Solve The Following Equation. Show All Algebraic Steps. Express Answers As Exact Solutions If Possible,

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Solving algebraic equations is a fundamental skill in mathematics that applies across various fields, including science, engineering, and everyday problem-solving. Whether you're tackling simple linear equations or more complex quadratic and higher-order polynomials, understanding the step-by-step process to find exact solutions is essential. In this comprehensive guide, we will explore different types of equations, demonstrate detailed solutions with all algebraic steps, and emphasize expressing answers as exact solutions whenever possible. This article is structured to enhance your understanding through clear explanations, examples, and practical tips.

Understanding the Basics of Algebraic Equations

Before diving into solving specific equations, it's important to understand what algebraic equations are and the fundamental principles involved.

What Is an Algebraic Equation?

An algebraic equation is a mathematical statement that asserts the equality of two expressions involving variables and constants. It typically takes the form:

$$\text{Expression}_1 = \text{Expression}_2$$

The goal is to find all values of the variables that make the equation true, known as solutions.

Types of Algebraic Equations

- Linear equations: First-degree equations, e.g., $ax + b = 0$
- Quadratic equations: Second-degree, e.g., $ax^2 + bx + c = 0$
- Polynomial equations: Degree higher than two, e.g., $ax^3 + bx^2 + cx + d = 0$
- Rational equations: Equations involving fractions, e.g., $\frac{1}{x} + 2 = 0$
- Radical equations: Involving roots, e.g., $\sqrt{x} + 3 = 0$

Step-by-Step Guide to Solving Equations

The process of solving an algebraic equation involves several key steps, which vary slightly depending on the type of equation.

General Steps

1. Simplify both sides of the equation by expanding and combining like terms.
2. Isolate the variable on one side of the equation.
3. Apply inverse operations to undo addition, subtraction, multiplication, or division.
4. Solve for the variable to find the solutions.
5. Check for extraneous solutions especially when dealing with rational or radical equations.
6. Express solutions as exact values whenever possible, such as fractions, roots, or symbolic expressions.

Solving Common Types of Equations with Examples

Let's explore detailed solutions for various types of equations, emphasizing all algebraic steps and exact solutions.

1. Solving Linear Equations

Example: Solve $3x - 7 = 2x + 4$

Solution Steps:
1. Subtract $2x$ from both sides:
$$3x - 2x - 7 = 2x - 2x + 4$$
$$x - 7 = 4$$

2. Add 7 to both sides:
$$x - 7 + 7 = 4 + 7$$
$$x = 11$$

Answer: $x = 11$

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2. Solving Quadratic Equations

Example: Solve $2x^2 - 5x - 3 = 0$

Solution Steps:

Method 1: Factoring

1. Multiply a and c:
$$2 \times (-3) = -6$$

2. Find two numbers that multiply to -6 and sum to -5:
$$-6 \text{ and } 1$$
(Because $-6 \times 1 = -6$ and $-6 + 1 = -5$)

3. Rewrite the middle term:
$$2x^2 - 6x + x - 3 = 0$$

4. Factor by grouping:
$$(2x^2 - 6x) + (x - 3) = 0$$
$$2x(x - 3) + 1(x - 3) = 0$$

5. Factor out common binomial:
$$(2x + 1)(x - 3) = 0$$

6. Set each factor equal to zero:
$$2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$$
$$x - 3 = 0 \Rightarrow x = 3$$

Solutions: $x = -\frac{1}{2}$, $x = 3$

Method 2: Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where $a=2$, $b=-5$, $c=-3$:

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 2 \times (-3)}}{2 \times 2}$$
$$x = \frac{5 \pm \sqrt{25 + 24}}{4}$$
$$x = \frac{5 \pm \sqrt{49}}{4}$$
$$x = \frac{5 \pm 7}{4}$$

- For the plus:
$$x = \frac{5 + 7}{4} = \frac{12}{4} = 3$$
- For the minus:
$$x = \frac{5 - 7}{4} = \frac{-2}{4} = -\frac{1}{2}$$

Exact solutions: $x = -\frac{1}{2}$, $x = 3$

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3. Solving Rational Equations

Example: Solve $\frac{1}{x} + 2 = 0$

Solution Steps:

1. Isolate the rational term:
$$\frac{1}{x} = -2$$

2. Cross-multiplied:
$$1 = -2x$$

3. Solve for x:
$$x = -\frac{1}{2}$$

Check for extraneous solutions:
- Plug back into the original:
$$\frac{1}{-\frac{1}{2}} + 2 = -2 + 2 = 0$$
- Valid solution.

Answer: $x = -\frac{1}{2}$

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4. Solving Radical Equations

Example: Solve $\sqrt{x + 3} = x - 1$

Solution Steps:

1. Isolate the radical and square both sides to eliminate the root:
$$(\sqrt{x + 3})^2 = (x - 1)^2$$
$$x + 3 = (x - 1)^2$$

2. Expand the right side:
$$x + 3 = x^2 - 2x + 1$$

3. Bring all terms to one side:
$$0 = x^2 - 2x + 1 - x - 3$$
$$0 = x^2 - 3x - 2$$

4. Solve quadratic:
$$x^2 - 3x - 2 = 0$$

Using quadratic formula:
$$x = \frac{3 \pm \sqrt{(-3)^2 - 4 \times 1 \times (-2)}}{2}$$
$$x = \frac{3 \pm \sqrt{9 + 8}}{2}$$
$$x = \frac{3 \pm \sqrt{17}}{2}$$

Potential solutions:
$$ x = \frac{3 + \sqrt{17}}{2} \quad \text{and} \quad x = \frac{3 - \sqrt{17}}{2} $$

5. Check solutions in original equation:

- For $x = \frac{3 + \sqrt{17}}{2}$:

Calculate $\sqrt{x + 3}$:
$$ x + 3 = \frac{3 + \sqrt{17}}{2} + 3 = \frac{3 + \sqrt{17} + 6}{2} = \frac{9 + \sqrt{17}}{2} $$

Since $\sqrt{9 + \sqrt{17}}$ is complicated, approximate:
$$ \sqrt{17} \approx 4.123 $$
$$ x \approx \frac{3 + 4.123}{2} \approx \frac{7.123}{2} \approx 3.5615 $$
$$ x + 3 \approx 3.5615 + 3 = 6.5615 $$
$$ \sqrt{x + 3} \approx \sqrt{6.5615} \approx 2.561 $$

Now check RHS:
$$ x - 1 \approx 3.5615 - 1 = 2.5615 $$

Close match, so this solution is valid.

- For $x = \frac{3 - \sqrt{17}}{2}$:

Similarly, approximate:
\[x \approx \frac

Frequently Asked Questions

Solve the equation 2x + 5 = 15. Show all algebraic steps and express the answer as an exact solution.

Step 1: Subtract 5 from both sides: $2x + 5 - 5 = 15 - 5$, which simplifies to $2x = 10$.
Step 2: Divide both sides by 2: $2x / 2 = 10 / 2$, giving $x = 5$.
Answer: $x = 5$.

Solve the quadratic equation x^2 - 4x - 5 = 0. Show all steps and provide exact solutions.

Step 1: Identify coefficients: $a=1$, $b=-4$, $c=-5$.
Step 2: Use the quadratic formula: $x = [-b \pm \sqrt{b^2 - 4ac}] / 2a$.
Step 3: Calculate discriminant: $\Delta = (-4)^2 - 4(1)(-5) = 16 + 20 = 36$.
Step 4: Find roots: $x = [4 \pm \sqrt{36}] / 2 = [4 \pm 6] / 2$.
Solution 1: $x = (4 + 6)/2 = 10/2 = 5$.
Solution 2: $x = (4 - 6)/2 = (-2)/2 = -1$.
Answer: $x = 5$ or $x = -1$.

Solve the equation $3(x - 2) = 2x + 4$. Show all algebraic steps and express the exact solutions.

Step 1: Expand the left side: $3x - 6 = 2x + 4$.
Step 2: Subtract $2x$ from both sides: $3x - 2x - 6 = 4$, which simplifies to $x - 6 = 4$.
Step 3: Add 6 to both sides: $x = 4 + 6 = 10$.
Answer: $x = 10$.

Solve the equation $|2x - 3| = 7$. Show all steps and provide exact solutions.

Step 1: Set up two cases:
Case 1: $2x - 3 = 7$
Case 2: $2x - 3 = -7$

Step 2: Solve each case:
Case 1: $2x = 7 + 3 = 10 \rightarrow x = 10/2 = 5$.
Case 2: $2x = -7 + 3 = -4 \rightarrow x = -4/2 = -2$.
Answer: $x = 5$ or $x = -2$.

Solve the exponential equation 2^{x+1} = 8. Show all algebraic steps and express the solution exactly.

Step 1: Rewrite 8 as a power of 2: 8 = 2^3.
Step 2: Set exponents equal: 2^{x+1} = 2^3.
Step 3: Since bases are the same, equate exponents: $x + 1 = 3$.
Step 4: Subtract 1 from both sides: $x = 3 - 1 = 2$.
Answer: $x = 2$.

Solve the inequality $3x - 4 > 2x + 1$. Show all algebraic steps and express the solution set.

Step 1: Subtract $2x$ from both sides: $3x - 2x - 4 > 1$, simplifying to $x - 4 > 1$.
Step 2: Add 4 to both sides: $x > 1 + 4 = 5$.
Answer: $x > 5$.

Solve the logarithmic equation log_{3}(x + 2) = 4. Show all steps and give the exact solution.

Step 1: Rewrite the logarithmic equation in exponential form: x + 2 = 3^4.
Step 2: Calculate 3^4: 3^4 = 81.
Step 3: Solve for x: $x + 2 = 81 \rightarrow x = 81 - 2 = 79$.
Answer: $x = 79$.

Additional Resources

Solve the Following Equation: Show All Algebraic Steps. Express Answers As Exact Solutions If Possible

When it comes to mastering algebra, one of the most fundamental skills is solving equations systematically and accurately. Whether you're a student brushing up on core concepts or an enthusiast looking to refine your problem-solving toolkit, understanding the detailed steps involved in solving equations is crucial. In this article, we will explore an example of solving an algebraic equation comprehensively, emphasizing the importance of showing all steps and expressing solutions precisely. Let’s dive into the process with clarity, precision, and expert insights.

Understanding the Goal: What Does It Mean to Solve an Equation?

Before we proceed, it's essential to clarify what "solving an equation" entails. An algebraic equation is a mathematical statement asserting the equality of two expressions, often containing variables. Solving the equation involves finding all the values of the variable(s) that satisfy this equality.

Key Points:
- The solution(s) are the value(s) of the variable(s) that make the equation true.
- Solutions should be expressed as exact as possible, not just decimal approximations, when feasible.
- Showing all algebraic steps ensures clarity, allows verification, and enhances understanding.

Sample Equation for In-Depth Solution

Let's consider the following algebraic equation as our example:

$$
2x^2 - 5x - 7 = 0
$$

Our goal: Find all real solutions for x that satisfy this quadratic equation.

Step-by-Step Solution Process

The process of solving this quadratic involves several methods, primarily:

- Factoring (if possible)
- Completing the square
- Applying the quadratic formula

Given the coefficients, the quadratic formula is most straightforward and universally applicable. We will demonstrate this method, showing all algebraic steps.

Step 1: Identify the coefficients

The standard quadratic form is:

$$
ax^2 + bx + c = 0
$$

For our equation:

$$
2x^2 - 5x - 7 = 0
$$

- $a = 2$
- $b = -5$
- $c = -7$

Step 2: Recall the quadratic formula

The quadratic formula for solving $ax^2 + bx + c = 0$ is:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

This formula computes the roots directly, provided the discriminant $D = b^2 - 4ac$ is non-negative.

Step 3: Calculate the discriminant (D)

Compute:

$$
D = b^2 - 4ac
$$

Plugging in the known coefficients:

$$
D = (-5)^2 - 4 \times 2 \times (-7)
$$

Calculate step-by-step:

1. Square b:

$$
(-5)^2 = 25
$$

2. Calculate $4ac$:

$$
4 \times 2 \times (-7) = 8 \times (-7) = -56
$$

3. Compute D:

$$
D = 25 - (-56) = 25 + 56 = 81
$$

Interpretation: Since the discriminant is positive ($D=81$), there are two real solutions.

Step 4: Write the solutions using the quadratic formula

Now, substitute the coefficients and the discriminant into the quadratic formula:

$$
x = \frac{-b \pm \sqrt{D}}{2a}
$$

$$
x = \frac{-(-5) \pm \sqrt{81}}{2 \times 2}

\]

Simplify numerator:

\[
x = \frac{5 \pm \sqrt{81}}{4}
\]

Calculate the square root:

\[
\sqrt{81} = 9
\]

So the solutions become:

\[
x = \frac{5 \pm 9}{4}
\]

Step 5: Find the two solutions

Solution 1:

\[
x = \frac{5 + 9}{4} = \frac{14}{4} = \frac{7}{2}
\]

Solution 2:

\[
x = \frac{5 - 9}{4} = \frac{-4}{4} = -1
\]

Exact solutions:

\[
\boxed{
x = \frac{7}{2} \quad \text{and} \quad x = -1
}
\]

Summary of the Solution

- The solutions are exact, expressed as fractions.
- Both roots are real and rational.
- The solutions satisfy the original quadratic equation:

$$
2x^2 - 5x - 7 = 0
$$

Additional Insights and Recommendations

Why show all steps?

- Demonstrates understanding of algebraic techniques.
- Facilitates verification.
- Helps identify potential errors at each step.

When to use different methods:

- Factoring: When the quadratic factors easily into binomials.
- Completing the square: Useful for understanding the structure or when quadratic formula isn't preferred.
- Quadratic formula: Most reliable, especially for complex or irrational roots.

Expressing solutions exactly:

- Prefer fractions over decimal approximations for clarity and precision.
- When roots are irrational, express as surds (e.g., $\sqrt{2}$) for exactness.

Additional Example: Solving a Different Quadratic

Suppose we have:

$$
x^2 - 4x + 3 = 0
$$

Step 1: Coefficients: $a=1$, $b=-4$, $c=3$.

Step 2: Discriminant:

\[
D = (-4)^2 - 4 \times 1 \times 3 = 16 - 12 = 4
\]

Step 3: Roots:

\[
x = \frac{-(-4) \pm \sqrt{4}}{2 \times 1} = \frac{4 \pm 2}{2}
\]

Step 4: Solutions:

\[
x = \frac{4 + 2}{2} = 3
\]
\[
x = \frac{4 - 2}{2} = 1
\]

Exact solutions: \(x=1, 3\)

Conclusion: The Power of Systematic Algebraic Solutions

Solving equations thoroughly and precisely is foundational in algebra. By meticulously following each step — from identifying coefficients, calculating the discriminant, applying the quadratic formula, to expressing solutions in their exact form — learners and practitioners ensure accuracy and deepen their understanding. The example explored here exemplifies how algebraic methods can be systematically employed to solve quadratic equations, emphasizing the importance of showing all steps and articulating solutions clearly.

Remember: The key to mastering algebra is practice, attention to detail, and an appreciation for the elegance of exact solutions. Whether tackling simple quadratics or more complex equations, these principles remain your guiding compass.

Happy solving!

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Express Answers As Exact Solutions If Possible

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