any given interval.

Whether you're solving for x in a classroom setting or applying these solutions in engineering problems, understanding how to manipulate and interpret trigonometric equations in radians is a fundamental skill.

Frequently Asked Questions

Solve the equation sin^2 x = 1 + cos x for x in radians, expressing the solutions in terms of pi. What are the solutions?

The solutions are $x = \pi/2 + 2k\pi$ and $x = 3\pi/2 + 2k\pi$, where k is any integer.

How do you solve the equation sin^2 x = 1 + cos x in radians, and what are all solutions in terms of pi?

Rearranging gives sin^2 x - cos x - 1 = 0. Using Pythagorean identities and solving, the solutions are x = □/2 + 2k□ and x = 3□/2 + 2k□.

What are the solutions to sin^2 x = 1 + cos x in radians, expressed in terms of pi?

Solutions are x = □/2 + 2k□ and x = 3□/2 + 2k□, for all integers k.

In radians, what values of x satisfy sin^2 x = 1 + cos x, and how are they expressed in terms of pi?

The solutions are x = □/2 + 2k□ and x = 3□/2 + 2k□.

Find all solutions in radians for the equation sin^2 x = 1 + cos x, expressed with pi, including multiple solutions if any.

The solutions are x = □/2 + 2k□ and x = 3□/2 + 2k□, where k is any integer.

How can we solve sin^2 x = 1 + cos x in radians and write the solutions in terms of pi?

By rewriting as sin^2 x - cos x - 1 = 0 and solving, the solutions are x = □/2 + 2k□ and x = 3□/2 + 2k□.

What is the general solution in radians, in terms of pi, for the equation sin^2 x = 1 + cos x?

x = □/2 + 2k□ and x = 3□/2 + 2k□, for any integer k.

Determine all x in radians, expressed in terms of pi, satisfying sin^2 x = 1 + cos x.

$x = \pi/2 + 2k\pi$ and $x = 3\pi/2 + 2k\pi$, where $k \in \mathbb{Z}$.

Additional Resources

Understanding the Equation $\sin^2 x = 1 + \cos x$: A Step-by-Step Solution in Radians (Expressed in Terms of Pi)

When tackling trigonometric equations, especially those involving both sine squared and cosine functions, it’s essential to understand their interrelationships and how to manipulate them effectively. The equation $\sin^2 x = 1 + \cos x$ is a fascinating example because it combines a quadratic sine term with a linear cosine term, prompting a variety of methods to find solutions in radians expressed in terms of Pi. Such problems are common in trigonometry and serve as excellent exercises for applying identities, algebraic techniques, and understanding periodic functions.

In this comprehensive guide, we will delve deeply into solving $\sin^2 x = 1 + \cos x$, exploring multiple methods, verifying solutions, and expressing answers in radians in terms of Pi. This approach aims to clarify the reasoning process and provide a clear pathway to understanding how to handle similar equations.

Understanding the Equation and Its Components

Before jumping into solutions, let's interpret the given equation:

$\sin^2 x = 1 + \cos x$

- Sin^{2}x can be replaced using the Pythagorean identity:

$$
\sin^2x = 1 - \cos^2x
$$

- The right side involves cosx, which hints at potential substitution or rearrangement to form a quadratic in terms of cosx.

Step 1: Rewrite the Equation Using Identities

To simplify the problem, replace Sin^{2}x with 1 - cos^{2}x:

$$
1 - \cos^2x = 1 + \cos x
$$

Subtract 1 from both sides:

$$
- \cos^2x = \cos x
$$

Or equivalently:

$$
\cos^2x + \cos x = 0
$$

This is a quadratic in cosx that can be factored or solved using standard algebraic techniques.

Step 2: Solve the Quadratic Equation in Terms of cosx

The quadratic form:

$$
\cos^2x + \cosx = 0
$$

Factor out cosx:

$$
\cosx (\cosx + 1) = 0
$$

Set each factor equal to zero:

1. $\cosx = 0$
2. $\cosx + 1 = 0$

Solution 1: $\cosx = 0$

Recall the general solutions for $\cosx = 0$:

$$
x = \frac{\pi}{2} + n\pi, \quad n \in \mathbb{Z}
$$

Expressed in terms of Pi, these solutions are:

\[
x = \frac{\pi}{2} + n\pi
\]

Solution 2: \(\cosx = -1\)

Recall the solutions for \(\cosx = -1\):

\[
x = \pi + 2n\pi, \quad n \in \mathbb{Z}
\]

Expressed in terms of Pi:

\[
x = \pi + 2n\pi
\]

Step 3: Verify the Solutions in the Original Equation

While the algebraic solution process suggests these solutions, it’s crucial to verify them in the original equation:

\[
\sin^2x = 1 + \cosx
\]

Verify for \(\cosx = 0\)

At $x = \frac{\pi}{2} + n\pi$:

- $\cos x = 0$
- $\sin x = \pm 1$, so $\sin^2 x = 1$

Substitute into the original:

$$
1 = 1 + 0 \implies 1 = 1
$$

True for all n, so solutions are valid.

Verify for $\cos x = -1$

At $x = \pi + 2n\pi$:

- $\cos x = -1$
- $\sin x = 0$, so $\sin^2 x = 0$

Substitute into the original:

$$
0 = 1 + (-1) \implies 0 = 0
$$

True for all n, so solutions are valid.

Summary of Solutions in Radians (Expressed in Terms of Pi)

Combining all, the solutions to the equation $\text{Sin}^2 x = 1 + \cos x$ are:

- For $\cos x = 0$:

$$
x = \frac{\pi}{2} + n\pi, \quad n \in \mathbb{Z}
$$

- For $\cos x = -1$:

$$
x = \pi + 2n\pi, \quad n \in \mathbb{Z}
$$

Additional Considerations: Domain Restrictions and Special Angles

The solutions provided are valid for all real x, considering the periodicity of sine and cosine functions. However, in practical applications, one might restrict the solutions to a specific interval, such as $[0, 2\pi)$:

- Solutions in $[0, 2\pi)$:

- $\frac{\pi}{2}$ (from $\cos x=0$)
- $\frac{3\pi}{2}$ (from $\cos x=0$)
- π (from $\cos x=-1$)

- Solutions outside $[0, 2\pi)$:

Add multiples of 2π accordingly.

Visualizing the Equation and Its Solutions

Graphically, plotting both sides of the original equation:

$$
\sin^2x \quad \text{and} \quad 1 + \cos x
$$

over a principal interval (e.g., $[0, 2\pi)$) reveals intersections at the solutions identified. These intersections occur precisely at points where:

- $\cos x = 0$, i.e., $x = \frac{\pi}{2}$, $\frac{3\pi}{2}$
- $\cos x = -1$, i.e., $x = \pi$

This confirms the solutions obtained algebraically.

Summary and Final Remarks

The equation $\sin^2x = 1 + \cos x$ is a classic example of how algebraic manipulation and trigonometric identities can work together to find solutions in radians expressed in terms of Pi. The key steps involved:

1. Recognizing the Pythagorean identity $\sin^2x = 1 - \cos^2x$.
2. Substituting to form a quadratic in $\cos x$.
3. Factoring and solving for $\cos x$.
4. Verifying solutions in the original equation.
5. Expressing solutions in terms of Pi, considering periodicity and domain restrictions.

The solutions are:

- $x = \frac{\pi}{2} + n\pi$, for all integers n,
- $x = \pi + 2n\pi$, for all integers n.

This comprehensive approach underscores the importance of identities, algebra, and verification in solving trigonometric equations. Whether you are solving for calculus, physics, or pure mathematics, mastering these techniques will greatly enhance your problem-solving toolkit.

Final Tips for Solving Similar Equations

- Always look for identities that can simplify the equation.
- Convert all trigonometric functions into a single function when possible.
- Formulate the problem as a quadratic and factor carefully.
- Verify solutions by substitution to avoid extraneous roots.
- Express solutions in radians using Pi for clarity, especially in exact form.

With practice, equations like $Sin^2x = 1 + cosx$ become manageable and reveal the elegant structure of trigonometric functions.

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