Solve For The Exact Solutions In The Interval [0, 2). List Your Answers Separated By A Comma, If It Has

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When tackling trigonometric equations or algebraic expressions within a specified interval, it’s essential to understand how to find exact solutions accurately. The interval [0, 2) is a common domain for many problems, especially those involving periodic functions like sine and cosine. In this article, we will explore methods to find the exact solutions in the interval [0, 2), explain the importance of understanding the properties of functions within this interval, and provide detailed steps to solve various types of equations. Whether you’re a student preparing for exams or a professional brushing up on problem-solving skills, this guide will help you grasp the core principles involved in finding exact solutions in the interval [0, 2).

Understanding the Interval [0, 2)

Definition and Significance

The interval [0, 2) includes all real numbers starting from 0 up to, but not including, 2. This interval is particularly significant in mathematics because many functions exhibit specific behaviors on this domain, especially those involving periodicity, symmetry, or certain boundary conditions.

- The interval is half-open, meaning it includes 0 but excludes 2.
- It is commonly used for solving equations involving functions like sine, cosine, tangent, and their inverses.
- Solutions within this interval often correspond to principal values, which are the most standard solutions.

Why Focus on Exact Solutions?

Exact solutions are precise mathematical expressions that satisfy the equation without approximation. They are crucial because:

- They provide the most accurate understanding of the solutions.
- They help in analyzing the behavior of functions precisely.
- They form the foundation for numerical approximations and further analysis.

Common Types of Equations and Methods to Solve Them

Solving Trigonometric Equations

Many problems in the interval [0, 2) involve trigonometric functions such as sine, cosine, and tangent. Let’s explore methods to solve these equations for their exact solutions.

1. Basic Trigonometric Equations

For equations like $\sin x = a$ or $\cos x = a$, the solutions depend on the value of a and the properties of the functions.

- Identify the general solutions based on known identities or inverse functions.
- Restrict the solutions to the interval [0, 2).

Example: Solve $\sin x = 0.5$ in [0, 2)

- The general solution for $\sin x = 0.5$ is:

$x = \arcsin(0.5) + 2\pi k$ or $x = \pi - \arcsin(0.5) + 2\pi k$, where k is an integer.

- Since $\arcsin(0.5) = \pi/6$, solutions become:

$x = \pi/6 + 2\pi k$, and $x = \pi - \pi/6 + 2\pi k = 5\pi/6 + 2\pi k$.

- Find solutions in [0, 2):

For $k = 0$:

$x = \pi/6 \approx 0.5236$, and $x = 5\pi/6 \approx 2.618$.

Since $2.618 > 2$, discard the second.

Answer: $\pi/6$

Therefore, the exact solution in [0, 2) is: $\pi/6$.

2. Equations Involving Multiple Trigonometric Functions

When equations involve multiple functions, such as $\sin x = \cos x$, use identities to simplify.

Example: Solve $\sin x = \cos x$ in [0, 2)

- Recognize that $\sin x = \cos x$ implies:

$\tan x = 1$

- The solutions to $\tan x = 1$ are:

$x = \pi/4 + \pi k$, where k is any integer.

- Find solutions in [0, 2):

For $k = 0$:

$x = \pi/4 \approx 0.7854$

For $k = 1$:

$x = \pi/4 + \pi \approx 0.7854 + 3.1416 \approx 3.9272 > 2$, discard.

- Answer: $\pi/4$

Solving Algebraic Equations in [0, 2)

Aside from trigonometric equations, algebraic equations also frequently appear within this interval.

1. Polynomial Equations

For polynomial equations like $x^2 = 1$, the solutions are straightforward.

- $x^2 = 1$

- $x = \pm 1$

- Solutions in [0, 2):

$x = 1$ (since -1 is outside the interval)

- Answer: 1

2. Rational Equations

For rational equations, such as $1/x = 0.5$, solve for x directly.

- $1/x = 0.5$

- $x = 1 / 0.5 = 2$

- Since the interval is [0, 2), $x = 2$ is not included (excluded), so no solutions here.

Note: Always check for extraneous solutions after solving.

Step-by-Step Approach to Find Exact Solutions in [0, 2)

To systematically find solutions within this interval, follow these steps:

Step 1: Simplify the Equation

- Use trigonometric identities or algebraic manipulation to simplify complex equations.
- For example, convert all functions to sine or cosine when possible.

Step 2: Find the General Solution

- For trigonometric functions, recall the unit circle and the known solutions.
- For algebraic equations, solve normally.

Step 3: Restrict Solutions to [0, 2)

- Determine the principal solutions.
- Add periodicity (like 2π for sine and cosine) to find all solutions within the interval.
- Discard solutions outside [0, 2).

Step 4: Verify and List Exact Solutions

- Confirm solutions satisfy the original equation.
- List solutions precisely, using exact forms such as π/6, π/4, etc.

Examples of Exact Solutions in [0, 2)

- sin x = 0.5 → x = π/6
- cos x = 0 → x = π/2
- tan x = 1 → x = π/4
- $x^2 = 1$ → x = 1

Note: Remember to consider the interval boundaries and the nature of each function.

Common Mistakes to Avoid

- Forgetting to convert general solutions to the specific interval.
- Ignoring solutions that fall outside [0, 2).
- Disregarding the domain restrictions of functions (like division by zero in rational equations).
- Assuming solutions are approximate instead of exact forms.

Conclusion

Finding exact solutions in the interval [0, 2) requires a clear understanding of the properties of functions involved, knowledge of general solutions, and careful restriction to the specified domain. Whether solving trigonometric equations like $\sin x = 0.5$ or algebraic equations such as $x^2 = 1$, the process involves simplifying, applying identities, and verifying solutions. By following systematic steps and being attentive to the interval boundaries, you can accurately list all solutions separated by commas.

Sample Final Answers:

$\pi/6, \pi/4, \pi/2, 1$

Remember, the key to success in solving these problems is thorough understanding and precise calculation, ensuring all solutions are exact and within [0, 2).

Frequently Asked Questions

How do you find the exact solutions of an equation within the interval [0, 2)?

To find exact solutions in [0, 2), you solve the equation algebraically and then select solutions that lie within the interval.

What types of equations commonly require solving for solutions in the interval [0, 2)?

Trigonometric, polynomial, and exponential equations often require solutions in specific intervals like [0, 2) for applications in physics and engineering.

How can I determine if a solution is within the interval [0, 2)?

Check if the solution's value is greater than or equal to 0 and less than 2; discard any solutions outside this range.

What methods are effective for solving equations in the interval [0, 2)?

Methods such as algebraic manipulation, substitution, factoring, and using inverse functions, along with graphing, are effective.

Can multiple solutions exist in the interval [0, 2) for a single equation?

Yes, an equation can have multiple solutions within [0, 2); each must be found and checked to ensure it lies in the interval.

How does the periodic nature of trigonometric functions affect solutions in [0, 2)?

Since trig functions are periodic, multiple solutions may occur within [0, 2), and solutions are found using the function's period and inverse functions.

Are solutions at the endpoints 0 and 2 included in the interval [0, 2)?

Only solutions at 0 are included; solutions at 2 are excluded since the interval is half-open [0, 2).

What is the significance of listing solutions separately in interval [0, 2)?

Listing solutions separately helps in understanding the number and nature of solutions within the specified interval for accurate analysis.

Additional Resources

Solve For The Exact Solutions In The Interval [0, 2): An In-Depth Analytical Approach

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Introduction

Mathematics, especially algebraic and trigonometric problem-solving, often involves finding solutions to equations within specific intervals. The interval \([0, 2)\) is a common domain used in many problems involving periodic functions, polynomial equations, or inequalities because it captures the first cycle of many trigonometric functions and a standard unit interval for polynomials.

Understanding how to solve equations precisely in this interval requires a combination of algebraic manipulation, understanding of function properties, and strategic application of inverse functions. This guide will deeply explore the process of solving equations to find exact solutions within $[0, 2)$, emphasizing clarity, methodology, and mathematical rigor.

Defining the Problem Space

Before diving into solutions, it's crucial to clarify what the typical equations look like and what is meant by "exact solutions."

- Exact solutions refer to solutions expressed in their simplest, most precise form, such as fractions, radicals, or known constants, rather than decimal approximations.
- The interval $[0, 2)$ indicates that solutions include all numbers starting from 0 up to, but not including, 2.

Examples of common equations to solve in $[0, 2)$:

- Trigonometric equations like $\sin x = a$, $\cos x = a$, or $\tan x = a$
- Polynomial equations of degree 2 or higher, e.g., $x^2 - 3x + 2 = 0$
- Rational equations, e.g., $\frac{1}{x} = 3$
- Transcendental equations combining functions, e.g., $\sin x = x/2$

Approach to Solving Equations in the Interval $[0, 2)$

1. Understanding the Function Behavior

- Identify the type of equation: Is it algebraic, trigonometric, exponential, or a combination?
- Determine the domain restrictions: For example, denominators cannot be zero; square roots must have non-negative radicands.
- Recognize periodicity: For trigonometric functions, solutions repeat every 2π or π depending on the function.

2. Isolate the Unknown Variable

- Rearrange the equation to isolate the variable when possible.
- For algebraic equations, factorization or quadratic formulas are often used.
- For trigonometric equations, express solutions in terms of inverse functions and known angles.

3. Use Inverse Functions and Known Identities

- Inverse functions: $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$
- Standard identities: For example,

- $\sin^2 x + \cos^2 x = 1$
- Double angle formulas: $\sin 2x = 2 \sin x \cos x$

4. Check the Interval Constraints

- Once potential solutions are found, verify they lie within $[0, 2)$.
- Remember that 2 is not included, so solutions equal to 2 are invalid.

Deep Dive into Specific Types of Equations

A. Trigonometric Equations

Example: Solve $\sin x = \frac{\sqrt{2}}{2}$ for $x \in [0, 2)$.

Step-by-step solution:

- Recognize that $\sin x = \frac{\sqrt{2}}{2}$ corresponds to angles where sine has this value.
- Known angles: $x = \frac{\pi}{4}$ and $x = \frac{3\pi}{4}$.
- Check the interval:

- $\frac{\pi}{4} \approx 0.785$, which is within $[0, 2)$.
- $\frac{3\pi}{4} \approx 2.356$, which is outside $[0, 2)$.

- Answer: The only solution in $[0, 2)$ is $\boxed{\frac{\pi}{4}}$.

More complex cases:

Suppose the equation is $\sin x = 1$. The solution is $x = \frac{\pi}{2}$ within $[0, 2)$.

If the equation is $\sin x = -\frac{\sqrt{2}}{2}$, solutions are $x = \frac{5\pi}{4}$ and $x = \frac{7\pi}{4}$, but only those less than 2 are valid:

- $\frac{5\pi}{4} \approx 3.927$ — outside $[0, 2)$.
- $\frac{7\pi}{4} \approx 5.497$ — outside $[0, 2)$.

Thus, no solutions for this last case within $[0, 2)$.

B. Polynomial and Rational Equations

Example: Solve $x^2 - 3x + 2 = 0$ over $[0, 2)$.

Solution:

- Factor: $(x - 1)(x - 2) = 0$.
- Solutions: $x = 1$ and $x = 2$.

Check the interval:

- \(x=1\) is within \([0, 2)\).
- \(x=2\) equals the upper bound but is not included in \([0, 2)\).

Answer: \(\boxed{1}\).

Note: For rational equations, ensure denominators are not zero and solutions satisfy the original equation after solving.

C. Transcendental Equations

Example: Solve \(\sin x = x/2\) over \([0, 2)\).

Approach:

- Recognize that this is a transcendental equation; algebraic solutions are unlikely.
- Use graphical analysis or iterative methods for approximate solutions.
- For exact solutions, look for points where both sides are equal and known.

Observation:

- At \(x=0\), \(\sin 0=0\) and \(0/2=0\), so \(x=0\) is a solution.
- For \(x>0\), compare \(\sin x\) and \(x/2\):

- At \(x=1\), \(\sin 1 \approx 0.84\), \(1/2=0.5\), so \(\sin x > x/2\).
- At \(x=2\), \(\sin 2 \approx 0.909\), \(2/2=1\), so \(\sin x < x/2\).

- The crossing point occurs somewhere between 1 and 2.

Exact solutions:

- The only exact solution in \([0, 2)\) is at \(x=0\).

Handling Special Cases and Constraints

- Boundary points: Because the interval is \([0, 2)\), solutions at 0 are included, but solutions at 2 are excluded.
- Multiple solutions: Equations may have multiple solutions, especially with periodic functions.
- Non-solvable equations: Some equations don't have exact solutions expressible in closed form. In such cases, specify solutions explicitly or state that solutions are irrational or transcendental.

Summary of Typical Exact Solutions in \([0, 2)\)

Equation Type	Exact Solutions	Notes
$\sin x = a$	$x = \sin^{-1} a + 2\pi n$, or $x = \pi - \sin^{-1} a + 2\pi n$	Adjust n to fit within $[0, 2)$
$\cos x = a$	$x = \cos^{-1} a + 2\pi n$, or $x= -\cos^{-1} a + 2\pi n$	Same as above
$\tan x = a$	$x = \tan^{-1} a + \pi n$	Find n such that $x\in [0, 2)$
Quadratic $ax^2 + bx + c= 0$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Check that solutions lie in $[0, 2)$
Polynomial higher degree	Factorization or formulas	Verify solutions within interval
Transcendental	Usually numeric or approximate, but certain points like $x=0$ may be exact	Use inverse functions or graphical methods

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Practical Tips for Solving in $[0, 2)$

- Always verify solutions: After solving algebraically, substitute back to verify they satisfy the original equation.
- Use known angle values: Memorize common angles and their sine, cosine, tangent values.
- Pay attention to periodicity: For trigonometric functions, solutions repeat every 2π or π.
- Check interval bounds carefully: Solutions at 0 are included; solutions at 2 are not

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