# Sketch The RegionR={(x,y):yx,0y}(b) Set Up The Iterated Integral Which Computes The Volume Of The Solid

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## Introduction to Regions and Volumes in Multivariable Calculus

Understanding how to sketch regions and set up integrals is fundamental in multivariable calculus, especially when calculating volumes of solids bounded by various surfaces. The process involves translating a geometric description into a mathematical integral that accurately computes the volume enclosed by those surfaces. In this article, we will explore the specific problem: sketching the region R defined by certain inequalities, and setting up the appropriate iterated integral to find the volume of the resulting solid.

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## Understanding the Region R: Definitions and Interpretation

### Interpreting the Given Description

The region R is described as:

> R = {(x, y): yx, 0 y} (b)

This notation appears to have some typographical or formatting issues, but based on common conventions in calculus, it likely refers to a region defined by inequalities involving x and y, such as:

- $y \leq x$ (or $y \leq \text{something}$)
- $0 \leq y \leq b$

or similar.

Given the context, a typical interpretation might be:

- \( 0 \leq y \leq b \)
- \( y \leq x \)

or perhaps the region bounded by:

- \( 0 \leq y \leq b \)
- \( y \leq x \leq \text{some function} \)

To proceed, assume the region R is bounded by the inequalities:

- \( 0 \leq y \leq b \)
- \( y \leq x \leq a \) for some positive constants \( a \) and \( b \)

Alternatively, if the region is bounded by \( y = x \) and the x-axis, between specific limits.

Note: For clarity, we will interpret the region R as the area in the xy-plane where:

- \( 0 \leq y \leq b \)
- \( y \leq x \leq a \)

This is a common scenario involving a triangular or rectangular region.

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# Sketching the Region R

## Step 1: Identify the Boundaries

To sketch the region R, identify the boundary lines:

- The line \( y = 0 \) (x-axis)
- The line \( y = b \) (horizontal line at y = b)
- The line \( x = y \) (diagonal line passing through the origin with slope 1)
- The vertical boundary at \( x = a \)

Assuming \( a \geq y \), the region is bounded between:

- The x-axis \( y = 0 \)
- The line \( y = b \)
- The line \( x = y \) (for the lower boundary)
- The vertical line \( x = a \) (for the right boundary)

Important: The specific shape depends on the values of \( a \) and \( b \). For illustration, suppose \( 0 \leq y \leq b \), and \( y \leq x \leq a \).

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## Step 2: Plot the Boundaries

- Draw the x-axis, from left to right.
- Draw the line $y = b$, a horizontal line at $y = b$.
- Draw the line $y = x$, passing through the origin with slope 1.
- Draw the vertical line $x = a$.

The region R is the set of all points within the intersection of these boundaries.

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## Step 3: Describe the Region

Depending on the relative positions of $a$ and $b$:

- If $a \geq b$, the region is a trapezoid or triangle bounded by the lines above.
- The region includes all points where:

$$
0 \leq y \leq b, \quad y \leq x \leq a
$$

which forms a region between the y-axis, the line $y = b$, the line $y = x$, and the vertical line $x = a$.

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# Setting Up the Iterated Integral for the Volume of the Solid

## Understanding the Solid's Geometry

Suppose the solid is formed by revolving the region R around a specified axis, or by considering a function defined over R to generate a volume.

For example, if the volume is computed as the integral of a function $f(x, y)$ over R, then:

$$
V = \iint_R f(x, y) \, dA
$$

where $dA$ is the differential area element.

Alternatively, if the problem involves revolving R about an axis (say, the x-axis), the volume can be computed using methods such as the disk or washer method, which necessitate setting up integrals accordingly.

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## Step 1: Decide the Order of Integration

Two common options exist:

- Integrate with respect to $x$ first, then $y$:

$$
V = \int_{y = 0}^{b} \int_{x = y}^{a} f(x, y) \, dx \, dy
$$

- Integrate with respect to $y$ first, then $x$:

To do this, find the bounds of $y$ for each fixed $x$.

If $y \leq x$ and $0 \leq y \leq b$, then for each $x$, $y$ varies from 0 up to $y = x$ (when $x \leq b$), or up to $y = b$ (for $x \geq y$).

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## Step 2: Write the Integral Expression

Assuming the first order (dy dx):

$$
V = \int_{y = 0}^{b} \left( \int_{x = y}^{a} f(x, y) \, dx \right) dy
$$

Alternatively, for the second order (dx dy), determine the bounds for $y$ as functions of $x$:

- For $0 \leq x \leq a$:

$$
y \text{ varies from } 0 \text{ to } \min(b, x)
$$

which simplifies to:

$$
V = \int_{x=0}^{a} \left( \int_{y=0}^{\min(b, x)} f(x, y) \, dy \right) dx
$$

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# Special Cases and Adjustments

Depending on the exact region and the function involved, the limits may need to be adjusted.

- If the region is bounded by $y = 0$, $y = b$, $x = y$, and $x = a$, the volume integral can be expressed with specific bounds accordingly.
- If the region is rotated about an axis, the integral setup involves the disk or washer method, which requires expressing the radius as a function of the variable of integration.

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# Examples of Setting Up the Integral

## Example 1: Volume Under a Surface $z = f(x, y)$ Over R

Suppose the volume is under the surface $z = x + y$, over the region R described above.

The volume:

$$
V = \iint_R (x + y) \, dA
$$

using the order $dy dx$:

$$
V = \int_{x=0}^{a} \int_{y=0}^{\min(b, x)} (x + y) \, dy \, dx
$$

or in the order $dy dx$:

$$
V = \int_{y=0}^{b} \int_{x=y}^{a} (x + y) \, dx \, dy
$$

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## Example 2: Volume of a Solid of Revolution

If the region R is revolved about the x-axis, the volume can be computed using the disk method:

$$
V = \pi \int_{x=x_{min}}^{x_{max}} [R(x)]^2 \, dx
$$

where $R(x)$ is the radius of the disk at position $x$.

Given the region, the radius might be $y$, which varies from 0 to $y = x$ or $y = b$, depending upon the bounds.

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# Summary of Steps to Set Up the Volume Integral

1. Identify the region R:
- Determine the inequalities defining R.
- Sketch the region for visual understanding.
2. Determine the bounds for integration:
- Decide the order of integration (dy dx or dx dy).
- Find the limits for each variable based on the region.
3. Express the integrand $f(x, y)$:
- This depends on the problem context (e.g., height function, surface, etc.).
4. Write the

# Frequently Asked Questions

## What does the region R = {(x, y) | y x, 0 y} represent in the context of setting up an iterated integral?

The region R describes the area in the xy-plane where y is between 0 and x, typically representing a triangular region under the line y = x and above y = 0.

## How do you determine the limits of integration when setting up the iterated integral for the volume of a solid over region R?

You analyze the boundaries of R to find the limits: for example, if y varies from 0 to x and x varies from a starting point to an endpoint, you set those as the limits of integration accordingly.

## What is the significance of choosing the order of integration (dy dx vs. dx dy) when computing volume over region R?

Choosing the order depends on the shape of the region; selecting the order that simplifies the limits makes the integral easier to evaluate. For R, integrating with respect to y first or x first depends on the region's boundaries.

## How do you set up the iterated integral to compute the volume of the solid over region R if the height function is given as a function of x and y?

The volume is computed as a double integral of the height function over R: V = ∬_R h(x, y) dy dx or dx dy, with limits determined by the region boundaries.

## What steps are involved in sketching the region R and translating it into limits for the iterated integral?

First, plot the boundary lines y = 0 and y = x, identify the intersection points, determine the range of x and y within the region, and then write the limits accordingly for the integral setup.

# Additional Resources

Sketch The Region R = { (x,y) : yx, 0 y } (b) Set Up The Iterated Integral Which Computes The Volume Of The Solid

Understanding how to translate geometric regions into integrals is a cornerstone of multivariable calculus, especially when calculating the volume of solids bounded by curves and surfaces. Today, we delve into a specific problem: sketching the region R defined by an inequality involving variables x and y, and then setting up the appropriate iterated integral to find the volume of a solid formed over this region. This process not only enhances spatial visualization skills but also reinforces the methodical approach required to tackle integral calculus problems involving multiple variables.

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The Significance of Sketching Regions in Multivariable Calculus

Before jumping into the calculations, it's essential to appreciate the importance of visualizing the region R. In multivariable calculus, integrating over a region requires a clear understanding of its shape and boundaries. Sketching the region:

- Clarifies the bounds of integration.
- Helps identify the most convenient order of integration (dy dx vs. dx dy).
- Prevents errors in setting up the integral limits.

When dealing with regions defined by inequalities involving multiple variables, a precise sketch becomes invaluable. It transforms an abstract algebraic inequality into a tangible geometric shape, serving as a roadmap for the subsequent integral setup.

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Understanding the Region R: Decoding the Inequality

The region R is given by the set:

\[ R = \{ (x, y) : yx, 0 y \} (b) \]

At first glance, the notation appears incomplete, but with context and typical conventions in calculus, we can interpret this as:

$$ R = \{ (x, y) : 0 \leq y \leq x \} $$

or possibly:

$$ R = \{ (x, y) : 0 \leq y \leq x \text{ for } x \text{ in some interval} \} $$

Given the standard inequalities, the most common interpretation is that R is the region bounded below by y=0 and above by y=x, over some interval of x.

Assuming this, the region R is the set of points lying between the x-axis and the line y = x, over a specified interval of x-values, say from x=a to x=b. For a complete understanding, we need to clarify the bounds for x.

Suppose the region R is defined over $x \in [a, b]$, with:

$$ R = \{ (x, y): a \leq x \leq b, 0 \leq y \leq x \} $$

This forms a wedge-shaped region between the y=0 axis and the line y=x.

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## Sketching the Region: Visual Strategy and Steps

To sketch R accurately, follow these steps:

1. Draw the coordinate axes: x-axis and y-axis.
2. Plot the boundary lines:
- The x-axis: y=0.
- The line y = x.
3. Identify the interval for x:
- If the problem specifies x in [a, b], mark these points on the x-axis.
- For simplicity, assume $a=0$ and $b$ is some positive number, say 3.
4. Shade the region:
- Between x=a and x=b, shade the area between y=0 and y=x.
- The region is bounded on the left by x=a and on the right by x=b.
- Vertically, it's limited below by y=0 and above by y=x.

The resulting shape is a right-angled wedge, expanding as x increases, bounded by the line y = x.

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## Choosing the Proper Orientation for Integration

When setting up the integral, the order of integration is crucial. There are two common options:

- Integrate with respect to y first (dy dx):
- For a fixed x, y varies from 0 up to y=x.
- The limits are:

- $y$ from 0 to x.
- $x$ from a to b.

- Integrate with respect to x first (dx dy):
- For a fixed y, x varies from y to b (assuming $b \geq y$).
- The limits are:
- $x$ from y to b.
- $y$ from 0 to b (or the maximum y in the region).

Given the shape, integrating with respect to y first is often simpler because the bounds are expressed directly as $y \leq x$.

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## Setting Up the Iterated Integral for Volume Calculation

Suppose we are asked to find the volume of a solid lying over the region R, with a height function $h(x, y)$. For simplicity, assume the height function is $h(x, y) = 1$—meaning we're calculating the area of R (or the volume if the height is 1), or more generally, integrating a function over R.

The general form of the iterated integral is:

$$
\text{Volume} = \int_{x=a}^{b} \int_{y=0}^{y=x} h(x, y) \, dy \, dx
$$

Or, if the volume involves a different height function $h(x, y)$:

$$
V = \int_{a}^{b} \int_{0}^{x} h(x, y) \, dy \, dx
$$

Key points for setting up the integral:

- Confirm the bounds: $y$ from 0 to x.
- Confirm the x-bounds: from a to b.
- Ensure the integrand matches the problem's specifics.

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## Example: Computing a Specific Volume over the Region R

Let’s assume the height function is $h(x, y) = 1$, meaning we're calculating the area of region R, which can be viewed as the volume of a solid with uniform height 1.

The integral becomes:

$$
V = \int_{x=0}^{3} \int_{y=0}^{y=x} 1 \, dy \, dx
$$

Calculating the inner integral:

$$
\int_{0}^{x} 1 \, dy = y \big|_{0}^{x} = x
$$

Then, the outer integral:

$$
V = \int_{0}^{3} x \, dx = \frac{x^{2}}{2} \big|_{0}^{3} = \frac{9}{2} = 4.5
$$

This result indicates that the area (or the volume of a shallow solid with height 1) over the region R from x=0 to x=3 is 4.5 square units.

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## Extending the Concept: Generalizing for Different Boundaries and Functions

While the example above assumes specific bounds and a simple height function, the methodology applies broadly:

- Adjust the x-bounds according to the problem's context.
- Modify the y-limits if the region is bounded differently.
- Incorporate the height function $h(x, y)$ for volume calculations of more complex solids.

For instance, if the solid is bounded by a surface $z = f(x, y)$, then the volume is:

$$
V = \int_{a}^{b} \int_{0}^{x} f(x, y) \, dy \, dx
$$

or tailored for different inequalities.

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## Visualizing the Result and Applying It in Real Problems

Understanding the process of sketching the region and setting up the integral is vital in fields like physics, engineering, and computer graphics. For example:

- In physics, calculating the volume of a material with a known cross-sectional shape.
- In engineering, designing components with specific boundary constraints.
- In computer graphics, rendering 3D objects based on boundary equations.

The ability to move seamlessly from a geometric description to a precise integral setup empowers professionals and students to analyze complex shapes and volumes effectively.

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## Final Remarks: The Power of Geometric Intuition in Calculus

Mastering the skill of sketching regions and translating them into integrals unlocks a deeper understanding of multivariable calculus. It reinforces the connection between algebraic inequalities and geometric intuition, making complex problems more approachable. Whether you're calculating the volume of a physical object or exploring abstract mathematical spaces, these foundational skills serve as essential tools.

In conclusion, by carefully sketching the region R defined by \( y \leq x \) and \( y \geq 0 \), over a specified x-interval, and then setting up the appropriate iterated integral, we can precisely compute the volume of the corresponding solid. This process exemplifies the elegance and utility of calculus in transforming visual and algebraic information into quantitative insights.

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